Economic applications of Matching Models

Summer School 'Variational problems in physics, economics, and geometry'

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- This presentation: marriage market only (although some hedonic)

A few relevant questions

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    - Several questions; in particular:
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        - How do we compare single-adult households and couples? What about intrahousehold inequality?

A few relevant questions (cont.)

2. College premium and the demand for college education
   Motivation: remarkable increase in female education, labor supply, incomes worldwide during the last decades.

FIGURE 3: FRACTION OF 30- TO 34-YEAR-OLDS WITH COLLEGE EDUCATION, COUNTRIES ABOVE MEDIAN PER CAPITA GDP AND BELOW PER CAPITA GDP, BY SEX

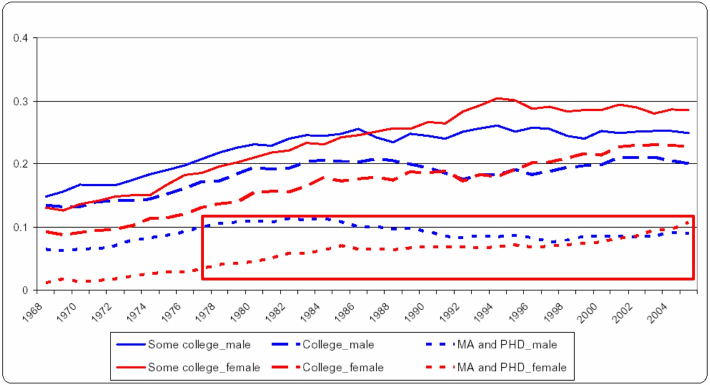
Source: See Figure 1.

Source: Becker-Hubbard-Murphy 2009

College premium and the demand for college education

In the US:

Figure 13: Completed Education by Sex, Age 30-40, US 1968-2005



Source: Current Population Surveys.

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   - How can we model that?
   - Testable predictions?
   - Do they fit the data?

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3. Abortion and female empowerment

- Roe vs. Wade (1973): de facto legalization of abortion in the US
- General claim (feminist literature): important source of 'female empowerment'
- Question: what is the mechanism?
- In particular, what about women:
  - who do want children
  - who would not use abortion (e.g. for religious reasons), etc.

Roadmap

1. *Matching models: general presentation*
2. The case of Transferable Utility (TU)
3. Applications:
    - Intra-household allocation: back-of-the-envelope computations
    - Roe vs Wade and female empowerment
    - Women's demand for highest education
4. Extensions

Matching models: three main families

1. Matching under NTU (Gale-Shapley)
   Idea: no transfer *possible* between matched partners
2. Matching under TU (Becker-Shapley-Shubik)
   - Transfers possible without restrictions
   - Technology: constant 'exchange rate' between utiles
   - In particular: (strong) version of interpersonal comparison of utilities
   - $\rightarrow$ requires restrictions on preferences
3. Matching under Imperfectly TU (ITU)
   - Transfers possible
   - But no restriction on preferences
   - $\rightarrow$ technology involves variable 'exchange rate'

... plus 'general' approaches ('matching with contracts', from Crawford-Knoer and Kelso-Crawford to Milgrom-Hatfield-Kominers and friends)

... and links with: auction theory, general equilibrium.

Similarities and differences

- All aimed at understanding who is matched with whom
- Only the last 2 address how the surplus is divided
- Only the third allows for impact on the group's aggregate behavior

Formal structure: Common components

- Compact, separable metric spaces X, Y ('women, men') with *finite* measures F and G. Note that the spaces may be *multidimensional*
- Spaces X, Y often 'completed' to allow for singles: $\bar{X} = X \cup \{\emptyset\}, \bar{Y} = Y \cup \{\emptyset\}$
- A *matching* defines of a measure h on $X \times Y$ (or $\bar{X} \times \bar{Y}$) such that the marginals of h are F and G
- The matching is *pure* if the support of the measure is included in the graph of some function ϕ
  Translation: matching is *pure* if $y = \phi(x)$ a.e.
  $\rightarrow$ no 'randomization'

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  - TU: life much easier (GQL $\rightarrow$ equivalent to surplus maximization) ... ... but price to pay: couple's (aggregate) behavior does *not* depend on 'powers', therefore on equilibrium conditions

Implications (crucial for empirical implementation)

- NTU: stable matchings solve

$$u(x) = \max_z \{U(x,z) | V(x,z) \geq v(z)\}$$

and

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for some pair of functions u and v.

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- TU: stable matchings solve

$$u(x) = \max_z \{s(x,z) - v(z)\} \text{ and } v(y) = \max_z \{s(z,y) - u(z)\}$$

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- ITU: stable matchings solve

$$u(x) = \max_z \{F(x,z,v(z))\} \text{ and } v(y) = \max_z \{F^{-1}(z,y,u(z))\}$$

for some pair of functions u and v.

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Transferable Utility (TU)

Definition

A group satisfies TU if there exists monotone transformations of individual utilities such that the Pareto frontier is an hyperplane
$u(x) + v(y) = s(x, y)$ *for all values of prices and income.*

Note that:

- TU is a property of a *group* (not an individual)
- TU is an *ordinal* property; it does *not* require linear, quasi-linear of convex preferences
  $\rightarrow$ in particular, can be applied to risk sharing!

Transferable Utility on the Marriage Market

Application to the Marriage Market
$\longrightarrow$ Basic question: when assuming TU, what restrictions on preferences?

- Need a model of household decision
  $\longrightarrow$ here: collective model; indeed
  - assumes efficiency (which matching models do)
  - encompasses unitary, bargaining, 'equilibrium', 'separate spheres',... as particular cases
- Public and private consumptions; utilities $u_i\left(q_i, Q\right)$
- TU if and only if 'Generalized Gorman' (Chiappori, Gugl 2014): conditional indirect utility is affine in (private) expenditures, with identical coefficients
- Then common model: x, y incomes and $s\left(x, y\right) = H\left(x + y\right)$

Basic result

- If a matching is stable, the corresponding measure satisfies the *surplus maximization problem*, which is an *optimal transportation problem* (Monge-Kantorovitch):
  Find a measure h on $X \times Y$ such that the marginals of h are F and G, and h solves
  $$\max_h \int_{X \times Y} s(x, y) \, dh(x, y)$$
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- Dual problem: dual functions $u(x), v(y)$ and solve
  $$\min_{u,v} \int_X u(x) \, dF(x) + \int_Y v(y) \, dG(y)$$
  under the constraint
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  under the constraint
  $$u(x) + v(y) \geq s(x, y) \quad \text{for all } (x, y) \in X \times Y$$
- In particular, *the dual variables u and v describe an intrapair allocation compatible with a stable matching*

Links with hedonic models

- Hedonic models: defined by set of buyers X, sellers Y, products Z
- Buyers: utility $u(x,z) - P(z)$ which is maximized over z
- Sellers: profit $P(z) - c(y,z)$ which is maximized over z
- Equilibrium: $P(z)$ such that markets clear ($\rightarrow$ measure over $X \times Y \times Z$)
- Canonical correspondence between QL hedonic models and matching models under TU (Chiappori, McCann, Nesheim 2010). Specifically, consider a hedonic model and define surplus:

$$s(x,y) = \max_{z \in Z} (U(x,z) - c(y,z))$$

Let η be the marginal of α over $X \times Y$, $u(x)$ and $v(y)$ by

$$u(x) = \max_{z \in K} U(x,z) - P(z) \text{ and } v(y) = \max_{z \in K} P(z) - c(y,z)$$

Then (η, u, v) defines a stable matching. Conversely, to each stable matching corresponds an equilibrium hedonic price schedule.

Proof

Start from:

$$u(x) + v(y) \geq s(x, y) \geq U(x, z) - c(y, z) \quad \text{on } X \times Y \times Z,$$

hence

$$c(y, z) + v(y) \geq U(x, z) - u(x) \quad \text{on } X \times Y \times Z$$

and

$$\inf_{y \in Y} \{c(y, z) + v(y)\} \geq \sup_{x \in X} \{U(x, z) - u(x)\} \quad \text{on } Z.$$

Take any $P(z)$ such that

$$\inf_{y \in Y} \{c(y, z) + v(y)\} \geq P(z) \geq \sup_{x \in X} \{u(x, z) - u(x)\} \quad \text{on } Z.$$

Supermodularity and assortative matching

One-dimensional:

- s is supermodular if whenever $x \geq x'$ and $y \geq y'$ then

$$s(x,y) + s(x',y') \geq s(x,y') + s(x',y)$$

- Then stable matching is *assortative;* indeed, surplus maximization
- Interpretation: *single crossing* (Spence - Mirrlees). Assume that s is C^1 then

$$s(x,y) - s(x',y) \geq s(x,y') - s(x',y')$$

and $\partial s/\partial x$ increasing in y; if s is C^2 then

$$\frac{\partial^2 s}{\partial x \partial y} \geq 0$$

- Of course, similar results with submodularity ($\partial s/\partial x$ decreasing in y)
- In both case, $\partial s/\partial x$ monotonic in y; if strict then *injective*

Supermodularity and assortative matching

- Problem: both super- (or sub-) modularity and assortative matching are typically one-dimensional
- Generalization (CMcCN ET 2010):

Definition

A surplus function $s : X \times Y \longrightarrow [0, \infty[$ is said to be $X-$*twisted* if there is a set $X_L \subset X_0$ of zero volume such that $\partial^x s(x_0, y_1)$ is disjoint from $\partial^x s(x_0, y_2)$ for all $x_0 \in X_0 \setminus X_L$ and $y_1 \neq y_2$ in Y.

- Then the stable matching is unique and *pure*

Definition

The matching is pure if the measure μ is born by the graph of a function: for almost all x there exists exactly one y such that x matched with y.

$\longrightarrow$ excludes 'mixed strategies'

Roadmap

1. Matching models: general presentation
2. The case of Transferable Utility (TU)
3. *Applications:*
   - *Intra-household allocation: back-of-the-envelope computations*
   - Roe vs Wade and female empowerment
   - Women's demand for highest education
4. Extensions

Intra-household allocation

Simple framework:

- One-dimensional heterogeneity (income, actual or potential)
- Surplus: convex function of total income $\rightarrow s(x, y) = H(x+y)$
  Note that supermodular $\rightarrow$ assortative matching: if F and G respective CDFs,

$$\begin{aligned} 1 - F(x) &= 1 - G(y) \Rightarrow x = \phi(y) = F^{-1}[G(y)] \\ &\Rightarrow y = \psi(x) = G^{-1}[F(x)] \end{aligned}$$

- Income distributions: 'linear shift': $F(t) = G(\alpha t - \beta)$ for some $\alpha < 1, \beta > 0$
  In particular, ϕ and ψ affine:

$$\psi(x) = \alpha x - \beta, \quad \phi(y) = \frac{y + \beta}{\alpha}$$

- Works pretty well in practice, even with $\beta = 0$

Intra-household allocation

Then:

- Stability:

$$u(x) = \max_y \left(s(x,y) - v(y)\right)$$

therefore

$$\begin{aligned} u'(x) &= \frac{\partial s}{\partial x}(x, \psi(x)) = H'(x + \psi(x)) \text{ and } v'(y) = H'(y + \phi(y) \\ &\Rightarrow u(x) = K' + \frac{1}{1+\alpha} H(x + \psi(x)), \\ v(y) &= K + \frac{\alpha}{1+\alpha} H(\phi(y) + y) \end{aligned}$$

- Pinning down K and K':
  - the sum is known (from the surplus function)
  - if more women than men, the last married woman is indifferent between marriage and singlehood

Intra-household allocation

Consider an upward shift in female income: y becomes ky with $k > 1$. Then:

- same matching patterns,
- but changes in the redistribution of surplus:

$$\frac{\partial v_k}{\partial k} = \frac{\alpha y}{\alpha + 1} H'(y + x) + \frac{\alpha}{(\alpha + 1)^2} H(y + x) \quad \text{and}$$

$$\frac{\partial u_k}{\partial k} = \frac{y}{\alpha + 1} H'(y + x) - \frac{\alpha}{(\alpha + 1)^2} H(y + x)$$

- Note the 2 components: increased total surplus and redistribution!

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Abortion and female empowerment

Background

- 73: Roe vs Wade

Abortion and female empowerment

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    where u distributed over $[0, U]$ $\rightarrow$ single women have a child if
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  - Couples: may have a child; unwanted children possible, proba. p

Abortion and female empowerment

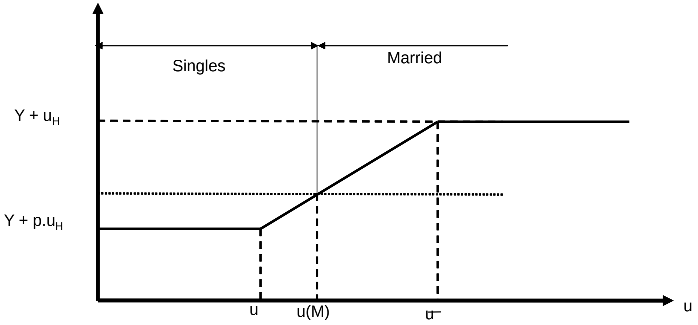
- Couples: benefit of a child $u_H + u$, cost $y - y' \rightarrow$ married couple plans to have a child if

$$u \geq y - y' - u_H = \underline{u}$$

- Therefore:
  - women of 'high' type ($u \geq \bar{u}$) always choose to have a child
  - women of 'intermediate' type ($\underline{u} < u < \bar{u}$) choose to have a child only when married, and need compensation $y - y' - u$
  - women of 'low' type ($u \leq \underline{u}$) never choose to have a child (may have unwanted child)

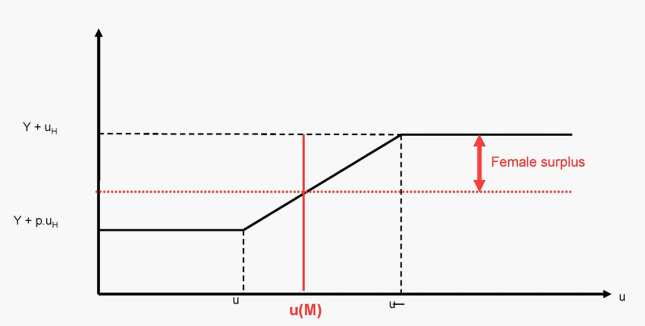
Abortion and female empowerment

Matching: Maximum husband's utility as a function of the wife's taste
Assumption: more women than men



Three possible regimes

1. Males very scarce → no surplus for women
2. Males scarce → marginal woman intermediate, determines surplus
3. Males abundant → maximum female surplus



Impact of birth control

- Definition: changes the probability of unwanted pregnancies

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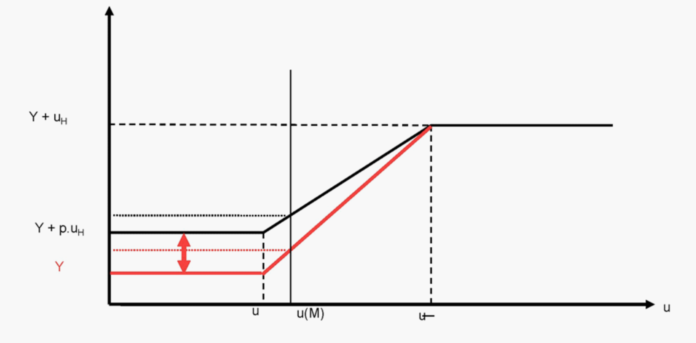
- ... but changes in allocation of surplus for *all* couples

Impact of birth control

Graph:



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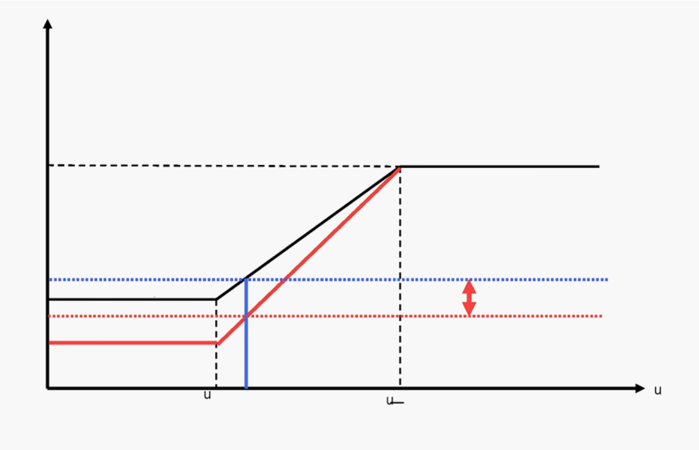

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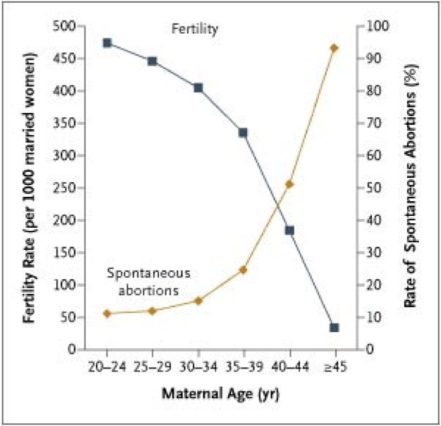
Reproductive capital and women's demand for higher education

Source: Corinne Low's dissertation (2014)

- Basic remark: sharp decline in female fertility between 35 and 45

Rates of Infertility and Miscarriage Increasing Sharply with Age



Source: Heffner 2004, "Advanced Maternal Age: How old is too old?"

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Spousal Income vs Age at Marriage (1955-1966 birth cohort, 2010 ACS)

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  - Con: delayed entry → loss of 'reproductive capital'
- Impact on marital prospects?

Model

- Two commodities, private consumption and child expenditures; utility:

$$u_i = c_i \left(Q + 1\right), \ \ i = h, w$$

and budget constraint (y_i denotes i's income)

$$c_h + c_w + Q = y_h + y_w$$

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and budget constraint (y_i denotes i's income)

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- Transferable utility: any efficient allocation maximizes $u_h + u_w$; therefore surplus with a child

$$s\left(y_h, y_w\right) = \frac{\left(y_h + y_w + 1\right)^2}{4}$$

and without a child ($Q = 0$)

$$s\left(y_h, y_w\right) = y_h + y_w$$

therefore, if π probability of a child:

$$s\left(y_h, y_w\right) = \pi \frac{\left(y_h + y_w + 1\right)^2}{4} + \left(1 - \pi\right)\left(y_h + y_w\right)$$

Populations

- Men: differ in income $\longrightarrow y_h$ uniform on $[1, Y]$

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  - how is the surplus distributed?
  - what is the impact on (ex ante) investment?

Resolution

- Two stage: invest in stage 1, match in stage 2

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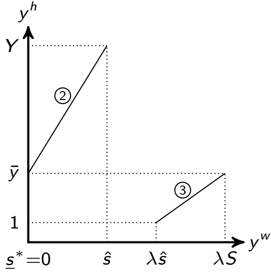
  - Regime 1: negative assortative matching (can be discarded)

1. Negative assortative



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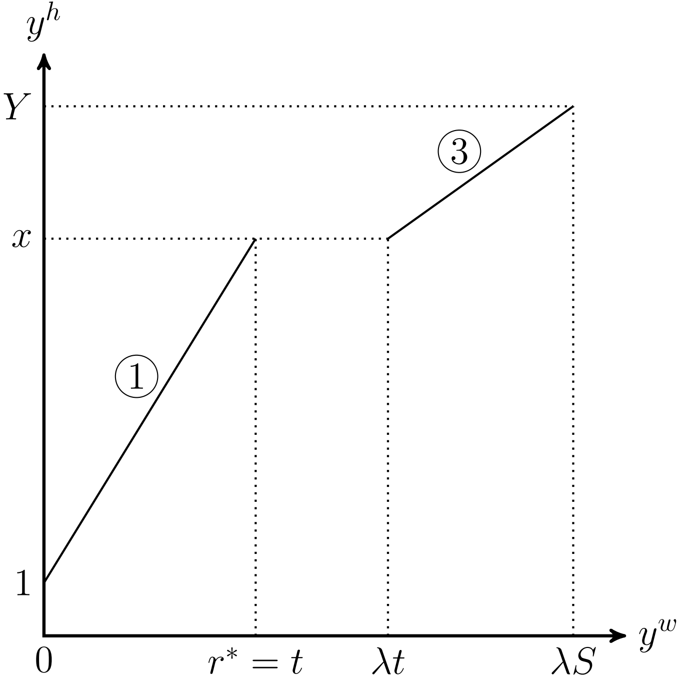

y^h
Y
x
1
0
$r^* = t$
λt
λS
y^w
①
③

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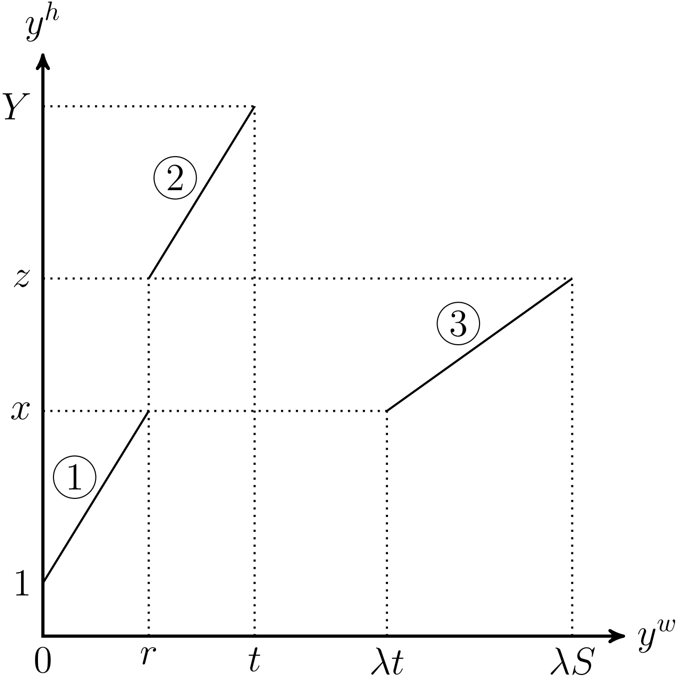
  - Regime 3: intermediate


y^h
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1
0
r
t
λt
λS
y^w
1
2
3

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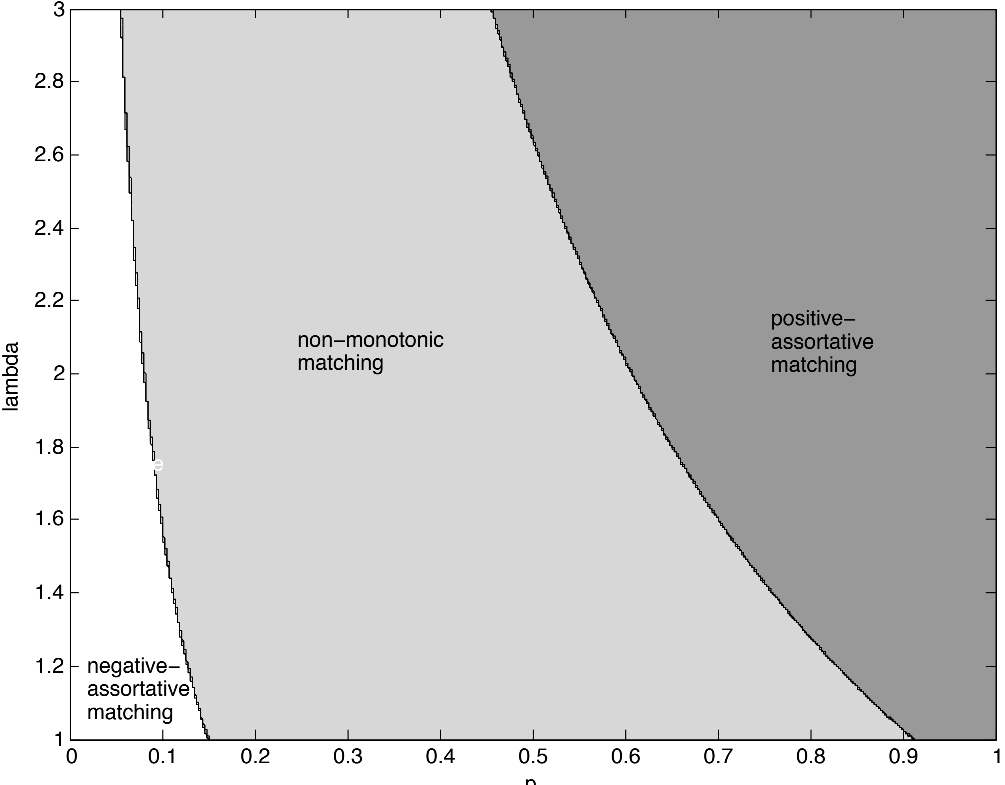
Stage 1: investment choice

→ Graph



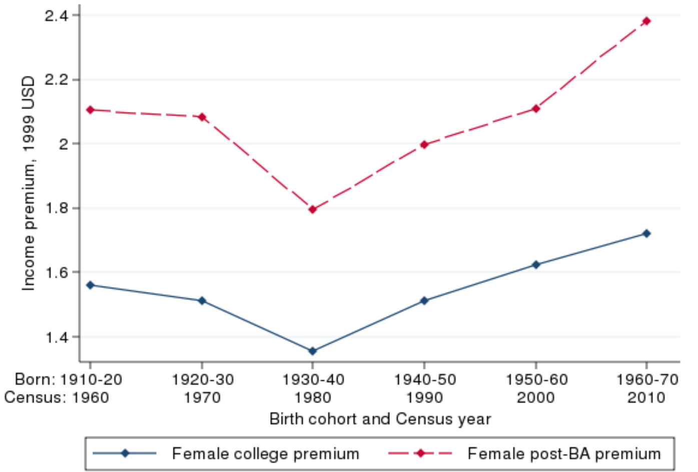
lambda
p
negative–
assortative
matching
non–monotonic
matching
positive–
assortative
matching

Empirical predictions

Basic intuition: we have moved from 'λ small, P/p large' to 'λ large, P/p not too large'
Why?

- Increase in λ: dramatic increase in 'college + premium'

Wage income premium over women with some college



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- Decrease in P/p: two factors

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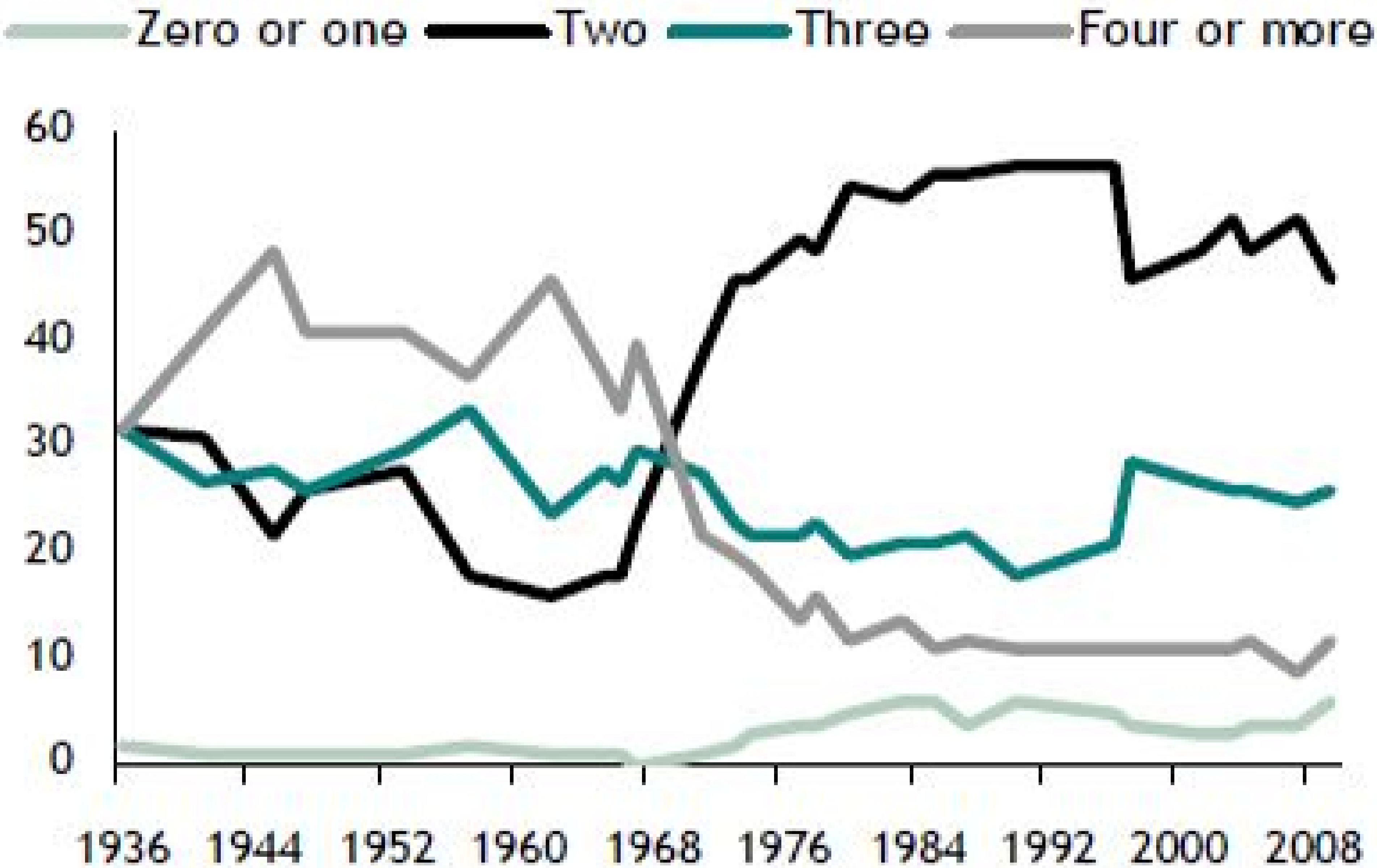
(%)



Notes: "Don't know/refused" responses not shown. Respondents were asked: "What is the ideal number of children for a family to have?"

Source: Gallup, 1936-2007; Pew Research Center, 2009

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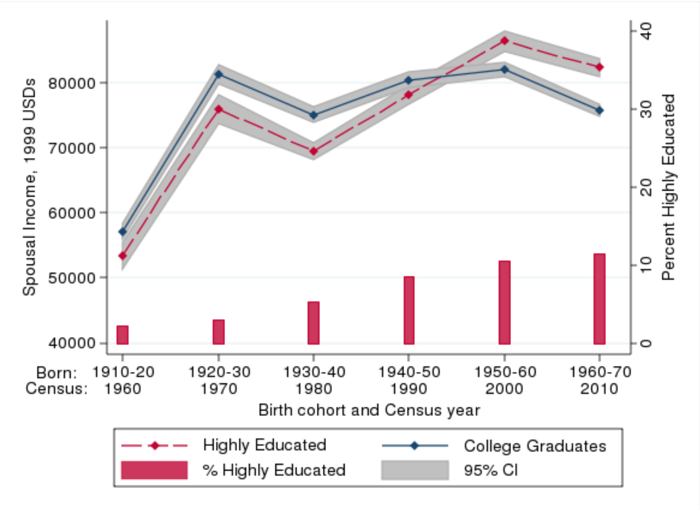
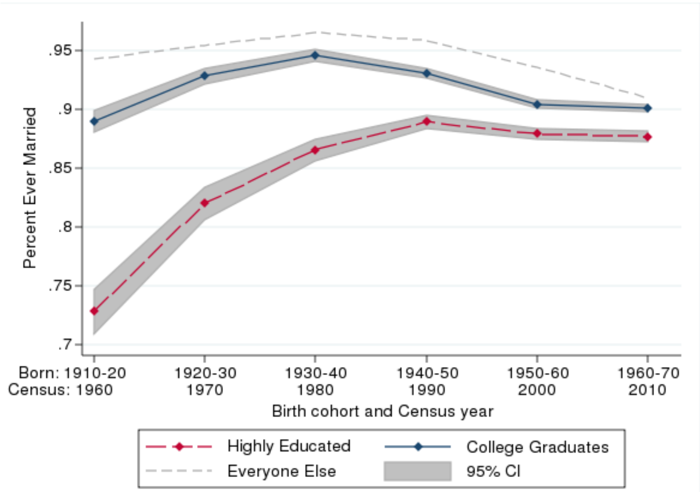
- What about data?

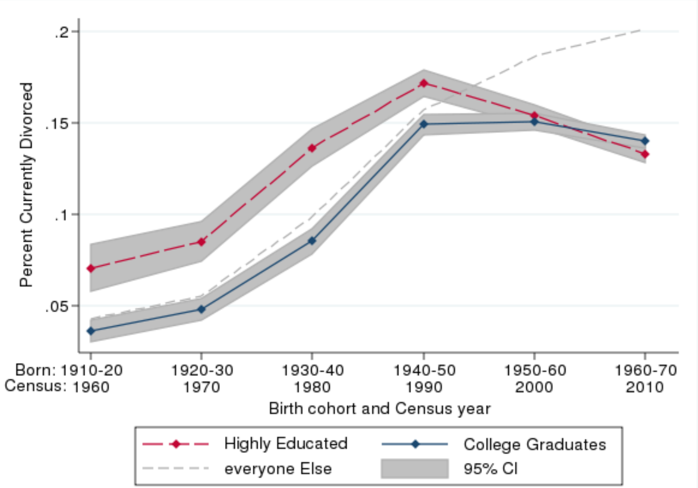
Spousal income by wife's education level, white women 41-50



Marriage rates by education level, white women 41-50



Currently divorced rates by education level, white women 41-50



Generalization: the 'true' bidimensional model

Source: Chiappori, McCann, Pass (in progress)

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$$u(x_1, x_2) = \max_y s(x_1, x_2, y) - v(y)$$

Assume purity, then $y = f(x_1, x_2)$ and envelope theorem:

$$\begin{aligned} \frac{\partial u}{\partial x_1} &= \frac{\partial s}{\partial x_1}(x_1, x_2, f(x_1, x_2)) \\ \frac{\partial u}{\partial x_2} &= \frac{\partial s}{\partial x_2}(x_1, x_2, f(x_1, x_2)) \end{aligned}$$

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- CDR give the pdf in f

$$\frac{\partial^2 s}{\partial x_1 \partial y} \frac{\partial f}{\partial x_2} = \frac{\partial^2 s}{\partial x_2 \partial y} \frac{\partial f}{\partial x_1}$$

Generalization: the ‘true’ bidimensional model

Actually, if ϕ defined by

$$f\left(x_1, x_2\right) = y \longrightarrow x_2 = \phi\left(x_1, y\right)$$

then DE in ϕ:

$$\frac{\partial \phi}{\partial x_1} = \frac{\frac{\partial^2 s\left(x_1, \phi\left(x_1, y\right), y\right)}{\partial x_1 \partial y}}{\frac{\partial^2 s\left(x_1, \phi\left(x_1, y\right), y\right)}{\partial x_2 \partial y}}$$

In our case:

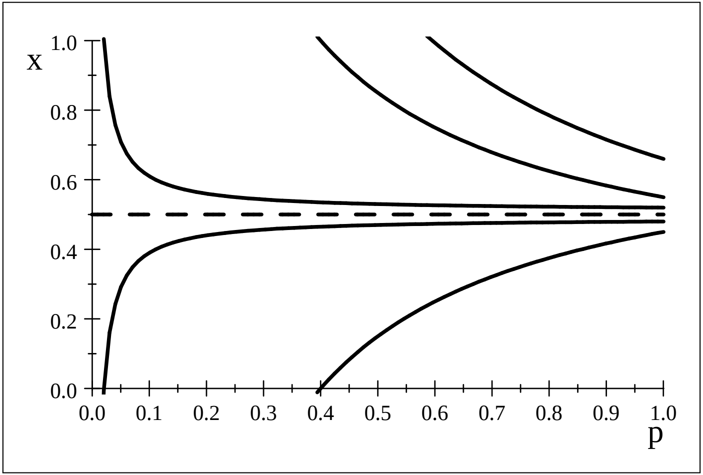
$$\frac{\partial \phi}{\partial p} = -\frac{1}{p}\left(\phi\left(p, y\right) + y - 1\right)$$

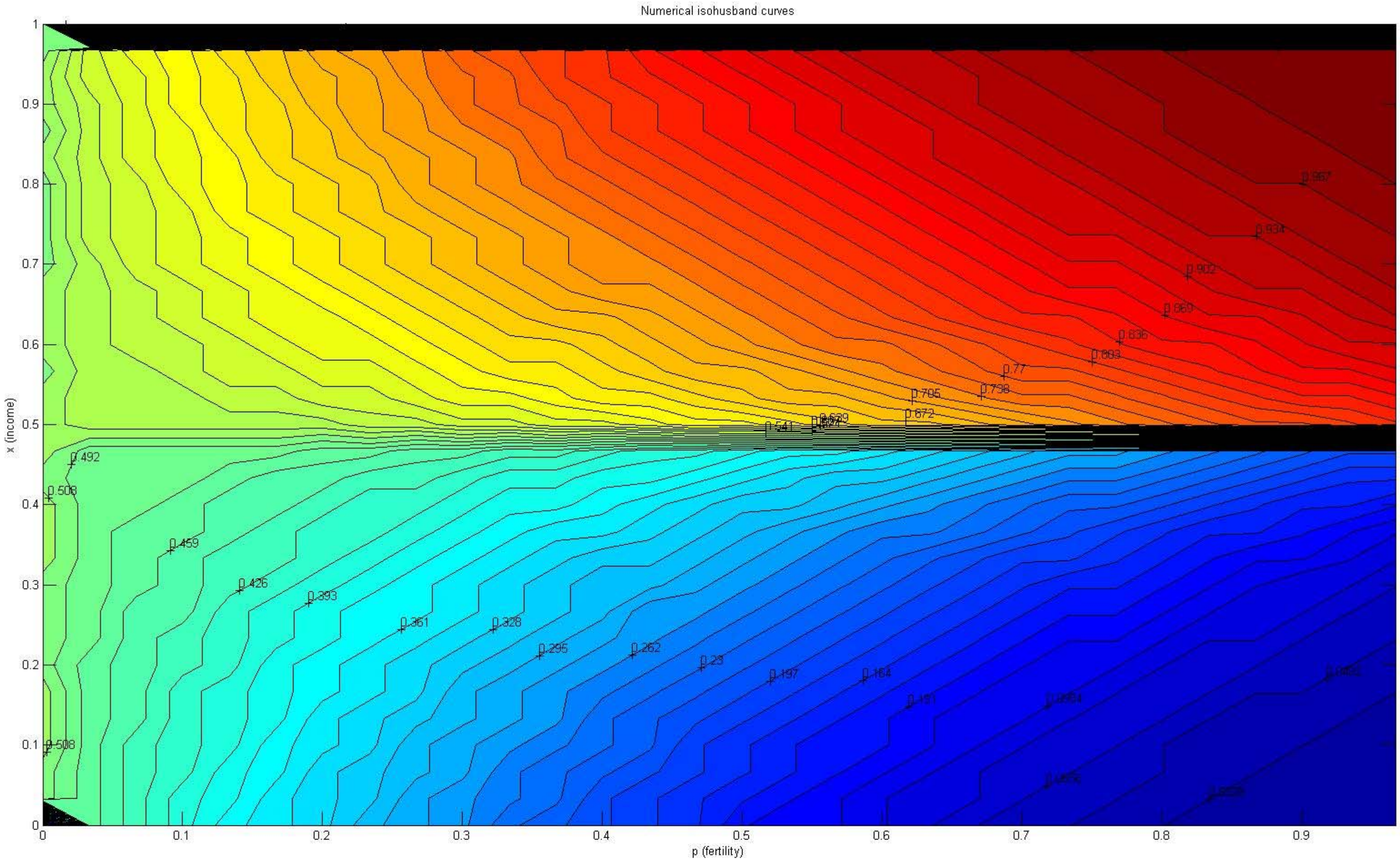
gives

$$\phi\left(p, y\right) = 1 - y + \frac{K\left(y\right)}{p}$$

and $K\left(y\right)$ pinned down by the measure conditions

The uniform case: iso-husband curves





Numerical isohusband curves
x (income)
p (fertility)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0.967
0.934
0.902
0.836
0.77
0.738
0.705
0.672
0.492
0.508
0.459
0.426
0.393
0.361
0.328
0.295
0.262
0.23
0.197
0.164

A stochastic version

Finally, how can we capture traits that are unobservable (to the econometrician)?
$\longrightarrow$ Usual idea: unobserved heterogeneity represented by a random component (say, in the surplus function)
$\longrightarrow$ A simple framework:

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  - both:

$$\varepsilon_{i,j} = \alpha_i^J + \beta_j^I + \eta_{ij}$$

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  - Estimation by logits; then one can compute

$$G(I) = E\left[\max_J U^{\bar{I},J} + \alpha_i^J \mid i \in I\right]$$

and $G(I) - G(I')$ is the marital premium from getting I instead of I'

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- Results

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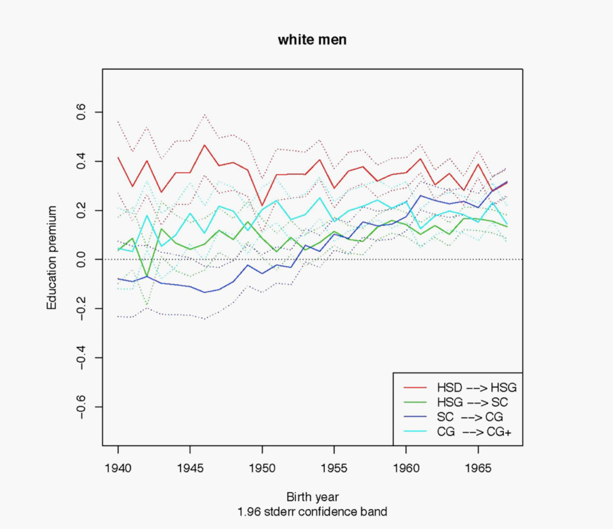

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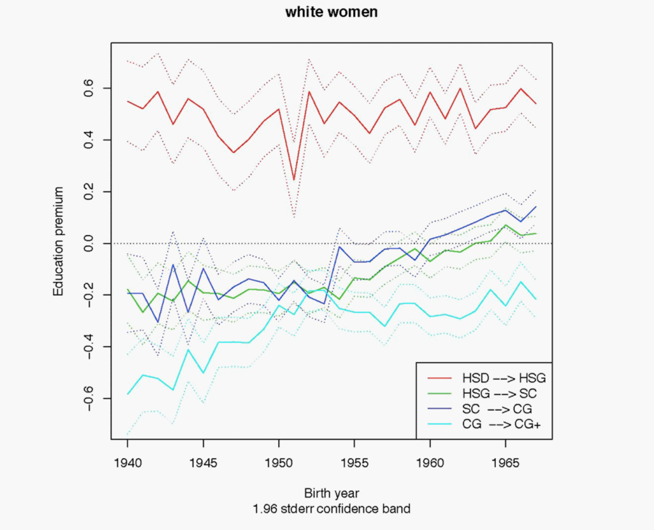

Conclusion

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3. Applied theory: many applications (abortion, female education, divorce laws, children, ...)
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