Macroeconomic instability and microeconomics financial fragility: a stock-flow consistent approach with heterogeneous agents[1]

Laura de Carvalho and Corrado Di Guilmi

Mathematics for New Economic Thinking
Toronto, October 31 2013

[1] Financial support by INET is gratefully acknowledged

- Features:
  - This paper introduces heterogeneous microeconomic behavior into a demand-driven stock-flow consistent (SFC) model.
  - Analytical aggregation of heterogeneous agents by means of statistical mechanics tools (Aoki 2006, Di Guilmi 2008, Foley JET 1994, Landini and Uberti CE 2008).
- Main objective: study the link between the **financial micro-variables** and the macroeconomic fluctuations (Minsky).

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Stock-flow consistent modeling: basic steps

"*I have found out what economics is; it is the science of confusing stocks with flows*" (Kalecki)

- Set up sectors' balance sheets (stocks);
- model the behavioral equations and the social accounting matrix (flows);
- build a dynamical system from the behavioral equations and identify the steady state;
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A social accounting matrix

Table 2.6 Transactions flow matrix

	Households (1)	Production firms: Current (2)	Production firms: Capital (3)	Banks: Current (4)	Banks: Capital (5)	Government (6)	Central Bank: Current (7)	Central Bank: Capital (8)	Σ
Consumption	$-C$	$+C$							0
Investment	$-I_h$	$+I$	$-I_f$						0
Govt. exp.		$+G$				$-G$			0
Wages	$+WB$	$-WB$							0
Profits, firms	$+FD_f$	$-F_f$	$+FU_f$						0
Profits, banks	$+FD_b$			$-F_b$	$+FU_b$				0
Profit, central Bk						$+F_{cb}$	$-F_{cb}$		0
Loan interests	$-r_{l(-1)} \cdot L_{h(-1)}$	$-r_{l(-1)} \cdot L_{f(-1)}$		$+r_{l(-1)} \cdot L_{(-1)}$					0
Deposit interests	$+r_{m(-1)} \cdot M_{h(-1)}$			$-r_{m(-1)} \cdot M_{(-1)}$					0
Bill interests	$+r_{b(-1)} \cdot B_{h(-1)}$			$+r_{b(-1)} \cdot B_{b(-1)}$		$-r_{b(-1)} \cdot B_{(-1)}$	$+r_{b(-1)} \cdot B_{cb(-1)}$		0
Taxes – transfers	$-T_h$	$-T_f$		$-T_b$		$+T$			0
Change in loans	$+\Delta L_h$		$+\Delta L_f$		$-\Delta L$				0
Change in cash	$-\Delta H_h$				$-\Delta H_b$			$+\Delta H$	0
Change, deposits	$-\Delta M_h$				$+\Delta M$				0
Change in bills	$-\Delta B_h$				$-\Delta B_h$	$+\Delta B$		$-\Delta B_{cb}$	0
Change, equities	$-(\Delta e_f \cdot p_{ef} + \Delta e_b \cdot p_{eb})$		$+\Delta e_f \cdot p_{ef}$		$+\Delta e_b \cdot p_{eb}$				0
Σ	0	0	0	0	0	0	0	0	0

Figure: Source: Godley and Lavoie, Monetary Economics, Palgrave McMillian, 2007

- Traditional *aggregative* SFC models: no role for **heterogeneity** and **interaction** among agents.
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Microfoundation methods I: agent based modeling

- Kinsella et al. (EEJ2011), Seppecher (MD2012): analysis of financial instability due to intra-sector flows;
- Bezemer (wp2011): heterogeneous *balance sheets*;
- drawback: agent-based models can only be solved numerically:
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Microfoundation methods II: statistical mechanics

- **Meso-foundation**: "*The point is that precise behavior of each agent is irrelevant. Rather we need to recognize that microeconomic behavior is fundamentally stochastic.*" (Aoki and Yoshikawa 2006);
- how it works:
  - **reduction in the heterogeneity** by grouping the agents in clusters;
  - identifying the mean-field variables (**mean-field approximation**);
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Steps of the analysis

1. Set up and numerically simulate the ABM with N heterogeneous firms belonging to two different categories (borrowing and hedge);
2. identifying the *mean-field* variables within each group (*mean-field approximation*) by taking the average of firms' variables;
3. set up and solve the master equation (ME) to obtain the dynamics of the number of firms in each group;
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- Two types of firms
  - **borrowing firms** ($\mathbf{z = 1}$): cannot finance all investment with internal finance and issue equities and bonds;
  - **hedge firms** ($\mathbf{z = 2}$): finance all investment with retained profits.
- investment function:

$$i_z^j(t) = \alpha\, h(t) + \beta_z\, a^j(t) + \epsilon\, u^j(t) \tag{1}$$

i: investment; a: retained profits; u: capacity utilization ratio; $h(t) = Pe(t)E(t)/P\,K(t)$, Pe: stock price; K: aggregate capital; $\alpha, \beta_z, \epsilon > 0$.

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- Total demand: consumption from salaries, profits and capital gains + investment;
- demand allocated among firms according to their size plus a uniformly distributed and idiosyncratic *preferential attachment* shock s with $\mathbb{E}[s] = 0$;
- price: constant mark-up μ on the cost of labor; accordingly, output shares of labor (Ψ) and profit (Π) are given exogenously;
- A firm fails if $a^j / K^j \leq c$, where c is a constant.

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Stochastic evolution of the two types of firms

1. Quantify the mean-field variables (reduction of heterogeneity);
2. identify the transition probabilities η and ζ as functions of the shock s and the firms' balance sheets variables;
3. define the ME:

   $\frac{dP(N_1,t)}{dt}$ =(probability of flow of firms into the borrowing state) - (probability of flow of firms out of the borrowing state)

4. the solution of the ME yields the dynamics of n_1:

$$\dot{n}_1(t) = \eta m(t) - (\eta + \zeta)[m(t)]^2 + \sigma \, dV(t) \qquad (3)$$

m:trend; dV is a stationary Wiener increment and $\sigma = \frac{\eta\zeta}{(\eta+\zeta)^2}$.

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- Solution of the ME: dynamics of N_1;
- total investment:

$$\dot{K}(t) = I(t) = N_1 i_1 + N_2 i_2 = N\alpha h(t) + N_1[\beta_1 a_1(t) + \epsilon u_1(t)] + N_2[\beta_2 a_2(t) + \epsilon u_2(t)] \quad (4)$$

- dynamics of aggregate debt:

$$\dot{B}(t) = N_1 \{\varpi [i_1(t) - a(t) - m_1(t)]\} \quad (5)$$

- total number of shares:

$$\dot{E}(t) = N_1 \left\{(1 - \varpi)\left[i_1^j(t) - a^j(t) - m_1^j(t)\right] / Pe(t)\right\} \quad (6)$$

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	Households	Firms: Current	Firms: Capital	Banks	Total
Consumption	$-C$	$+C$			0
Investment		$+I$	$-[N\alpha h + N_1(\beta_1 a_1 + \epsilon u_1) + N_2(\beta_2 a_2 + \epsilon u_2)]$		0
Wages	$+\Psi pQ$	$-\Psi pQ$			0
Profits	$+\Theta(\Pi pQ - rB)$	$-(N_1 a_1 + N_2 a_2)$	$(1-\Theta)(\Pi pQ - rB) + r(N_1 m_1 + N_2 m_2)$		0
Loan interests		$-rB$		rB	0
Deposit interests	$r(M_\Psi + M_\Theta)$	$+r(N_1 m_1 + N_2 m_2)$		$-r(M_\Psi + M_\Theta + N_1 m_1 + N_2 m_2)$	0
Change in loans			$+N_1[\bar{\omega}(i_1 - a - m_1)]$	$-\dot{B}$	0
Change in deposits	$-(\dot{M}_\Psi + \dot{M}_\Theta)$		$-(N_1\dot{m}_1 + N_2\dot{m}_2)$	$\dot{M}$	0
Change in equities	$-Pe\dot{E}$		$+N_1[(1-\bar{\omega})(i_1 - a - m_1)]$		0
Total	0	0	0	0	0
Capital Gains	$-\dot{P}eE$		$+\dot{P}eE$		0

Figure: Matrix of flows

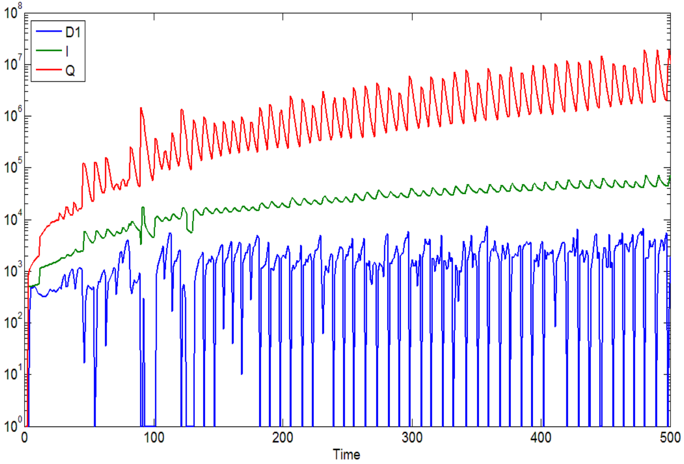


Figure: Dynamics of aggregate demand, investment and debt.

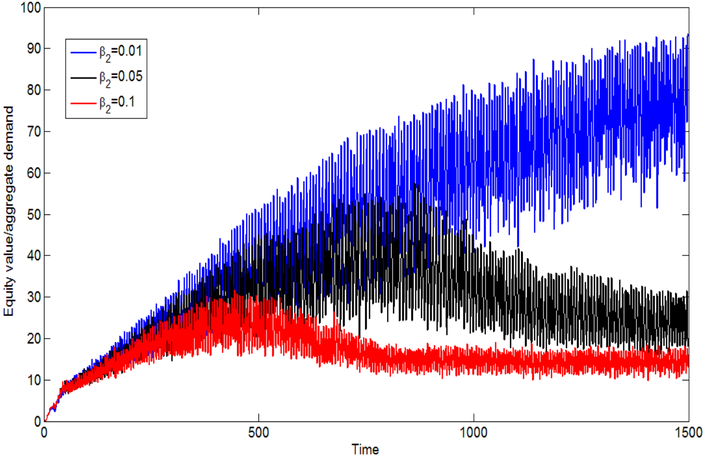


Figure: Dynamics of equity value to aggregate demand ratio for different values of β_2 ($\beta_1 = 0.05$).

Dynamics of leverage

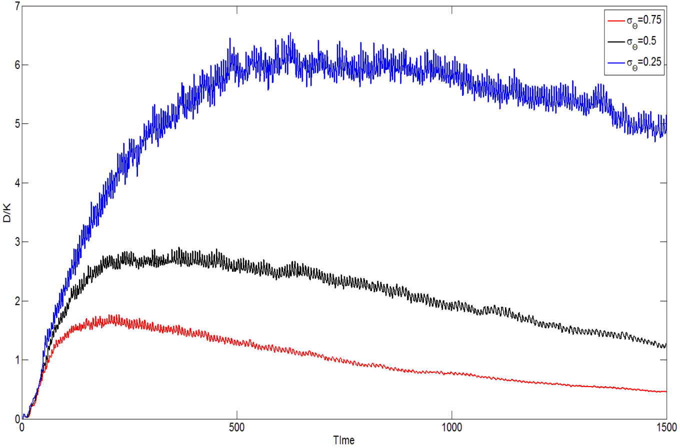
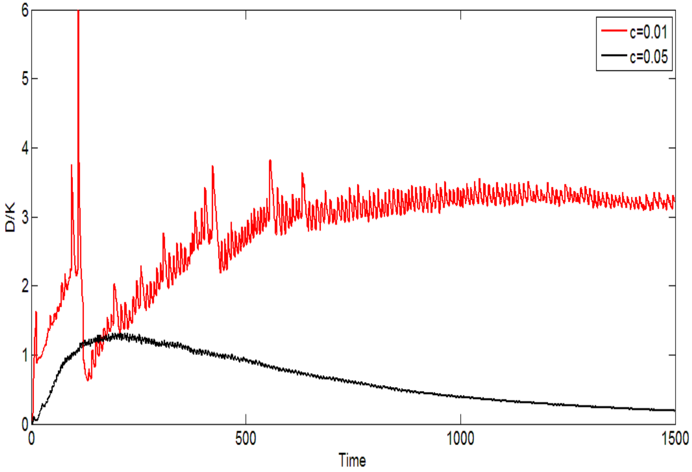


Figure: Dynamics of the debt/capital ratio for different values of σ_Θ.



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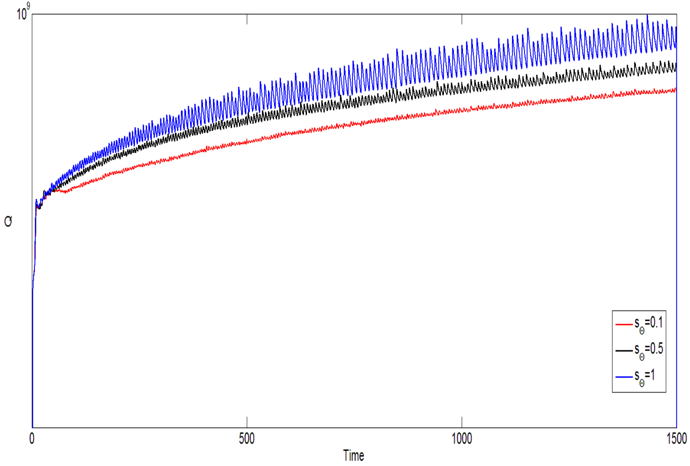

Figure: Dynamics of aggregate demand for different values of s_{Θ}.

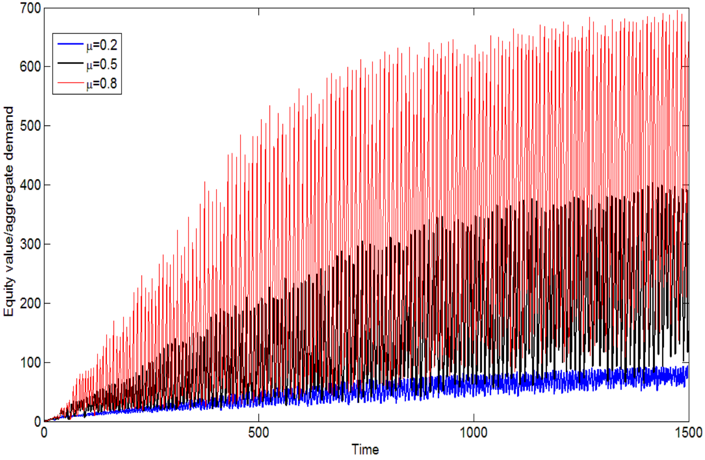

Figure: Dynamics of equity value to aggregate demand ratio for different values of μ.

- **Heterogeneity** in firms' behavior influences the dependence of the real sector to the financial sector (financialization and leverage ratio).
- the progressively larger weight of the **financial sector** can avoid the paradox of thrift and affects growth, amplitude of fluctuations and distribution of income.
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Future research

- A more refined study of the conditions under which bubbles and busts are generated in the present setting;
- introduction of a variable mark-up and the possibility for households to shift between the two categories of profit-earners and income-earners;
- a more sophisticated analysis of the analytical solution of the model, to be achieved by the study of the stability properties of the dynamical system.

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Thank you!

- All firms adopt the same Leontief-type technology with constant coefficients. As a consequence, the demand for labor at full capacity can be residually quantified once the stock of capital is determined by investment decisions in the previous periods. The supply of labor is infinitely elastic.

$$\bar{q}^j(t) = 1/\gamma \; k^j(t) \tag{7}$$

$\gamma > 0$;

- Total output Q is divided between aggregate consumption C and investment I:

$$pQ(t) = (1-s_\Psi)Y_\Psi(t) + (1-s_\Theta)Y_\Theta(t) + (1-\sigma_\Theta)G_\Theta(t) + I(t)$$

- Accordingly

$$pQ(t) = \frac{1+\mu}{s_\Psi - \mu[1-\Theta(1-s_\Theta)]}[I(t) + A(t)] \quad (8)$$

where

$$A(t) = r[(1-s_\Psi)M_\Psi(t) + (1-s_\Theta)(M_\Theta(t) - \Theta B(t))] + (1-\sigma_\Theta)G_\Theta(t)$$

- Any excess of retained profits over investment will be held by the firm in the form of money m

$$\dot{m}^j(t) = a^j(t) - i^j(t) \quad (9)$$

- where savings S are defined as the difference between households' disposable income and consumption levels $S(t) = Y(t) - C(t)$, and $G(t') = [Pe(t') - Pe(t)]E(t)$.
- The demand for money is residually determined as

$$M_h(t) = W(t) - Pe(t)E(t) \quad (10)$$

- Given that only profit earners demand for share, we have that

$$\dot{M}_\Psi(t) = Y_\Psi(t) - C_\Psi(t) \quad (11)$$

and, accordingly

$$M_\Theta(t) = M_h(t) - M_\Psi(t) \quad (12)$$

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- Probability for a firm of transitioning from one state to another:

$$\eta^j(t) = Pr[s(t) \geq \Gamma_1], \tag{13}$$

$$\zeta^j(t) = Pr[s(t) < \Gamma_2]. \tag{14}$$

where

$$\Gamma_z = \frac{i_z(t) - m_z(t) - [\Pi p q_z(t) - r(b_z(t) - m_z(t)]}{(1-\Theta)\Pi p q_z(t)} \frac{K(t)}{K(t) - K_z(t)}. \tag{15}$$

$$\begin{aligned} \frac{dP(N_1,t)}{dt} = \; & \omega_+(t)P(N_1-1)(t) + \omega_-(t)P(N_1+1)(t) \\ & + [\omega_+(t) + \omega_-(t)]\, P(N_1)(t) \end{aligned} \tag{16}$$

- The solution method splits the state variable into 2 components
$$N_1 = Nm + \sqrt{N}s \tag{17}$$
m is the deterministic trend; s is the stochastic noise
- trend dynamics:
$$\frac{dm}{d\tau} = \eta m - (\eta + \zeta) m^2, \tag{18}$$
where $\tau = t/N$.
- stationary distribution of the Fokker-Planck equation:
$$\theta(s) = C \exp\left(-\frac{s^2}{2\sigma^2}\right) \quad : \quad \sigma^2 = \frac{\eta\zeta}{(\eta+\zeta)^2}. \tag{19}$$
- dynamics of n_1:
$$\frac{n_1(t)}{dt} = \eta m - (\eta + \zeta) m^2 + \sigma \, dV(t) \tag{20}$$
dV is a stationary Wiener increment.

Plots

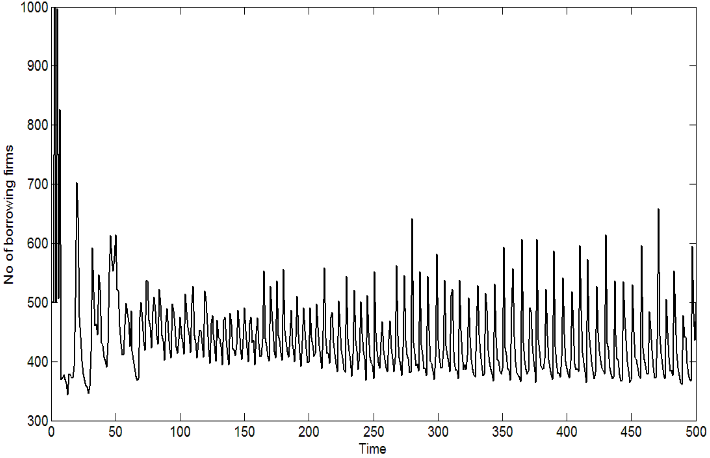

Figure: Dynamics of the number of borrowing firms (total number of firms: 1,000).

Plots

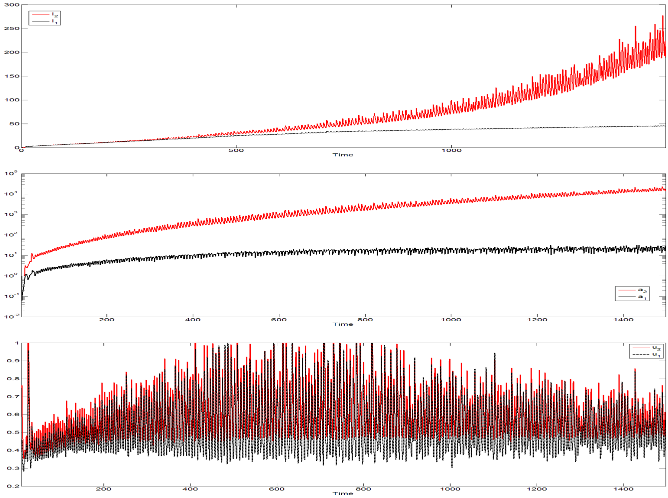

Figure: Investment (top), accumulated profit (center) and capacity utilization (bottom) for average borrowing and hedge firms.

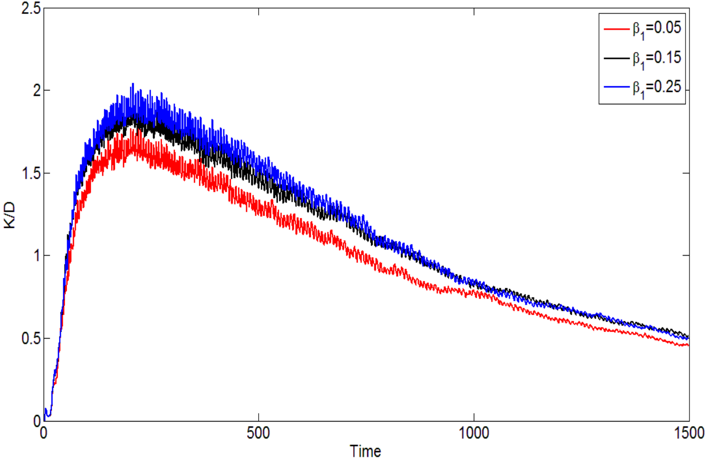

Figure: Dynamics of the debt/capital ratio for different values of β_1.

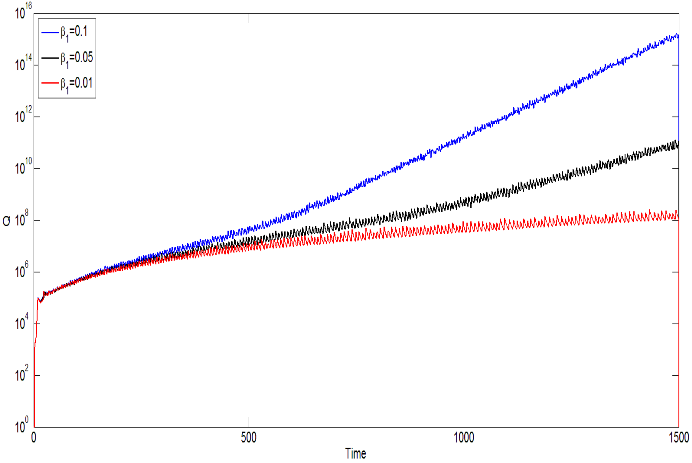

Figure: Dynamics of aggregate demand for different values of β_2.

Plots

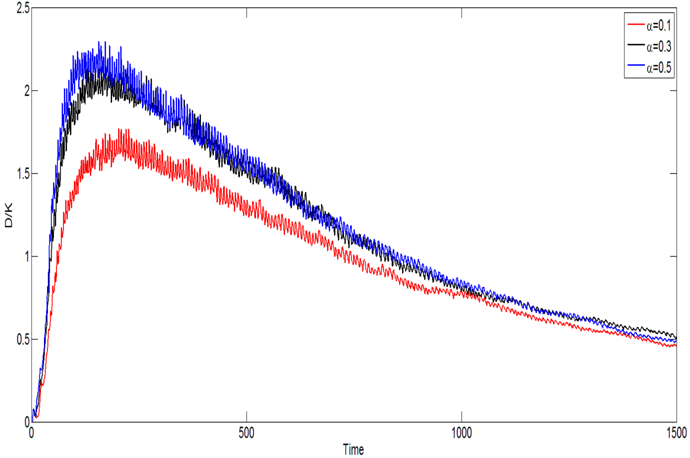

Figure: Dynamics of the debt/capital ratio for different values of α.

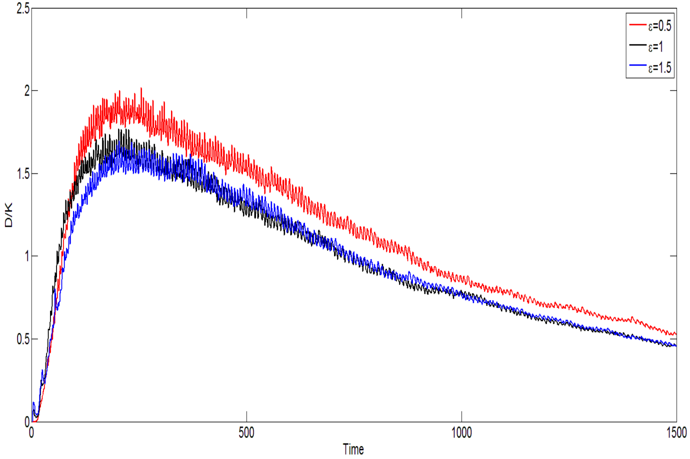

Figure: Dynamics of the debt/capital ratio for different values of ϵ.

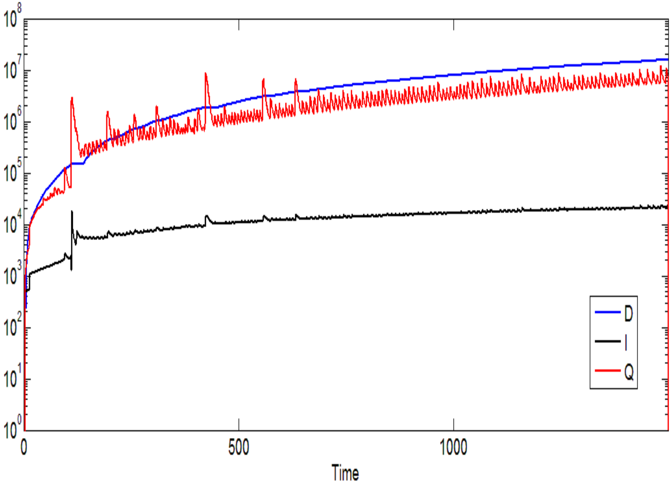

Figure: Dynamics of debt, investment and aggregate demand for $c = 0.01$.

Plots

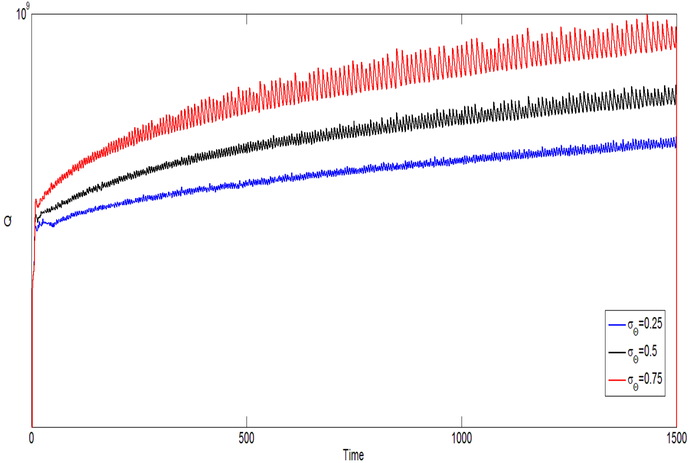

Figure: Dynamics of aggregate demand for different values of σ_Θ.

Plots

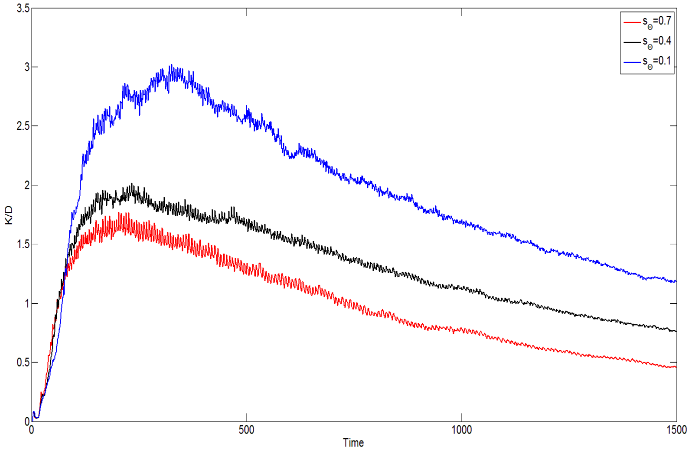

Figure: Dynamics of the debt/capital ratio for different values of s_Θ.

Plots

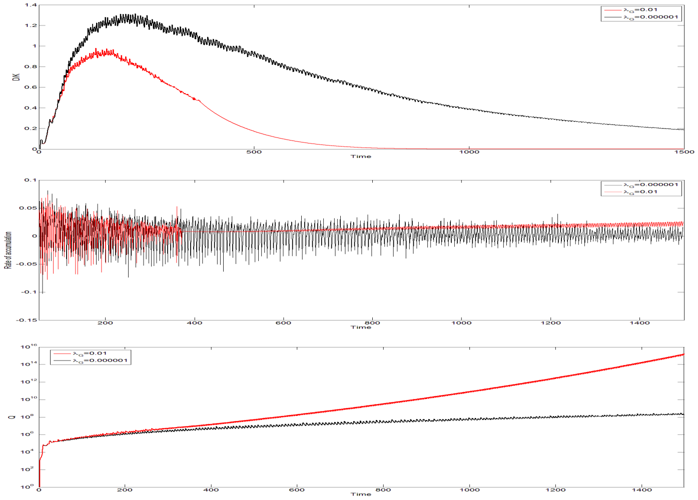

Figure: Debt to capital ratio (top), capital accumulation (center) and aggregate demand (bottom)for different values of λ_G.