Part III: Variable Quantity Swaps

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Introduction

- General problem
- Load Swaps
- Econometric Models
- Recent Macro Effects
- Unit Contingent Swaps

General Problem

Setup

- Consider a short position to an end-user, which can consume the commodity as needed, at a fixed contract price of p_f.
- The terminal payoff for delivery period n is:

$$\pi_{\text{short}} \equiv Q_n \left[p_f - p_n \right]$$

where:

- Q_n is the (random) demand at time n
- p_n is the associated spot price.
- Here n could be indexing an arbitrary sequence of delivery periods—monthly, daily or hourly, as is often the case in power.

General Problem

Typical Hedge

- Q_n is a random variable.
  - Viewed collectively $\{Q_\cdot\}$ is a stochastic process.
  - Econometric analysis is usually performed to yield estimates for $\bar{Q}_n \equiv E[Q_n]$ in addition to other statistical attributes.
- The typical hedge invoked by many practitioners is to forward purchase the expected quantity, yielding a terminal payoff of:

$$\pi_{\text{hedge}} \equiv \bar{Q}_n [p_n - F_n]$$

  where F_n is the forward price at which the hedge was transacted.

  - Usually this forward price is constant over a set of delivery times n due to the nature of monthly ratable forward contracts.
  - We will see shortly, hedging using the expected demand as the notional is often far from optimal.

General Problem

Portfolio Payoff

- The portfolio payoff is the sum the structure payoff and the hedge:

$$\pi_{\text{final}} \equiv \left[Q_n - \bar{Q}_n\right] [p_f - p_n] + \bar{Q}_n [p_f - F_n]$$

  - The second term above is a constant.
  - The first term is where the action is.
  - Demand is positively correlated with price, rendering the expected value of the first term negative.
  - Correlation risk is against the holder of this position.

- If forwards and options markets on Q_n were traded, then π_{final} could be treated as a quanto.
  - Some desks construct forwards and volatilities for Q_n, estimate some correlations and treat such structures as quantos.
  - The problem with this approach is that derivatives on Q, aside from the rare structured load swap, are simply not traded; they are never discussed by brokers and are certainly not listed or cleared.

Volumetric risk is totally uncommoditized.

Load Swaps

Context

- Many utilities by choice or by regulation solicit contracts to serve pools of customers.
  - Such contracts often result from large load auctions / RFPs.
  - Contracts refer to tranches of customers of varying behavior (industrial, commercial, residential).
  - Delivery obligations extend beyond energy to reliability products.
- Load obligations are natural hedges for owners of generation.
  - OTC hedging of sizable generation positions can take months and can involve large transaction costs.
  - A load obligation is often a poor match to a generation owners portfolio.
    - Baseload (e.g. nuclear) generation is essential a flat 7x24 delivery of power.
    - Actual load varies dramatically.

Load Swaps

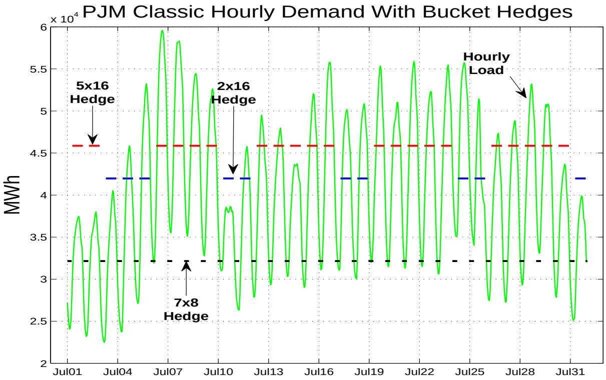
Embedded Risks

- The risks inherent to load swaps are:
  - Short time scale behavior:
    - Demand varies daily or hourly for power, and for some classes of customers is heavily weather dependent.
    - Demand is positively correlated with price.
    - Vanilla bucket hedges are only partially effective as hedges.
  - Attrition (migration) risk:
    - Utility customers in competitive markets have the option to leave utility service for another retail provider.
    - As with prepayment options in mortgages, customers have optionality—if a competitor can provide a lower price then the customer can leave the tranche.
  - Load growth estimates:
    - Historically power demand has systematically increased on the order of 1-2% annually.
    - Performance of hedging strategies depends heavily on estimates of load growth.

Load Swaps

Context

- The following figure shows historical PJM Load (July 07) with block hedges.



Load Swaps

Context

- The settlement value for the unhedged structure in contract month m and bucket $\mathcal{B}$ is:

$$\sum_{h \in \mathcal{B}(m)} \left[\bar{L}_h \left(p_f - p_h \right) + \left(L_h - \bar{L}_h \right) \left(p_f - p_h \right) \right]$$

  - We have switched to hourly indexing.
  - L_h is the hourly load.
  - $\bar{L}_h$ is the expected value.

- We will start by examining the first component above.
  - The notional is deterministic but varies by hour.
  - Only the price differential is random.
  - These are sometimes referred to as "fixed notional" or "hourly-shaped" swaps.

Load Swaps

Hourly-Shaped Swaps

- Hourly "Forward" Prices:
  - Hourly prices can (should?) be represented as:

    $$\bar{p}(d) = \bar{\alpha}_B p_B(d)$$

    where $\bar{p}$ denotes the hourly spot prices in bucket B.
  - By conservation of dollars $\bar{1}^t \bar{\alpha}_B = N_b$ where N_b is the number of hours in the bucket.
  - Estimation of $\bar{\alpha}$ — one obvious way:

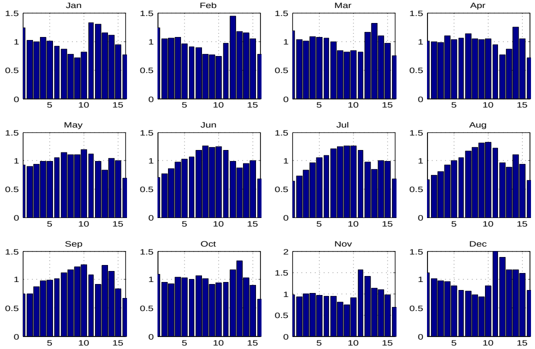
    $$\bar{\alpha}_B^c = \frac{1}{N_d} \sum_{m \in c} \sum_{d \in m} \frac{\bar{p}(d)}{p_B(d)}$$

    where $c \in [1, \ldots, 12]$ references calendar months and $N(d)$ is the number of days in the sample.

Load Swaps

Hourly-Shaped Swaps

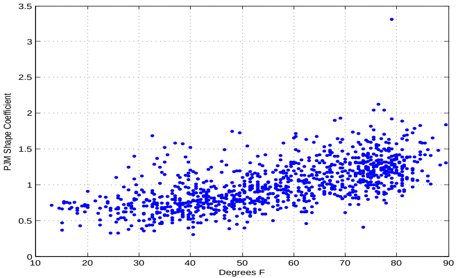
- Shaping Coefficients:
  - The following figure shows some sample "shaping coefficients" estimated in this way for the 5x16 buckets for PJM Western Hub spot prices.
  - Note the obvious high prices in summer months in the middle of the day.



Load Swaps

Hourly-Shaped Swaps

- Shaping Coefficients:
  - The following figure shows the shaping coefficient for PJM at 4PM.
  - The implication is that low temperatures (low prices) are associated with more benign shaping that high temperatures (high prices).



Load Swaps

Hourly-Shaped Swaps

- Shaping Coefficients:
  - For a fixed day (or set of days/hours) in a month:

$$p_{\text{fixed}} = \frac{\sum_h \bar{L}(h)\tilde{E}[p_h]}{\sum_h \bar{L}(h)}$$

  - What does $\tilde{E}[p_h]$ mean? It doesn't trade?
  - Common approach — use the "shaping" coefficients:

$$\tilde{E}[p_h] = \alpha_h \tilde{E}\left[p_B\right] = F(0,T)$$

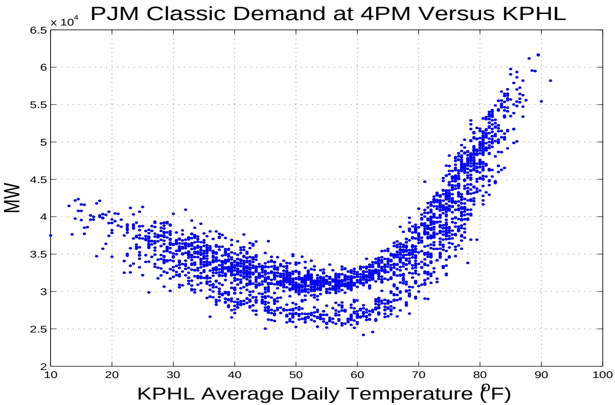
where $p_B \equiv \frac{1}{N_h}\sum_{h\in d} p_h$.

  - This assumes the same risk premia for hourly spot prices as for daily.
  - It also ignores the fact that $\bar{\alpha}$ depends on temperature and is, therefore, correlated to the daily price $p_B(d)$.

Load Swaps

Variable Notional

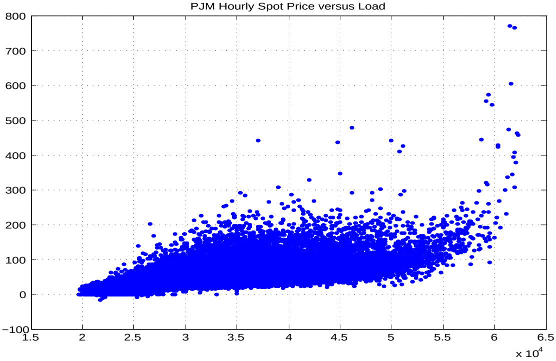
- Hourly quantities are now random.
  - $q(h) = L(h)$ is an actual realized hourly load.
  - This is PJM load at 4PM versus KPHL temperature.



Load Swaps

Variable Notional

- These are hourly spot prices versus load.



Load Swaps

Variable Notional

- A critical valuation issue is the correlation between loads and prices at the hourly level.
- The fact that L_h and p_h are positively correlated makes load swaps more expensive per MWh.
  - When loads are high, you are short relative to your expected load quantity hedge and prices are high.
  - Conversely, when loads are low you are long in a low price scenario.
- Load is not commoditized.
  - Some market participants use weather derivatives to hedge residual risks.

Load Swaps

Variable Notional

- Risks include:
  - $L(h) = \bar{L}(h) + \epsilon_h$
  - Changes in estimated growth rates (macro-dynamics) effect results.
  - Changes in the distribution of $L(h)$ due to customer migration.
- In a risk-neutral setting, fair value is equivalent to:

$$\tilde{E}\left[\sum_h L(h)\left(p_{\text{fixed}} - p_h\right)\right] = 0$$

which implies:

$$p_{\text{fixed}} = \frac{\tilde{E}\left[\sum_h L(h) p_h\right]}{\tilde{E}\left[\sum_h L(h)\right]}$$

- There are a variety of ways that practitioners try to estimate/parameterize this covariance.

Load Swaps

Variable Notional

- Stack Models Revisited
  - One form: $p_t = \Phi_t \left[L_t(1+\delta_t)|\bar{F}_t\right] + \epsilon_t$
  - If we omit δ_t (or build it into L_t) and ϵ_t we have:

$$p_t = \Phi_t \left[L_t|\bar{F}_t\right]$$

  - Assuming a single fuel (natural gas) and an exponential heatrate stack:

$$p_t = HG_t e^{\lambda L_t}$$

  - Assuming that:
    - L_t is normal(μ_L,σ_L)
    - G_t is independent of L_t

    then

$$\tilde{E}\left[L_t p_t\right] = HG(0,t)\tilde{E}\left[L_t e^{\lambda L_t}\right] = HG(0,t)\frac{d}{d\lambda}\tilde{E}\left[e^{\lambda L_t}\right]$$

Load Swaps

Variable Notional

- Stack Models Revisited
  - This yields:
$$\tilde{E}\left[L_t p_t\right] = \left(\mu_L + \lambda\sigma_L^2\right) e^{\mu_L + \frac{1}{2}\lambda\sigma_L^2}$$
  - Note also that the forward price for power is:
$$F(0,t) = \tilde{E}\left[p_t\right] = e^{\mu_L + \frac{1}{2}\lambda\sigma_L^2}$$
  - Therefore:
$$\frac{p_{\text{fixed}}}{F(0,t)} = \frac{F(0,t)\left(\mu_L + \lambda\sigma_L^2\right)}{\mu_L F(0,t)} = 1 + \lambda\frac{\sigma_L^2}{\mu_L}$$
  - This is a constant elasticity result: uplift depends upon convexity of the stack and the ratio of load variance to mean, but not upon fuel prices.
  - More interesting/realistic stacks yields more interesting behavior.

Load Swaps

Variable Notional

- Econometric Models:
  - Based on regressions of historical behavior of relevant underlying variables.
  - The results yield simulation methods to generate the joint distribution of future realizations of these variables.
  - These realizations yield physical measure distributions of:
    - The payoff Π of whatever the structure is that you are valuing.
    - Available hedges $\vec{\mathcal{H}}$ which trade at market prices $\vec{p}_{\mathcal{H}}$.
  - Standard portfolio analysis method can then be applied—for example, construction of minimum variance hedges:

$$\min \operatorname{var}\left[\Pi + \vec{w}^{\dagger}\left(\vec{\mathcal{H}} - \vec{p}_{\mathcal{H}}\right)\right]$$

Econometric Methods

Context

- - *Working Problem:* Calculate the mid-market fixed price p_f for a variable load swap of the form:
    - Pricing Date: 30Dec2011.
    - Delivery: Peak power for Jul12.
    - Spot price index: PJM Western Hub hourly real-time price.
    - Load index: PJM Classic Preliminary Load Index.[1]

[1] Preliminary load estimates are published by PJM within a few days of the delivery day based upon econometric analysis and samples of subsets of consumption. Final load estimates are published a few months later, and are typically very close to to the preliminary estimate, which makes the preliminary load index more suited to swaps with monthly settlements.

Econometric Methods

Valuation

- Historical Uplift—The ratio of the fair-value price to the vanilla bucket price.

  - Given historical data for a given month m and bucket B, the implied (in arrears) fair price p_f solves:

$$\sum_{h \in B(m)} [L_h p_f - L_h p_h] = 0$$

  - The historical uplift is the ratio of p_f to the average realized hourly price which yields:

$$U(m, B) = \frac{\sum_{h \in B(m)} L_h p_h}{\left(\sum_{h \in B(m)} L_h\right) \left(\frac{1}{N_{B(m)}} \sum_{h \in B(m)} p_h\right)}$$

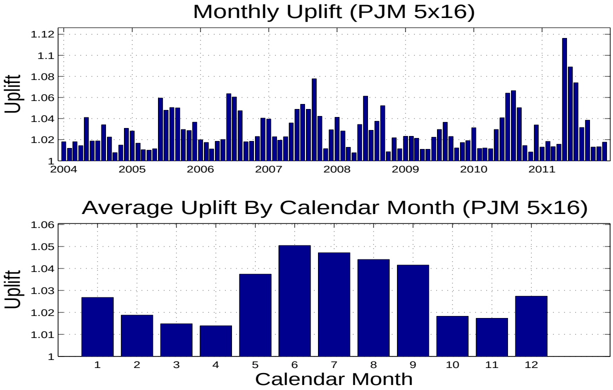
  where $N_{B(m)}$ is the number of hours in $B(m)$.

  - Note that if load L_h were constant, the uplift would be identically one—any departure from this value is due to the presence of empirical correlations between load and price.

Econometric Methods

Valuation

- Historical Uplift
  - The results for our working example are shown for the 5x16



Econometric Methods

Valuation

- Regressions and Simulations
  - Valuation procedure is to analyze the load/price dynamics to calculate a fair-value price.
  - To value this load transaction we need a method for constructing the joint distribution of:

    $$\bar{X}_d \equiv \left[\tau_d, \bar{L}_B(d), \bar{\mathcal{U}}_B(d), p_B(d)\right]$$

    where τ_d is daily temperature, $p_B(d)$ is the bucket (spot) price, and:

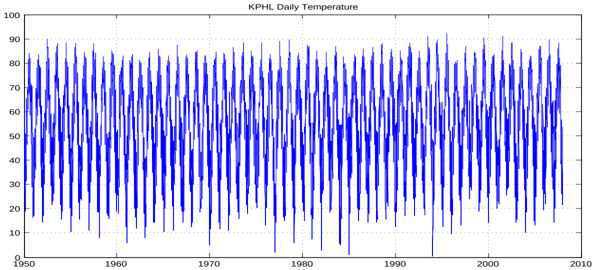
$$\bar{L}_B(d) \equiv \sum_{h \in B(d)} L_h \qquad \bar{\mathcal{U}}_B(d) \equiv \frac{\sum_{h \in B(d)} L_h p_h}{\left(\sum_{h \in B(d)} L_h\right)\left(\frac{1}{N_{B(d)}} \sum_{h \in B(d)} p_h\right)}$$

where $N_{B(d)}$ is the number of hours in $B(d)$.

Econometric Methods

Introduction to Weather

- For natural gas and power markets temperature is the main driver.
- This figure show historical temperature at KPHL.
- Key point:
  - Temperature has decades of reliable data. Estimation is robust.
  - Natural gas and power markets have much shorter data sets.
  - Weather-normalizing spot price behavior yields much more reliable estimates of spot price behavior than by analysis of spot prices directly.



Econometric Methods

Introduction to Weather

- Model:

$$\tau_d = \mu(d) + \sigma(d) X_d$$

where d denotes day, τ temperature and:

  - The mean is respresented as:

$$\mu(d) = \alpha_0 + \alpha_1(d - d_*) + \sum_{k=1}^{K} \left[c_k \cos\left(2\pi k \Phi(d)\right) + d_k \sin\left(2\pi k \Phi(d)\right) \right]$$

    where d_* is a reference date and $\Phi(d)$ is the fraction of the year corresponding to d: $\Phi(d) = \frac{d - BOY(d)}{365}$.
  - Similarly for $\sigma(d)$.
  - The residuals X_d is assumed (for now) to be a stationary process.

Econometric Methods

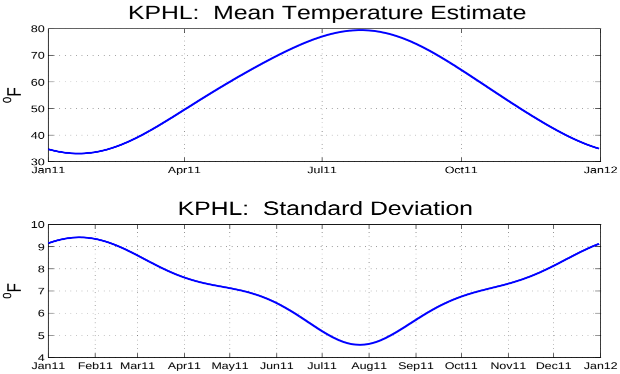
Introduction to Weather

- Estimation involves:
  - Choosing the number of modes K to keep.
  - Including or rejecting the presence of a systematic drift in the temperature ($\alpha_1 \neq 0$).
- Using an out-of-sample estimation criterion for each of these questions yields the following results for KPHL.
  - $\alpha_1 \approx 2.2e - 4$ (or approximately .08 degF / year).
  - $K = 3$ for both μ and σ.

Econometric Methods

Introduction to Weather

- The following figure shows the estimated seasonal mean for and standard deviation.



Econometric Methods

Introduction to Weather

- The residuals from the regressions are:

$$\hat{X}_d \equiv \frac{\tau_d - \mu(d)}{\sigma(d)}$$

.

- Of particular relevance is the auto-correlation function (ACF) which is defined to be:

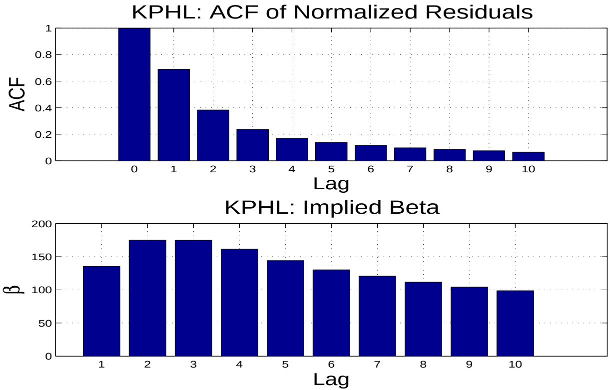
$$\rho(j) = E\left[X_d \cdot X_{d-j}\right]$$

.

- The next plot below shows the ACF and the log(ACF).
- Standard ARMA or seasonal bootstrap methods can be used to simulate temperature across one or many weather locations.

Econometric Methods

Introduction to Weather



Econometric Methods

Spot Prices

- Spot heatrates are the natural variables for power prices.
- Consider, for example, a regression form:

$$\log\left[\frac{p(d)}{p_{\mathrm{NG}}(d)}\right] = \alpha + \gamma p_{\mathrm{NG}}(d) + \sum_{k=1}^{K} \beta_k \theta(d)^k + \epsilon_d$$

where:

- The modified temperature is: $\theta(t) = \frac{e^{\lambda(t)}}{1+e^{\lambda(t)}}$ with $\lambda(t) \equiv \frac{\tau(t)-\tau_{\mathrm{ref}}}{w}$.
- Here τ_{ref} and w selected to be characteristic mean and width of temperatures realized over the entire data set.
- The fact that $\theta \in [0,1]$ results in regular behavior beyond the range of historical data.

Econometric Methods

Valuation

- Relevant Random Variables
  - Sample regression/simulation form:
  - For load by bucket:

$$\bar{L}_B(d) = \alpha + \beta d + \sum_{k=1}^{K_L} a_k \theta^k(d) + \sigma_L(d)\epsilon_L(d)$$

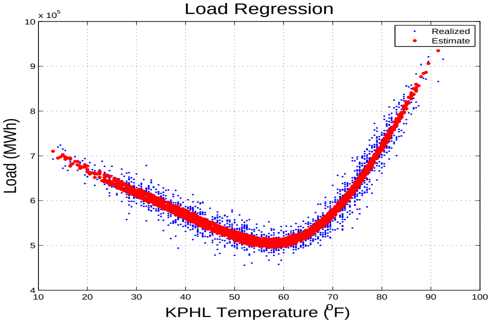
  - For load by bucket:

$$\bar{\mathcal{U}}_B(d) = \sum_{k=1}^{K_U} b_k \theta^k(d) + \sigma_U(d)\epsilon_U(d)$$

Econometric Methods

Valuation

- Typical setup:
  - Temperatures and Henry Hub natural gas prices are simulated (the latter often risk neutral).
  - Natural gas basis is simulated, followed by spot heat rates.
  - Load variables are simulated, correlated with $[\tau_d, p_d]$.
  - This shows the results for the $\bar{L}_B(d)$.



Econometric Methods

Valuation

- Hedging Construct
  - Load swaps are negotiated in terms of the fixed price p_f that an acquirer of a short load position requires for assuming the obligation.
  - This will result in an iterative aspect to valuation.[2]
  - For a specified transaction price p_f, the minimum variance hedge are the hedge weights $\bar{w}_* (p_f)$ obtained from:

$$\min \operatorname{var} \left[\sum_{d \in m} \left(p_f \bar{L}_B(d) - \sum_{h \in B(d)} L_h p_h \right) + \vec{w}^\dagger \left(\vec{\mathcal{H}} - \vec{p}_{\mathcal{H}} \right) \right]$$

    where $\bar{\mathcal{H}}$ are the payoffs of whatever basket of hedges we choose to consider, and $\bar{p}_{\mathcal{H}}$ the prevailing market price.

  - The optimal hedge $\bar{w}_* (p_f)$ is a function of p_f, which we don't know yet.

[2] For mean/variance criteria this can be solved exactly as a solution to a linear system; for utility based approaches iteration is required. We adopt the later for generality.

Econometric Methods

Valuation

- Hedging Construct
  - Returning to our working problem, on the pricing date 30Dec2011 we will start by setting $p_f^{(1)}$ as the fair value peak bucket in Jul12 without any hedges:

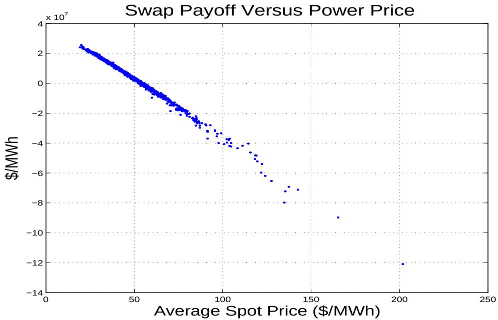
$$p_f^{(1)} = \frac{E\left[\sum_d \sum_{h \in B(d)} L_h p_h\right]}{E\left[\sum_{d \in m} \bar{L}_B(d)\right]}$$

  - This is our initial estimate for the mid-market fixed price p_f.
  - Using simulations of implied by the regressions yields: $p_f^{(1)} =$ \$53.07/MWh.

Econometric Methods

Valuation

- Hedging Construct
  - This figure shows the load swap with $p_f^{(1)} = \$53.07/\text{MWh}$ versus bucket power prices as per the simulations.
  - This motivates simply using a single 5x16 swap as the hedge basket.



Econometric Methods

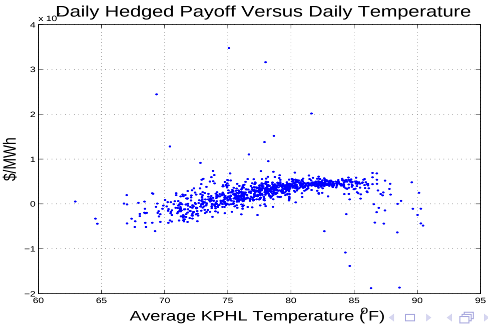
Valuation

- Results
  - The optimal hedge $w_*(p_f^{(1)})$=1.11MWh of power per MWh of expected load; this is signficantly more than the näive hedge of using the expected notional.
  - Iterating the fair value equation yields fixed point for p_f of $57.18/MWh.
    - The forward price was $53.60/ MWh;
    - The uplift for the peak bucket in Jul12 is 1.067.
  - This hedge accomplished a lot.
    - The standard deviation of the load swap payoff is $21.11/MWh;
    - That of the hedged portfolio with this forward purchase was reduced to $1.73/MWh.

Econometric Methods

Valuation

- Results
  - Could additional hedging be done?
  - The following figure shows residuals versus daily temperature.
    - The same plot of monthly residuals versus monthly temperatures shows minimal structure.
    - This plot shows that customized weather trades could reduce risk at extreme levels of temperature.



Variable Load Swaps

Other Commodities/Structures

- Power markets provide hourly demand data which facilitates analysis.
- Variable quantity risk exists in other markets, notably natural gas.
  - High natural gas demand and high spot prices tend to occur at low temperatures.
  - This has resulted in the introduction of natural gas swaps which "trigger" on a temperature level.
  - For example, a swap with settlement in month m defined by:

$$\sum_{d \in m} \mathbf{1}_{\{\tau_d \leq \tau_*\}} \max\left[p_d - K, 0\right]$$

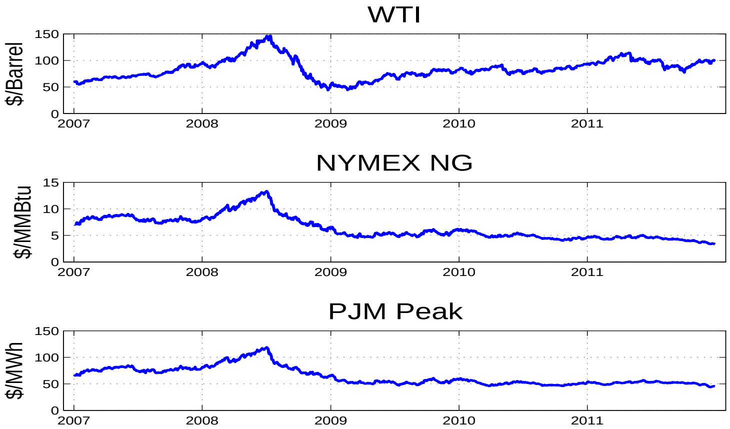
provides protection to the supplier for spot prices levels p_d in excess of the strike if the daily temperature τ_d is below the trigger τ_*.

- The methodologies above can be deployed in such settings.

Variable Load Swaps

Recent Macro Effects

- If you had to represent the recent economic turmoil using one plot from the commodities markets it would be this following rolling cal strips for WTI, NG and PJM.



Variable Load Swaps

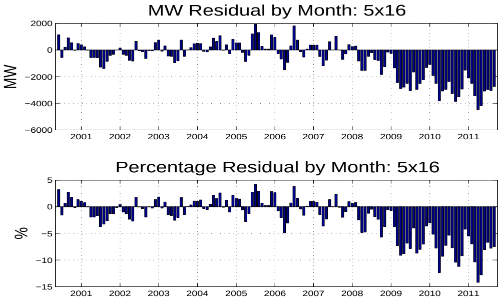
Recent Macro Effects

- The consequences had many associated effects.
- Variable quantity swaps became riskier due to:
  - High spreads between transaction strike and current market prices.
  - Departures of expected quantity from norm.
- Margining provisions associated with hedges for retail suppliers were extreme.

Variable Load Swaps

Recent Macro Effects

- Micro-convexity Risk:
  - Hourly load-price correlation discussed above has been modeled extensively.
- Macro-convexity Risk:
  - What was not anticipated by most if not all market participants was what transpired since late 2008.
  - The following plot shows fully weather-normalized load residuals given a forecast made in Aug2008.



Variable Load Swaps

Recent Macro Effects

- Macro-convexity Risk: (cont)
  - Note that the departure from norm is several times more severe than post the tech-bubble era.
  - The drop in expected load coincided with the universal drop in energy prices.
  - Hedging at originally expected volumes meant that you were left holding a significantly long position in a falling price environment.
  - This negative convexity is more severe than the occasional hot/cold day previously discussed as it effects large volumes and there is no law of large numbers to save you.
  - To add to the problem, many customers have the option to leave to an alternative supplier.
  - This is akin to mortgage prepayment optionality, and resulted in even longer positions for many retail suppliers.

Unit Contingent Swaps

Setup

- Generation owners desiring to hedge the value of power produced and/or fuel consumed will often prefer to make the swap quantity dependent on the actual quantity generated.
- For a baseload generator (e.g. a nuke) the standard hedge is a vanilla swap with terminal payoff (from the perspective of the hedge provider who is buying the power) for a given month m of:

$$\sum_{h \in m} Q \left[p_h - p_{fixed} \right]$$

where:

  - Q is the contractually specified hourly quantity which is the same for every hour.
  - p_h is the hourly realized spot price.
  - p_{fixed} is the contractually fixed payment price.

Unit Contingent Swaps

Setup

- The PV from the perspective of the hedge provider is:

$$d_m Q_{tot} \left[F_m - p_{fixed} \right]$$

  where

  - d_m is the current discount factor to the settlement date of the month m.
  - $Q_{tot} = \sum_{h \in m} Q$.

Unit Contingent Swaps

Valuation and Risk

- The unit contingent version has a terminal payoff

$$\sum_{h \in m} \hat{Q}_h \left[p_h - p_{fixed} \right]$$

  where

  - $\hat{Q}_h$ is the actual quantity of power generated by the facility.

- Valuation of this structure is anything but straightforward as the random variables $\hat{Q}_h$ are not commoditized (if they were this would resemble a quanto).

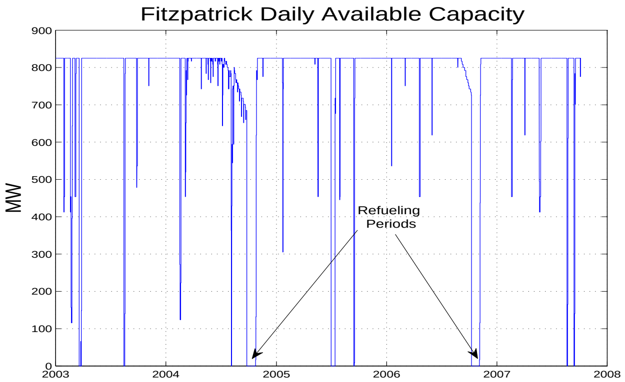
Unit Contingent Swaps

Valuation and Risk

- The following figure shows the daily output from a publicly available datasource for a particular nuclear generator in upstate NY.
  - Note the periodic refueling outages (which are planned) as well as the random derates and variations in output that constitute daily versions of $\hat{Q}_h$.
  - At an hourly level (data which is typically proprietary) the actual generation varies around this picture due to thermal fluctuations among other factors.

Unit Contingent Swaps

Valuation and Risk



Unit Contingent Swaps

Valuation and Risk

- The standard hedging approach is to sell the expected generation output through vanilla swaps.
  - Expected output is based up analysis of historical performance.
  - Consider a UC contract for 1Y of power at 500MW at an initial fixed price of \$100/MWh.
  - This is contract is of moderate size, consisting of approximately 4.4m MWh or a total notional value of approximately \$440m.
  - When energy prices dropped by over 65% to say \$35/MWh before the deal starts realizing, the uncertainty associated with actual versus expected generation is significant.
  - Every 1% increase in availability, results in a loss of roughly \$3m.

Summary

- Energy structures with variable quantity (always) involve uncommoditized risks.
  - Variable demand with no demand swaps.
  - Variable supply (generation) with no generation swaps.
- This is a fundamentally incomplete markets setting—uncommoditized risks are untradable.
- Econometric or structure models with viable hedges and distributional estimates of the final portfolio payoff are the currently viable approaches to valuation and hedging of such structures.