Part II: Tolling Deals

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Basics of Tolling Deals

Deal Structure

- Tolling deal is a derivative rendition of a power generator.
  - Typically the fuel is natural gas.
- The Basic Structure:

$$\tilde{E}\left[\sum_t d(0,t)\max\left(F_P(t,t) - H_* F_G(t,t) - V, 0\right)\right]$$

where:

- t denotes delivery day (this is discrete time).
- F_P denotes the forward (or spot) price of power for a particular delivery bucket (e.g. 5x16) and F_G denotes the price of natural gas; both are typically at liquid pricing hubs.
- H_* is the heatrate (conversion rate between gas and power).
- V is VOM (variable operation and maintenance).

Basics of Tolling Deals

Deal Structure

- The primary purpose of a toll is to annuitize the value of either a soon to be purchased asset or a soon to be built asset in order to facilitate borrowing.

- The typical structure is:
  - Daily day-ahead manual exercise into a standard power buckets.
  - Monthly "capacity payments" (as opposed to upfront premium).
  - Monthly settlement.

  so that the valuation is really:

$$V = \tilde{E}\left[\sum_{m} d(0,T_m) Q_m \sum_{d \in B(m)} \max\left(F_P(d,d) - H_* F_G(d,d) - V, 0\right)\right]$$

  - $B(m)$ denotes the days in month m with delivery in the relevant power bucket B.

Power Market Mechanics

Linear Instruments

- Fixed price swaps settle on the monthly average realized spot price for a prescribed bucket.
- Eastern US:
  - Peak bucket is 5x16 (M-F 7AM to 11PM).
  - Offpeak (the "offpeak wrap") is the complement.
  - Often circumstances require trading (or at least viewing) the offpeak wrap as 2x16 and 7x8. These are highly illiquid swaps.
  - Standard contract size: 50MW.
- Western US:
  - Peak bucket is 6x16 (M-Sat 7AM to 11PM).
  - Offpeak (the "offpeak wrap") is the complement.
  - Standard contract size: 25MW.
- Texas (ERCOT):
  - Peak bucket is 5x16 (M-F 6AM to 10PM).
  - Offpeak (the "offpeak wrap") is the complement.
  - Standard contract size: 50MW.

Power Market Mechanics

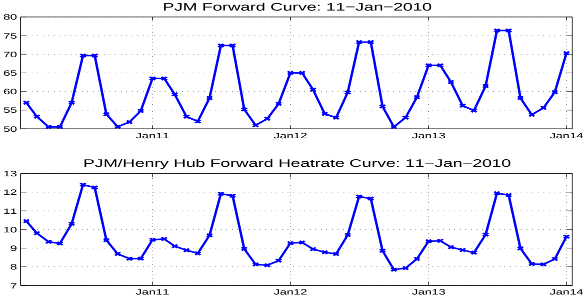
Linear Instruments: Heatrates

- The term "heatrate" refers to a ratio of power price to a natural gas price.
  - The ratio can pertain to market prices, either spot or forward ("market heatrates").
  - It can also refer (as we'll see shortly) to engineering specs of a generator.
- Power swaps often trade as heat-rates:
  - A power buy/sell is associated with a sell/buy of a specified volume of natural gas, both over the same delivery period.
  - The trade will be quoted in heatrate units: $\frac{F_P(t,T)}{F_G(t,T)}$, where P and G denote power and gas respectively.

Power Market Mechanics

Linear Instruments: Heatrates

- The following figure shows PJM forward prices and PJM/Henry Hub forward heatrates.
  - The seasonality in both arises from (well-founded) expectations that loads will be higher in summer and to a lesser extent winter months.
  - This forces expected clearing prices higher up the stack into more expensive units.



Power Market Mechanics

Options

- Annual Swaptions
  - Same structure as natural gas in mechanics/valuation.
- Monthly options:
  - Same structure as natural gas.
  - Standard expiry: '-2b'
- Daily Fixed Strike
  - Financially settling and usually manually exercised day-ahead ('-1b')
  - Example: Cash settled value of an auto-exercised call:

$$\sum_{d \in m} \max\left[F(d,d) - K, 0\right]$$

  - Example: Cash settled value of standard exercise:

$$\sum_{d \in m} 1_{\{E_d\}}\left[F(d,d) - K\right]$$

    where E_d denotes exercise events.
  - Dominant vol exposure: daily (spot).

Power Market Mechanics

Options

- Monthly Options:
  - Exercises at time (T_e) before the beginning of the delivery month into either:
    - A physical forward with delivery during the contract month at a price that is the strike K;
    - Cash settlement based on the value $F_m(T_e) - K$.
  - For example, the value of a call (in either case) is: $d(0,T)\tilde{E}\left(\max\left[F_m(T_e) - K, 0\right]\right)$.

- Daily Options:
  - A set of daily options usually exercising one business day before delivery.
    - These are usually financial settling on the spread between spot price and the strike: $F(t,t) - K$ where t indexes the delivery day.
  - For example, the value of a call is: $d(0,T)\tilde{E}\left(\sum_{t\in m}\max\left[F(t,t) - K, 0\right]\right)$.
  - The notation $t \in m$ denotes the active delivery days in the option during month m. For peak (5x16) power options the sum would span the (roughly 20-22) business days in the particular month.

Valuing Tolls—A Monthly Example

Working Problem

- Consider a tolling deal with monthly exercise with the following terms:
  - Pricing date: 11Jan2010
  - Underlyings are PJM Western Hub 5x16 and Henry Hub natural gas.
  - Heatrate $H_* = 11.0$
  - Delivery monthly Jul10
  - Exercise is standard penultimate settlement of NYMEX NG contract.
    - Note: for simplicity we will assume that PJM monthly options also expire '-4b').
  - Notional: 400 MW

Valuing Tolls—A Monthly Example

Relevant Data

- Remark on Notional:
  - The number of NERC business days in the Jul10 is 21.
  - So the total notional of this toll is $21 \cdot 16 \cdot 400 = 134,400$ MWh.
- The relevant forward prices are:
  - $X_0 \equiv F_P(0, T) = 69.650$
  - $Y_0 \equiv H_* F_G(0, T) = 11.0 \cdot 5.603 = 61.633$
- The discount factor is 0.999 .
- Note: The market heatrate is: $H = \frac{69.650}{5.603} = 12.431.$
  - This is higher than the deal heatrate $H_* = 11.0$
  - The option is in-the-money.

Vol Backwardation:

- We do not have a vol backwardation issue as the respective legs are expiring (by assumption) at their vanilla expiry.

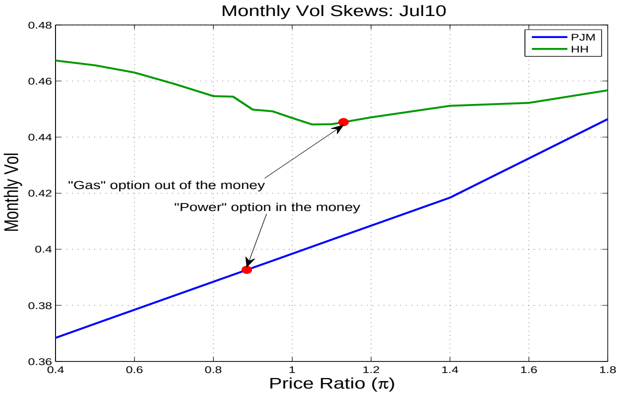
Valuing Tolls—A Monthly Example

Skew: What vols do we pick?

- There is ambiguity as the vol skews for each underlying are defined in terms of fixed strike options.
- We have floating strikes in the sense that the "other" leg is itself a commodity price.
- A common approach is to use the underlying price of the opposing leg to define moneyness.
- For an option on $F_1 - F_2 - K$ view:
  - F_1 in reference to $F_2 + K$: $\pi_1 \equiv \frac{F(0,T_2)+K}{F(0,T_1)}$.
  - F_2 in reference to $F_1 - K$: $\pi_2 \equiv \frac{F(0,T_1)-K}{F(0,T_2)}$.
- For this problem:
  - $\pi_X \equiv \frac{H_* F_G(0,T)}{F_P(0,T)}$ and $\pi_Y \equiv \frac{F_P(0,T)}{H_* F_G(0,T)}$.
  - In our working problem $\pi_X = 0.885$ and $\pi_Y = 1.130$.
- The results of this vol lookup are: $\sigma_X = 0.393$ and $\sigma_Y = 0.445$.

Valuing Tolls—A Monthly Example

Skew: What vols do we pick?



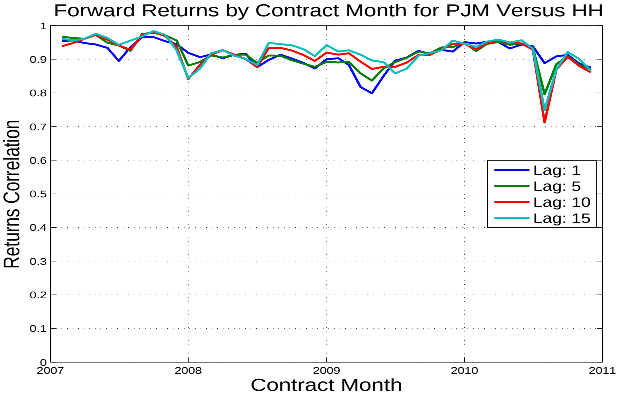
Valuing Tolls—A Monthly Example

Correlation

- What correlation should we use?
  - Monthly tolling deals are uncommon and there is no broker market from which to calibrate this parameter.
  - The following figure shows the returns correlation by contract month estimated over a 1Y trailing window using several time-scales (lags) for returns:
    - Specifically returns on lag L are defined as $\log\left[\frac{F(d,T)}{F(d-L,T)}\right]$.
    - The purpose of considering returns for $L > 1$ is to both view returns on time scales on which actual hedging activity may occur as well as to smooth out possible anomalies in historical marks.

Valuing Tolls—A Monthly Example

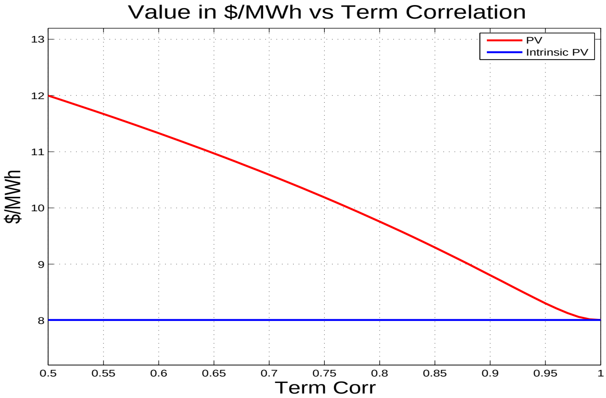
Correlation



Valuing Tolls—A Monthly Example

Correlation

- The results for the this working problem are shown in the figure below for a range of correlations.



Valuing Tolls—A Monthly Example

Delta and Gamma

- Deltas—recall:

$$\frac{\partial V}{\partial X} = N(d_1) \qquad \frac{\partial V}{\partial Y} = -N(d_2)$$

- Converting to our forward tolling setting yields:

$$\frac{\partial V}{\partial F} = d(\tau)QN(d_1) \qquad \frac{\partial V}{\partial G} = -d(\tau)QH_* N(d_2)$$

- Gamma—similarly:

$$\Gamma = d(\tau)Q\frac{N'(d_1)}{\hat{\sigma}}\begin{pmatrix} \frac{1}{F(0,T)} & -\frac{1}{G(0,T)} \\ -\frac{1}{G(0,T)} & \frac{F(0,T)}{G^2(0,T)} \end{pmatrix}$$

- Γ is positive-definite because:

$$\bar{\alpha}^t\Gamma\bar{\alpha} = d(\tau)Q\frac{N'(d_1)}{\hat{\sigma}F(0,T)}\left[\alpha_1 - \frac{F(0,T)}{G(0,T)}\alpha_2\right]^2$$

.

- When Δ-hedged, all directions point up.

Valuing Tolls—A Monthly Example

Delta and Gamma

- It is useful to diagonalize the matrix to see where the convexity is:
  - The eigenvalues are:

$$\lambda_1 = 0$$
$$\lambda_2 = d(\tau)Q\frac{N'(d_1)}{\hat{\sigma}}\left[\frac{1}{F(0,T)} + \frac{F(0,T)}{G(0,T)^2}\right]$$

  - The eigenvector corresponding to $\lambda_1 = 0$ is:

$$\vec{v}_1 = \begin{bmatrix} \frac{F(0,T)}{G(0,T)} \\ 1 \end{bmatrix} \quad (1)$$

  - There is no convexity when market heat rate $H(t,T)$ is constant.
  - The second eigenvector is:

$$\vec{v}_2 = \begin{bmatrix} 1 \\ -\frac{F(0,T)}{G(0,T)} \end{bmatrix} \quad (2)$$

  - Convexity is maximal for price changes in which $\Delta G = -H\Delta F$.
  - This direction is not particularly relevant to empirical natural gas and power dynamics.
  - In this direction, a \$1 increase in *forward* power prices is associated with a roughly \$12 drop in natural gas prices; hardly an expected event.

Valuing Tolls—A Monthly Example

Vega

- The valuation above is identical to a call option on X struck at Y_0 with the modification that the term volatility is given by the modified form:

$$\hat{\sigma}^2 = T\left[\sigma_X^2 + \sigma_Y^2 - 2\rho\sigma_X\sigma_Y\right]$$

- As with a call option, $\frac{\partial V}{\partial \hat{\sigma}}$ is positive:

$$\frac{\partial V}{\partial \hat{\sigma}} = XN'(d_1)\frac{\partial d_1}{\partial \hat{\sigma}} - YN'(d_2)\frac{\partial d_2}{\partial \hat{\sigma}} = XN'(d_1)$$

.

- The chain rule clearly yields:

$$\frac{\partial V}{\partial \sigma_X} = \frac{\partial V}{\partial \hat{\sigma}}\frac{\partial \hat{\sigma}}{\partial \sigma_X}$$

- This means that:

$$\mathrm{sign}\left[\frac{\partial V}{\partial \sigma_X}\right] = \mathrm{sign}\left[\frac{\partial \hat{\sigma}}{\partial \sigma_X}\right] = \mathrm{sign}\left[\sigma_X - \rho\sigma_Y\right]$$

Valuing Tolls—A Monthly Example

Vega

- By symmetry:

$$\text{sign}\left[\frac{\partial V}{\partial \sigma_Y}\right] = \text{sign}\left[\sigma_Y - \rho\sigma_X\right]$$

- The implication is that vega with respect to one of the underlyings will be negative if: min $\left[\frac{\sigma_X}{\sigma_Y}, \frac{\sigma_Y}{\sigma_X}\right] < \rho$.
- As we know from our working example, this situation is not uncommon as σ_X and σ_Y are often of comparable magnitude and ρ is often very close to unity.
- The intuition is simple: if the correlation of the two assets is high and if $\sigma_X > \sigma_Y$ then any increase in σ_Y "chews into" the volatility of the spread $\hat{\sigma}$.

Valuing Tolls—A Monthly Example

Vega

- By implicit differentiation:

$$\frac{\partial \hat{\sigma}}{\partial \sigma_X} = \frac{T^{\frac{1}{2}}\left(\sigma_X - \rho\sigma_Y\right)}{\left(\sigma_X^2 + \sigma_Y^2 - 2\rho\sigma_X\sigma_Y\right)^{\frac{1}{2}}}$$

  with a symmetric result for $\frac{\partial \hat{\sigma}}{\partial \sigma_Y}$.

- Therefore:

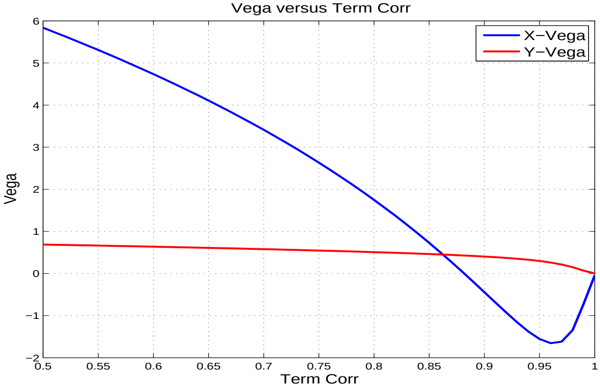
$$\frac{\partial V}{\partial \sigma_X} = XN'(d_1)\frac{T^{\frac{1}{2}}\left(\sigma_X - \rho\sigma_Y\right)}{\left(\sigma_X^2 + \sigma_Y^2 - 2\rho\sigma_X\sigma_Y\right)^{\frac{1}{2}}}$$

  which scales as $T^{\frac{1}{2}}$.

Valuing Tolls—A Monthly Example

Vega

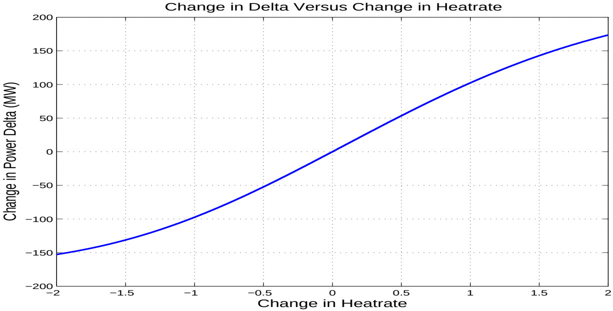
- In our working problem the correlation threshold for negative vega is $\frac{\sigma_X}{\sigma_Y} = 0.882$.
- We expect X-vega to be negative.
- The following plot shows both vegas as a function of correlation.



Hedging Frictions

Block-Size Impact on Hedging

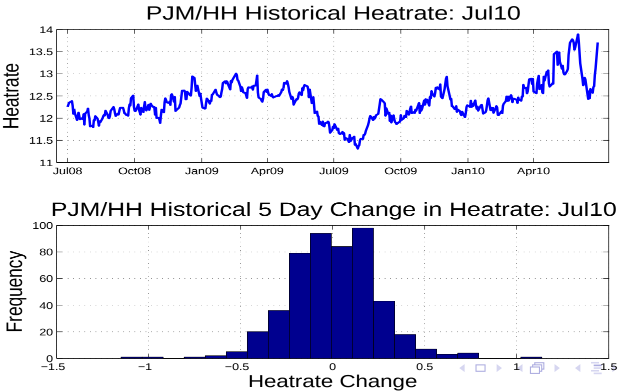
- This has implications for hedging.
  - The fact that you can only trade 50MW limits your ability to capture the modeled extrinsic value.
  - This limitation is an issue in all markets, but particularly so for power and natural gas.
  - The following plot shows the change in delta (converted to MW) as a function of a change in heatrate on the trade date.



Hedging Frictions

Block-Size Impact on Hedging

- How much do heatrates actually move?
  - The following plot shows heatrate history for the Jul10 contract as well as the distribution of 5 day heatrate changes.
  - The conclusion is that delta hedging a 400 MW toll is not going to be a very fruitful enterprise.



Daily Tolls

Working Problem

- Most tolls trade for tenors of 5-7 years.
  - This is well outside the liquidity window at which individual contract months trade.
  - The impact on hedge performance can be non-trivial—but has yet to estimated rigorously.
- In what follows we will switch to the following toll:
  - Pricing date: 11Jan2010
  - Underlyings are PJM Western Hub 5x16 and Henry Hub natural gas.
  - Heatrate: 8.0
  - Delivery period: 01Jan11 to 31Dec11
  - Standard exercise: '-1b'.
  - Notional: 400 .
- Note: A 2-factor Gaussian exponential framework was used for each with correlations specified to be roughly consistent with implied correlations.

Daily Tolls

Multi-Factor Approach

- In the absence of quoted markets for heatrate options, one might consider using estimated correlations as a benchmark.
- In the two-factor framework:

$$\frac{dF_k(t,T)}{F_k(t,T)} = \sum_{j=1}^{2} \left[\int_0^t \sigma_j^{(k)}(T) e^{-\beta_j (T-t)} dB_j^{(k)}(s) \right]$$

  where

  - $k = 1$ corresponds to power.
  - $k = 2$ corresponds to natural gas.

- Factor correlations:
  - $\mathrm{corr}\left[dB_1^{(1)}, dB_1^{(2)}\right] = \rho_{\mathrm{long}}$.
  - $\mathrm{corr}\left[dB_2^{(1)}, dB_2^{(2)}\right] = \rho_{\mathrm{short}}(t)$.

Daily Tolls

Multi-Factor Approach

- The terms "monthly vol" ($\bar{\sigma}_M$) and "daily vol" ($\bar{\sigma}_D$) will always refer to implied vols pertaining to the fixed strike options.
- Market dichotomy:
  - Power: Monthly and daily fixed strike options are the vanilla options.
  - Natural Gas: Monthly and the forward starter are the vanilla options.
- Calibration of the model above for each commodity involves setting $\{\sigma_j^{(k)}(T)\}_{j=1}^2$ to be consistent with $\bar{\sigma}_M^{(k)}$ and $\bar{\sigma}_D^{(k)}$ by contract month.

Daily Tolls

Spot Volatility

- Recall the caricature of spot (daily) prices processes with returns for the daily spot prices i.i.d. normal:

$$F(t,t) = F_m(T_m)e^{\zeta Z_t - \frac{1}{2}\zeta^2}$$

where

  - F_m denotes the contract month containing day t
  - Z_t is standard normal.
  - ζ is the spot volatility.

Daily Tolls

Valuation

- In this modeling framework each of the two underlying commodities is log-normally distributed:
  - Valuation eventually leads to Margrabe/quadrature.
  - The term correlation can be explicitly calculated analytically directly from the two-factor diffusions.

- In the caricature model:
  - Each underlying has a log-normal distribution with volatilities $\bar{\sigma}_{D,k}$ which satisfies:

$$\bar{\sigma}_{D,k}^2 T_D = \bar{\sigma}_{M,k}^2 T_M + \zeta_k^2$$

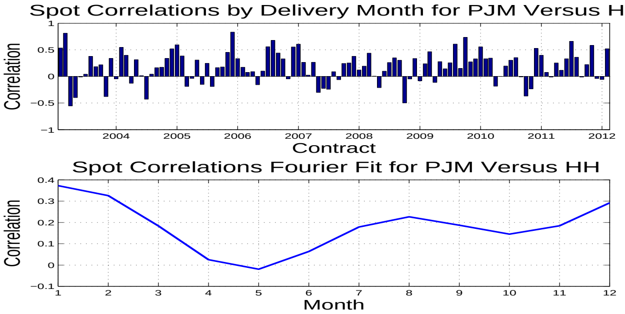
  - The term-correlation is given by:

$$\rho_{\mathrm{term}}\left(T_D\right) = \frac{\bar{\sigma}_{M,1}\bar{\sigma}_{M,2} T_M \rho_L + \zeta_1 \zeta_2 \rho_S}{\bar{\sigma}_{D,1}\bar{\sigma}_{D,2} T_D}$$

Daily Tolls

Valuation

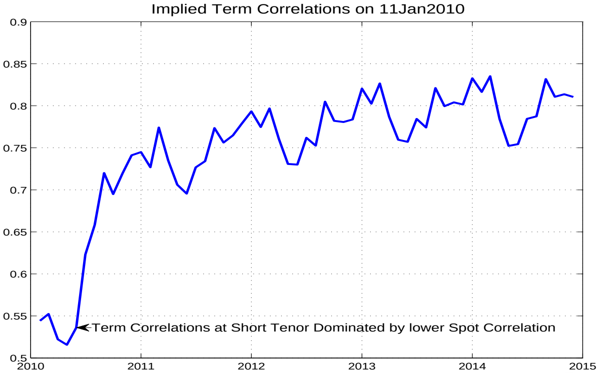
- Note the seasonality:
  - Higher correlation in winter months due to gas price spikes being the driver of power prices.
  - Weather drives summer price dynamics, independently of fuel prices.



Daily Tolls

Valuation

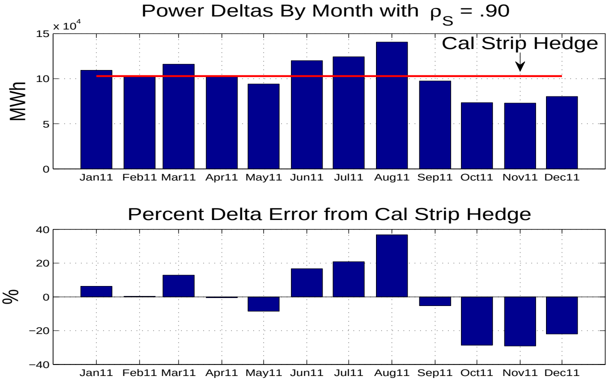
- Using ρ_S from statistical estimate yields the following term structure of correlation.
- Trading activity suggests implied values of ρ_S that are *much* higher than these.
- Why?



Hedging Frictions

Impact of Strips

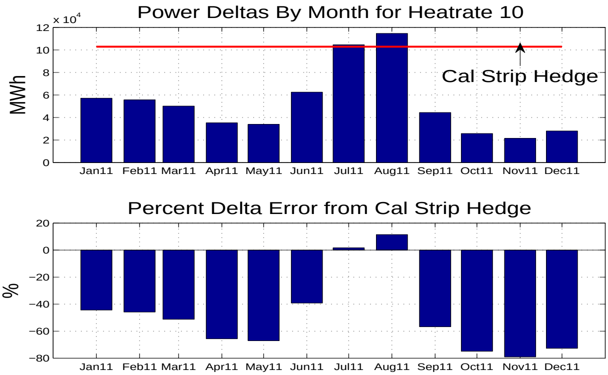
- The following figure shows deltas by month as well as the total cal strip delta, both in absolute terms as as a percentage difference for our toll.
- Note the nontrivial variation from the cal strip quantity.



Hedging Frictions

Impact of Strips

- The problem is exacerbated at higher heatrates, which "pushes" the delta into fewer months making the cal strip instrument an even more blunt instrument.
- The following figure shows the same results for an $H_* = 10$ heatrate.



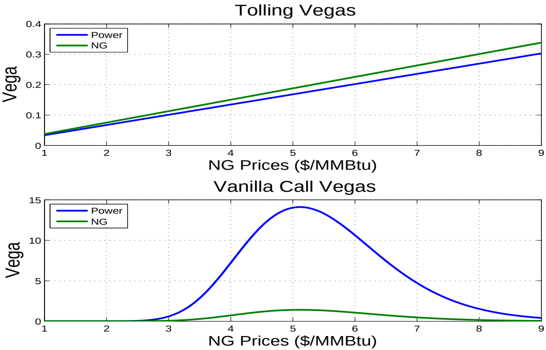
Hedging Frictions

Mismatch in Greek Behavior

- Hedging gamma and vega exposure for tolls is nontrivial:
  - Since vanilla options have a single expiration convention, with expiry near delivery, you have to choose between vega and gamma hedging.
  - Typically you choose to control gamma at shorter tenors and vega and longer tenors.
- The behavior of vega arising from tolling structures is different than that of the vanilla products.
  - The following figures show vega for a sample 8HR toll with tenor 1Y.
  - The underlying prices were $50/MWh and $5/MMBtu.
  - The vols were .50 for each leg and correlation was set at .90.

Hedging Frictions

Mismatch in Greek Behavior



- The top figure shows the power and gas vegas for the toll across a range of gas prices assuming that the power price remains at the market heatrate (10).
- The lower figure shows the same for ATM vanilla calls.
- Note the meaningfully different behavior both in slope and in magnitude.
- Costly rebalancings of vega hedges would be required to maintain neutrality.

Hedging Frictions

Spurious Risks

- Use of this approach results in exposures to volatility and correlations that can large and unmanagable.
- Are these induced by the choice of model?
- Rephrased:
  - Under this modeling paradigm are changes in volatilities and correlations (if liquidity permitted observation and calibration) off-setting?
  - Value is very dependent on the spot volatility which is driven by the spread between $\bar{\sigma}_D$ and $\bar{\sigma}_M$. Is this real?
  - The absence implied correlation data of a quality analogous to implied vols renders this unanswerable.
- The hedging program that follows is highly questionable.

Hedging Frictions

Spurious Risks

- Vega exposure is not confined to $\bar{\sigma}_D$ for the two commodities.
- The resulting vegas with respect to $\bar{\sigma}_M$ and $\bar{\sigma}_D$ for each of the commodities is largely driven by the implicit correlation effect.

$$\frac{\partial V}{\partial \bar{\sigma}_D} = \frac{\partial V}{\partial \bar{\sigma}_D}|_\rho + \frac{\partial V}{\partial \rho}|_{\bar{\sigma}_D} \frac{\partial \rho}{\partial \bar{\sigma}_D}$$

- The term correlation effect for vega hedging results in substantial positions:
  - Long/short or (short/long) positions in monthly/daily vega depending on relative values of the various component volatilities.

Hedging Frictions

Spurious Risks

- Note that the spread vol that enters Margrabe is:

$$\hat{\sigma}^2 = \left[\bar{\sigma}_{M,1}^2 + \bar{\sigma}_{M,2}^2 - 2\rho_L \bar{\sigma}_{M,1}\bar{\sigma}_{M,2}\right] T_M + \left[\zeta_1^2 + \zeta_2^2 - 2\rho_S \zeta_1 \zeta_2\right]$$

  from which we see that:

$$\frac{1}{2}\hat{\sigma}\frac{\partial \hat{\sigma}}{\partial \bar{\sigma}_{M,1}} = \left[\bar{\sigma}_{M,1} - \rho_L \bar{\sigma}_{M,2}\right] T_M + \frac{\partial \zeta_1}{\partial \bar{\sigma}_{M,1}} \left[\zeta_1 - \zeta_2 \rho_S\right]$$

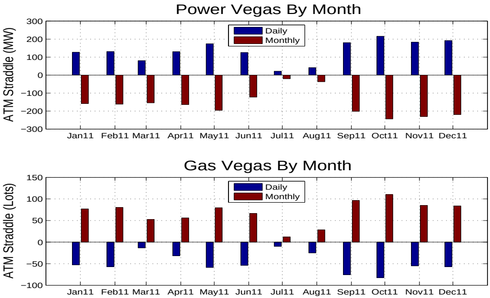
- The sign of the first term on the RHS is exactly like the monthly vega terms in the previous monthly example.

- We know that $\frac{\partial \zeta_1}{\partial \bar{\sigma}_{M,1}} < 0$ and $\frac{\partial \zeta_1}{\partial \bar{\sigma}_{D,1}} > 0$ and similarly for all other permutations of vols and commodity leg.

Hedging Frictions

Spurious Risks

- The figure below shows initial vega exposures in our example.
  - Implied vol hedging strategy involves substantial volume in monthly and daily options in both commodities.
  - Moreover, these hedges would also have to be unwound $t \uparrow T$.



Vol-Lookup Approximation

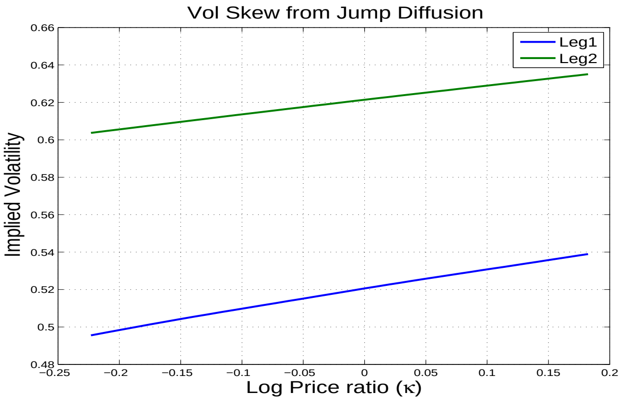
Simple Experiment

- How much error is sustained in the vol-lookup approximation.
- The following analysis considers a reasonable setup:
  - A spread option with tenor of $\frac{1}{4}$ of a year.
  - Power and gas forwards of \$100 and \$10 respectively
  - Heatrates H_* are varied on the interval $[8, 12]$.
- The procedure:
  - N=100,000 standard i.i.d normal deviates $\bar{Z}_n$ where each $\bar{Z}_n \in \Re^2$
  - Poisson jumps with arrival rate 8/year and size 2 were added to $Z_{n,1}$.
  - The $\{\bar{Z}_n\}_{n=1}^N$ were normalized to unit standard deviation and transformed to yield $\bar{X}_n$ with correlation .90.
  - These were normalized to have a standard deviation corresponding to implied vols of .50 and .60 respectively.

Vol-Lookup Approximation

Simple Experiment

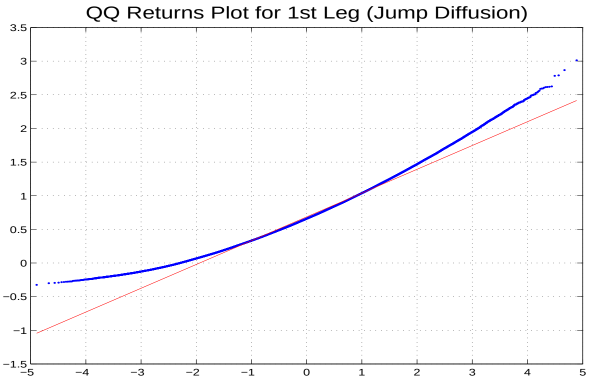
- The following plot shows the skew for the two legs.



Vol-Lookup Approximation

Simple Experiment

- The qq-plot of the returns is shown below.



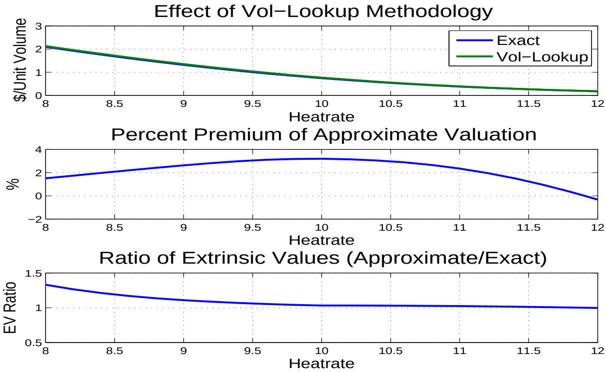
Vol-Lookup Approximation

Simple Experiment

- The following plot shows the results comparing exact (simulation) valuation to the vol-lookup methods as a function of the deal heatrate.
  - The Monte-Carlo error is typically well under $0.10.
  - The vol-lookup approximation is roughly 2% higher at low strikes.
  - The third plot shows the ratios of extrinsic values which is more ominous, with low heatrates seeing ratios above 1.5.

Vol-Lookup Approximation

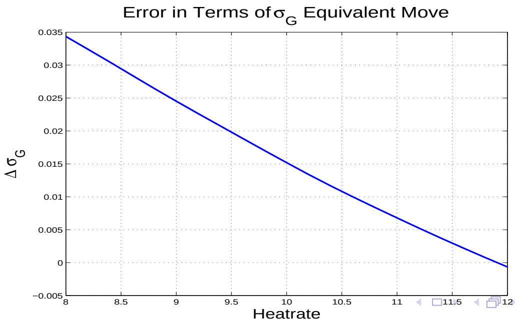
Simple Experiment



Vol-Lookup Approximation

Simple Experiment

- Vol Perspective:
  - Converting this difference in extrinsic value into a σ_G equivalent move by dividing the difference by the G-vega yields the following.
  - The errors can be meaningfully outside of the volatility bid-offer.
  - This has been a subject of some investigation (see C. Alexander and A Venkatramanan 2011).
  - However, a rigorous and efficient methods for bounding this error remain undeveloped.



Modeling Alternatives

Movitations

- The substantial difficulties in hedging heat rate options calls into question the entire modeling framework—arguably even the relevance of risk-neutral pricing methodologies.
- It is typical to proceed with methodologies similar to those discussed above, simply cranking the correlation up to levels which either:
  - Are consistent with what trading activity is observed.
  - Result in realized payoffs that dominate theta bleed.
- Is this a reasonable thing to do?

Modeling Alternatives

Heatrate Distributions

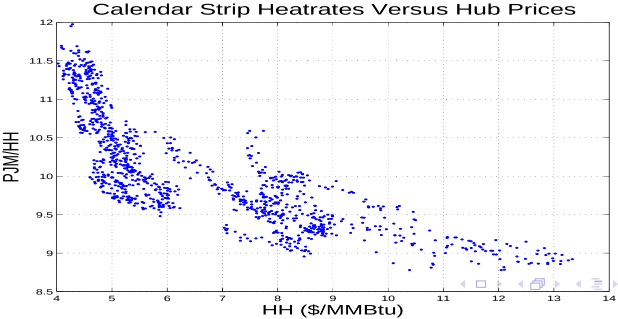
- Rolling calendar strip heatrates versus natural gas prices.
- Note the systematic decrease in heatrates as NG prices increase.
  - Bidding behavior of NG power generation:

    $$p_{\text{bid}} = H_* G(t,t) + K$$

    which would suggest:

    $$H(t,T) = H_* + K/G(t,T)$$

  - Switching: $G(t,T) \uparrow$ means cheaper sources of generation on the margin.
  - It also explains why monthly power vols are lower than for natural gas.



Modeling Alternatives

Heatrate Distributions

- What are the implications of the two-factor modeling heatrate distributions?
- For any fixed time t let:

$$
\begin{aligned}
\tilde{\sigma}_F &= \sigma_F t^{\frac{1}{2}} \\
\tilde{\sigma}_G &= \sigma_G t^{\frac{1}{2}}
\end{aligned}
$$

  where σ_F and σ_G are volatilities of the two underlying forward prices.

- We know that:

$$
H(t,T) \equiv \frac{F(t,T)}{G(t,T)} = \frac{F(0,T)}{G(0,T)} e^{\tilde{\sigma}_F Z_F - \frac{1}{2}\tilde{\sigma}_F^2 - \tilde{\sigma}_G Z_G + \frac{1}{2}\tilde{\sigma}_G^2}
$$

  where:

  - The normal deviates corresponding to the two underlyings at time t are Z_F and Z_G respectively.
  - The correlation between the two deviates is ρ.
  - For time-varying local volatility these are the implied volatilities for $[0, t]$.

Modeling Alternatives

Heatrate Distributions

- Writing: $Z_F = \rho Z_G + \sqrt{1-\rho^2}W$ where W is independent of Z we have:

$$H(t,T) = \frac{F(0,T)}{G(0,T)}\left[e^{\tilde{\sigma}_F\sqrt{1-\rho^2}W - \frac{1}{2}(1-\rho^2)\tilde{\sigma}_F^2}\right]\left[e^{(\rho\tilde{\sigma}_F - \tilde{\sigma}_G)Z_G - \frac{1}{2}(\rho^2\tilde{\sigma}_F^2 - \tilde{\sigma}_G^2)}\right]$$

- The heatrate $H(t,T)$ is not in general a martingale since the third term has an expected value:

$$\begin{aligned}\tilde{E}\left[e^{(\rho\tilde{\sigma}_F - \tilde{\sigma}_G)Z_G - \frac{1}{2}(\rho^2\tilde{\sigma}_F^2 - \tilde{\sigma}_G^2)}\right] &= e^{\frac{1}{2}\left[(\rho\tilde{\sigma}_F - \tilde{\sigma}_G)^2 - (\rho^2\tilde{\sigma}_F^2 - \tilde{\sigma}_G^2)\right]} \\ &= e^{\tilde{\sigma}_G(\tilde{\sigma}_G - \rho\tilde{\sigma}_F)}\end{aligned}$$

- Using the fact that:

$$G(t,T) = G(0,T)e^{\tilde{\sigma}_G Z_G - \frac{1}{2}\tilde{\sigma}_G^2}$$

we have:

$$Z_G = \frac{1}{\tilde{\sigma}_G}\log\left[\frac{G(t,T)}{G(0,T)}\right] + \frac{1}{2}\tilde{\sigma}_G$$

Modeling Alternatives

Heatrate Distributions

- Putting this all together we have:

$$\begin{aligned} H(t,T) &= \frac{F(0,T)}{G(0,T)}\left[e^{\tilde{\sigma}_F\sqrt{1-\rho^2}W-\frac{1}{2}(1-\rho^2)\tilde{\sigma}_F^2}\right] \\ &\quad \left[e^{(\rho\tilde{\sigma}_F-\tilde{\sigma}_G)\left(\frac{1}{\tilde{\sigma}_G}\log\left[\frac{G(t,T)}{G(0,T)}\right]+\frac{1}{2}\tilde{\sigma}_G\right)-\frac{1}{2}(\rho^2\tilde{\sigma}_F^2-\tilde{\sigma}_G^2)}\right] \\ &= \frac{F(0,T)}{G(0,T)}\left[e^{\tilde{\sigma}_F\sqrt{1-\rho^2}W-\frac{1}{2}(1-\rho^2)\tilde{\sigma}_F^2}\right]e^{\frac{1}{2}\rho\tilde{\sigma}_F(\tilde{\sigma}_G-\rho\tilde{\sigma}_F)}\left[\frac{G(t,T)}{G(0,T)}\right]^{\left(\rho\frac{\tilde{\sigma}_F}{\tilde{\sigma}_G}-1\right)} \end{aligned}$$

- This establishes that under the Margrabe paradigm $H(t,T)$ is related to $G(t,T)$ functionally as:

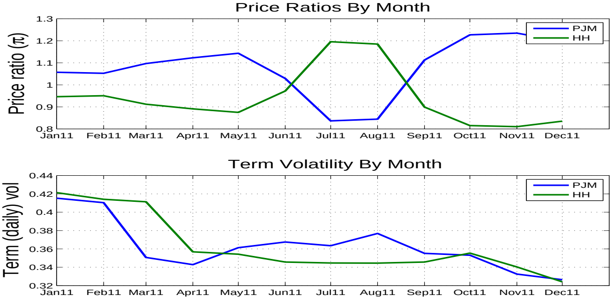
$$H(t,T) = cXG(t,T)^{\left(\rho\frac{\tilde{\sigma}_F}{\tilde{\sigma}_G}-1\right)}$$

where c is a constant and X is a unit mean log-normal random variable and "returns" variance $\tilde{\sigma}_F\sqrt{1-\rho^2}$.

Modeling Alternatives

Heatrate Distributions

- For $H(t, T)$ to be a decreasing function of $G(t, T)$ requires that: $\rho \frac{\tilde{\sigma}_F}{\tilde{\sigma}_G} < 1$.
- Issues:
  - In many months $\sigma_F > \sigma_G$.
  - Increasing ρ to near unity causes a breakdown in "known" behavior.
- Note the potential problems with the $H_* = 10$ toll.



Modeling Alternatives

Heatrate Distributions

- Key Points:
  - Simply cranking the term correlation up does not result in realistic distributions in many cases.
  - The liquidity in daily options for power is highly concentrated near-the-money.
  - For natural gas, liquidity in daily options is very limited in both strike and tenor.
  - Using "marked" vols is often little more than feeding "Curious George Draws a Vol Surface" into a multi-factor model.
- Some practitioners and researchers have gravitated to alternatives.

Modeling Alternatives

Econometric Models

- The goal is to generate a physical measure of all related prices and then either:
  - Construct hedges and calculate value directly.
  - Transform the distribution to be consistent with market tradables.
- Consider at regression form:

$$\log\left[\frac{p(d)}{p_{\mathrm{NG}}(d)}\right] = \alpha + \gamma p_{\mathrm{NG}}(d) + \sum_{k=1}^{K} \theta(d)^k + \epsilon_d$$

where:

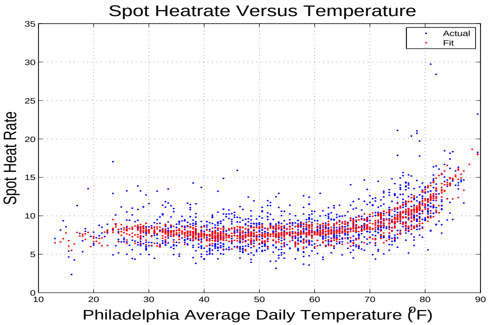
- The modified temperature is: $\theta(t) = \frac{e^{\lambda(t)}}{1+e^{\lambda(t)}}$ with $\lambda(t) \equiv \frac{\tau(t)-\tau_{\mathrm{ref}}}{w}$.
- Here τ_{ref} and w selected to be characteristic mean and width of temperatures realized over the entire data set.
- The fact that $\theta \in [0,1]$ results in regular behavior beyond the range of historical data.

Modeling Alternatives

Econometric Models

- The result is shown below.
- Coupled with:
  - Temperature simulations (more on this later).
  - Simulations for p_{NG}

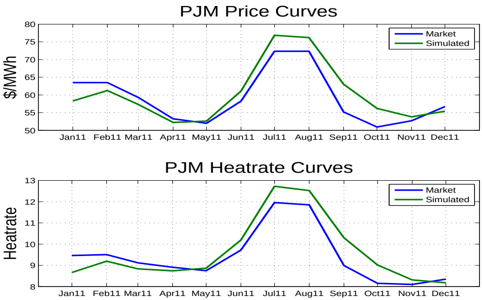
yields a joint distribution for: $\vec{\pi} \equiv [\tau, p_{\mathrm{NG}}, p]$.



Modeling Alternatives

Econometric Models

- Expected values of simulated prices need not equal forward prices.
  - The simulations are physical measure.
  - The difference is an estimate of a risk premium.



Modeling Alternatives

Econometric Models

- Valuation process:
  - Generate simulations for $\vec{\zeta}(d) \equiv [\tau, p_{\mathrm{NG}}, p](d)$.
  - Compute the simulated payoffs of:
    - The asset or trade in question: Π_A.
    - The set of potential hedges: $\bar{X}$
  - Compute an optimal static (initial) hedge—for example:

$$\vec{h}_* \equiv \underset{\vec{h}}{\operatorname{argmin}} \operatorname{var}\left[\Pi + \vec{h}^\dagger \vec{X}\right]$$

  - The resulting hedge portfolio: $\Pi_* \equiv \Pi + \vec{h}_*^\dagger \vec{X}$ usually has a non-trivial probability distribution.
  - The mid-price of the structure Π is (arguably) the sum of the expected residual and the market prices of the hedges $\vec{p}_X$.

$$E(\Pi_*) + \vec{h}_*^\dagger \vec{p}_X$$

  - Bid/offer can be constructed from the probability distribution (e.g. percentiles or standard deviations from the mean).

Modeling Alternatives

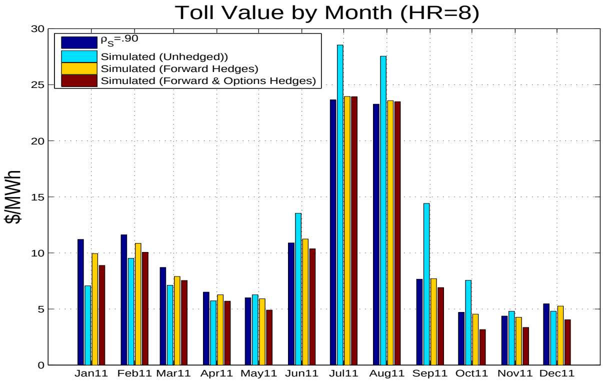
Econometric Models

- Valuation process:
  - If the structure payoff Π is spanned by the hedges then the resulting value is the usual mark-to-market.
  - Example: Suppose Π is just the payoff of a power swap $\Pi = Np(d)$.
    - The optimal hedge is $h_* = [0, 0, -N]$ ("sell N of the power swap").
    - The resulting variance of Π_* is zero (this is a perfect hedge).
    - If the forward power price is F then the value of Π is $N \cdot F$.
    - This approach yields the proper mark-to-market value if Π can be constructed from available hedging instruments.

Modeling Alternatives

Econometric Models

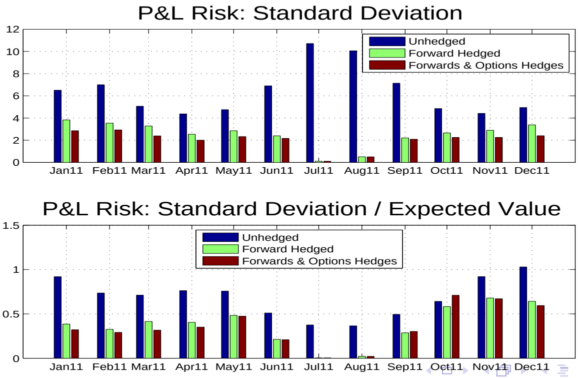
- Valuation process:
  - Returning to our working problem, the following figure shows results using forwards and forwards+ATM options as hedges by month.



Modeling Alternatives

Econometric Models

- Valuation process:
  - The following figure shows the risk reduction in the various scenarios.
  - The effect of options hedges on the residual risk was minimial.
  - Fixed strike options are not particularly useful at hedging tolls.



Modeling Alternatives

Econometric Models

- Valuation process:
  - The unit heatrate H_* is relevant—the higher the heatrate the less "swap-like" the toll.
  - The results can differ meaningfully from "standard" methods.

Heat rate	Financial	Simulated Unhedged	Simulated Forwards	Simulated Forwards & Options
$H_* = 8$	10.37	11.28	10.01	9.27
$H_* = 10$	4.06	4.16	3.24	2.43

Modeling Alternatives

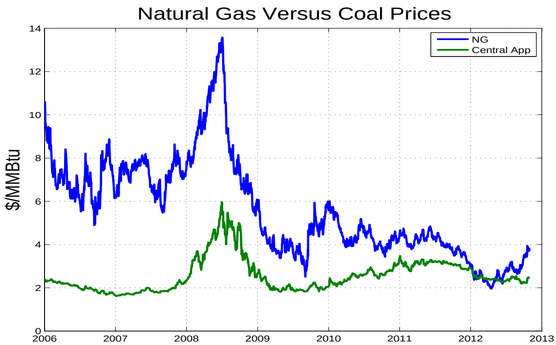
Econometric Models

- Strengths:
  - Construction of (arguably) realistic price processes and effective static hedges.
  - The use of static hedges underestimates the value in theory, but given hedge frictions discussed previously this is more of a "feature" than a "bug."
  - The residual post-hedging risk can be quantified and used to construct bids and offers.
- Weaknesses:
  - Dynamic hedging reflected only implicitly if options structures are included in the hedge basket.
  - Vulnerable to systemic changes / nonstationarity; e.g. coal switching, regulatory changes).
- Structural (stack) models are an alternative.

Modeling Alternatives

Stack Models: Motivations

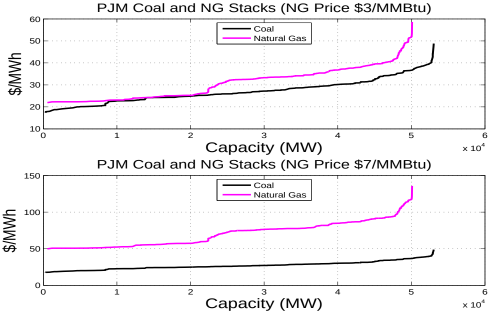
- Consider the following comparison of historical natural gas versus coal prices.
  - The structural drop in NG prices has put the traditionally more expensive combined cycle generators near parity with coal plants.
  - The traditional segregation of the two types of units, which are very different in attributes, is no longer a given.



Modeling Alternatives

Stack Models: Motivations

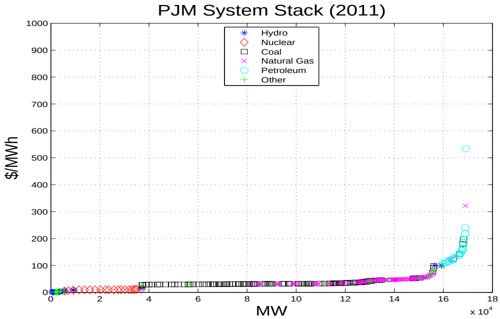
- The change in the "merit order" of the two sub-stacks for two price regimes is shown below.
  - The competition between the flexible natural gas units and the baseload coal units is likely to have a meaningful effect on heatrate behavior and correlation structure.
  - How can/should this be modeled?



Modeling Alternatives

Stack Models

- The Stack: Marginal cost of generation versus total capacity:
  - Capacities $[C_1, \ldots, C_N]$ sorted in increasing cost (\$/MWh) $[p_1, \ldots, p_N]$, the stack is a plot of p_n vs $\sum_{1 \le k \le n} C_k$.
  - The result is $p_{\mathrm{MC}} \approx \Phi\left[C | \bar{F}_t\right]$.
    - p_{MC} is the marginal cost utilizing total capacity C.
    - We have explicitly identified fuel dependence.



Modeling Alternatives

Stack Models

- The Basic Modeling Tenant:
  - Given a load (demand) L_t at time t and a version of the stack Φ_t at time t the spot price is:

$$p_t = \Phi_t \left[L_t(1+\delta_t) | \bar{F}_t \right] + \epsilon_t$$

    - δ are random variables reflecting uncertain availability (outages).
    - ϵ are random variables reflecting randomness in bidding.
    - Φ_t is time dependent and is probably not Φ (full availability).
  - Issues:
    - Calibration without over parameterizing the problem.
    - The power system is a grid with high-dimensional optimization setting locational prices; stack models are "stylized representations" of such intending to capture the "essence" of spot price phenomena.
  - Perceived advantages:
    - The distribution of p_t depends upon fuel prices in a sensible way.
    - Changes to the stack (new builds or retirements) are sensibly extrapolated into changes in the the distribution of p_t.

Modeling Alternatives

Stack Models

- Create and maintain a detailed database of the "region" in question.
- Calibration:
  - Assume parametric forms for δ and ϵ.
  - Generate the historical stack on a daily basis using prevailing fuel prices.
  - Assume a functional form for the seasonal availability of generation; e.g;

$$C_i^{\text{actual}}(t) = C_i \Psi(t)$$

where

$$\Psi(t) = \sum_{k=1}^{K} [\gamma_k \sin(2\pi kt) + \delta_k \cos(2\pi kt)]$$

  - Assumes all generation types sustain the same seasonal availability.
  - The form is then:

$$\Phi_t \left[C | \bar{F}_t \right] = \Phi \left[\frac{C}{\Psi_t} | \bar{F}_t \right]$$

  - Set the free parameters via MLE.

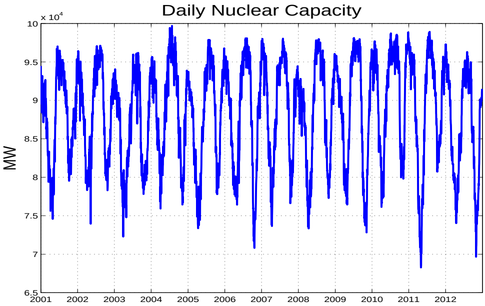
Modeling Alternatives

Stack Models

- You now have a model under "physical" measure for spot price dynamics:

$$p_t = \Phi\left[\frac{L_t(1+\delta_t)}{\Psi_t}|\bar{F}_t\right] + \epsilon_t$$

- Why incorporate $\Psi(t)$?
  - The following figure shows available U.S. nuclear capacity.
  - Maintenance follows lower seasonal demand in "shoulder" months.



Modeling Alternatives

Stack Models

- Two approaches: (as with econometric models)
  - Risk neutral: Create a risk-neutral measure by adjusting parameters and/or the distribution of L_t to hit market forwards and options prices.
    - Pricing is via Monte Carlo and is believed (by some anyway) to yield realistic distributions.
    - This does not resolve the issue of limited ability to hedge (recall peaking options).
  - Physical: Use the physical measure to construct minimum-variance hedges using instruments that trade now ($t = 0$).

Modeling Alternatives

Stack Models

- Simplified Stack Models:
  - The approach above requires extensive data—arguably too much detail.
  - An alternative: Each type of generation is its own "sub-stack" which can be represented analytically.
  - Stack Arithmetic:
    - Suppose that we group each generation by input fuel and represent the "sub-stacks" by $\Phi_j(C|F_j)$. Then:

$$\Phi^{-1}[p] = \sum_j \Phi_j^{-1}\left[\frac{p}{F_j}\right]$$

    - To see this note that $\Phi^{-1}[p]$ is the total capacity with cost $\leq p$.
    - The above simply totals all generation with cost $\leq p$.
  - Judicious choices of functional forms for Φ_j (notably exponential) can yield analytical tractability and enhanced numerical efficiency.[1]

[1] See, for example, Carmona, Coulon and Schwarz: "A Structural Model for Electricity Prices."

Modeling Alternatives

Stack Models

- Summary of Stack Model Implementation:
  - Build a stack:
    - Unit by unit from a database.
    - Via parametric forms invoking: $\Phi^{-1}[p] = \sum_j \Phi_j^{-1}\left[\frac{p}{F_j}\right]$
  - Generate simulations:
    - Temperature τ_d.
    - Hourly loads $\bar{L}_d$ conditional on τ_d.
    - Simulations for the input fuel prices $\bar{F}(t,T)$.
    - The resulting spot price distribution is obtained from:

$$\bar{p}_d = \Phi\left[\frac{\bar{L}_d(1+\delta_d)}{\Psi_t}|\bar{F}(d,d)\right] + \bar{\epsilon}_d$$

  - This yields a join distribution of $[\tau, L, \bar{F}, \bar{p}]$.
  - Calibration:
    - Physical measure: Deploy the above historically adjusting free parameters to hit realized spot prices.
    - Risk neutral: Adjust free parameters to “hit” market data.
  - Simulation: The joint distribution is used to to price and hedge.

Modeling Alternatives

Stack Models

- State-of-Affairs:
  - Building stack models from actual generation databases in a given region/ISO/zone yields models that:
    - Are difficult to calibrate.
    - Ignore the network nature of actual power systems.[2]
  - "Caricature" stack models have substantial advantages:
    - The relative tractability renders calibration potentially viable.
    - Arguably remains a "work in progress."

[2] The computational requirements for models that attempts to capture the entire system currently preclude calibration to historical weather, load and price data or forward market data.

Conclusions

- Multi-factor models applied to daily tolls in the risk-neutral setting have issues:
  - Spurious risk due to spot correlation effects.
  - Hedging programs that are difficult to affect.
  - Dependence on volatilities and correlations that are often not traded.
- This has spawned development alternatives:
  - Econometric models:
    - Calibrated to historical price behavior
    - Ostensibly more realistic price distributions.
    - Valuation can yield estimates of hedging slippage.
    - Difficult to embed anticipated systemic changes.
  - Structural models:
    - Intended to accommodate anticipated systemic changes.
    - Challenging to implement—still a work in progress.

On Deck

- Variable Quantity Swaps
- Natural Gas Storage