Part I: Correlation Risk and Common Methods

Glen Swindle

August 6, 2013

Outline

- Origins of Correlation Risk in Energy Trading
- Basic Concepts and Notation
- Temporal Correlation
  - Stack-and-Roll Hedging
  - Forward Yields, Inventory and Forward Dynamics
- Common Modeling Approaches
  - Comment on Spread Options
  - Reduced Form Models
  - Econometric and Structural Models

Origins

Correlation Risk—Hedging

- Correlation risk most commonly arises in practice in hedging simple illiquid positions with simple liquid positions.
  - Stack-and-roll hedging (Temporal):

    Long-tenor risk hedged with short-tenor instruments.
  - Basis (Locational):

    Hedging risks at illiquid delivery locations with liquid delivery locations.
  - Cross-commodity (Conversion):

    Hedging commodities with limited liquidity in swaps markets with closely related proxies having liquid markets.

Origins

Correlation Risk—Assets and Structured Transactions

- Most physical assets and structured commodities hedges of such involve the concept of transforming one commodity/delivery location/delivery time to another.
  - Storage (Temporal):

    Storing a commodity "now" for delivery "then."
  - Transport (Locational):

    Moving a commodity from a supply source to a demand sink.
  - Refining and Generation (Conversion):

    Transforming one commodity to another commodity or set thereof.

Origins

Correlation Risk—Customer Demand

- Demand/Price Risk
  - Demand for a quantity can vary over both short and long time-scales creating inherent correlation risks to the commodities supply chain.
  - Serving customers involves a settlement payoff of:

$$\sum_n \left[\bar{D}_n \left(p_f - p_n\right) + \left(D_n - \bar{D}_n\right)\left(p_f - p_n\right)\right]$$

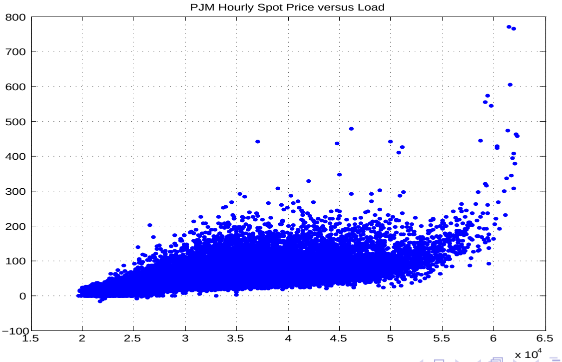
where:

- D_n is the demand in time period n.
- $\bar{D}_n$ is the expected demand;
- p_n is the spot price for period n.
- p_f is the fixed (contract) price.

Origins

Correlation Risk—Customer Demand

- Demand/Price Risk
  - Correlation between demand and price is always against the holder of the short position.
  - The following plots shows hourly spot power prices versus demand in PJM.



Origins

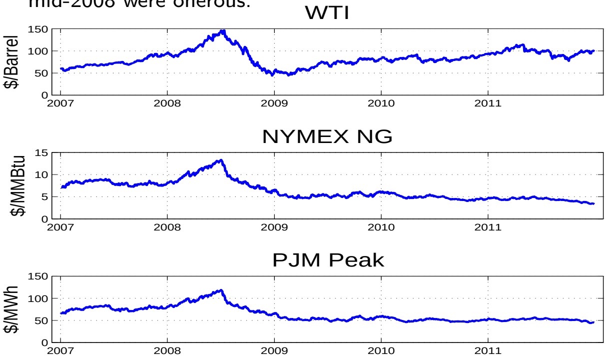
Correlation Risk—Commercial Operations

- Liquidity and Credit Risk
  - Whether exchange traded or OTC, hedging activities are usually accompanied by colateral posting requirements.
  - "Macro" relationships between demand and price on long time scales can cause substantial mismatches in colateral posting terms.
  - Example: Retail energy companies
    - Provide commodities to retail end-users (who typically are not margined)
    - Hedge this inherent short position via standard futures or OTC swaps markets (which are margined)
    - This mismatch in credit support can result in lethal colateral calls in highly volatile times.

Origins

Correlation Risk—Commercial Operations

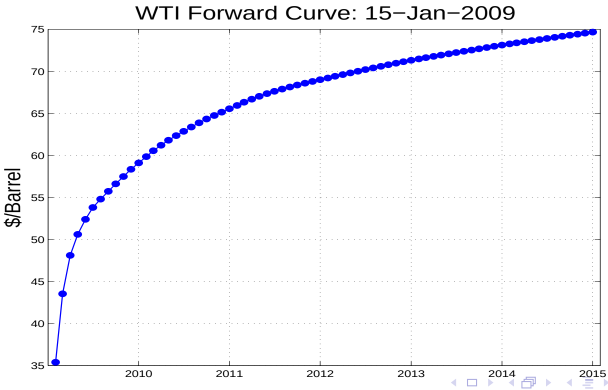
- Liquidity and Credit Risk
  - The following plots shows the rolling cal strip for NYMEX WTI, NG and PJM power prices.
  - The colateral calls against entities with long energy hedges put on in mid-2008 were onerous.



Basic Concepts and Notation

Forward Curves

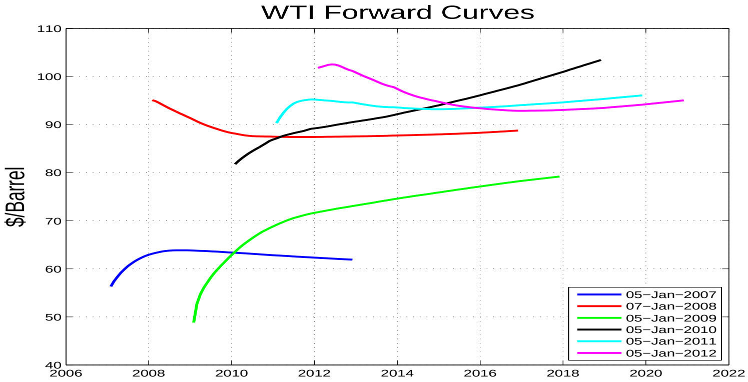
- The primitive underlying for commodities valuation is the term structure of prices for future delivery.
  - This figure shows the forward curve for WTI on 15Jan2009.
  - Each point represents the price for WTI delivered in subsequent months as of this pricing date.



Basic Concepts and Notation

Forward Curves

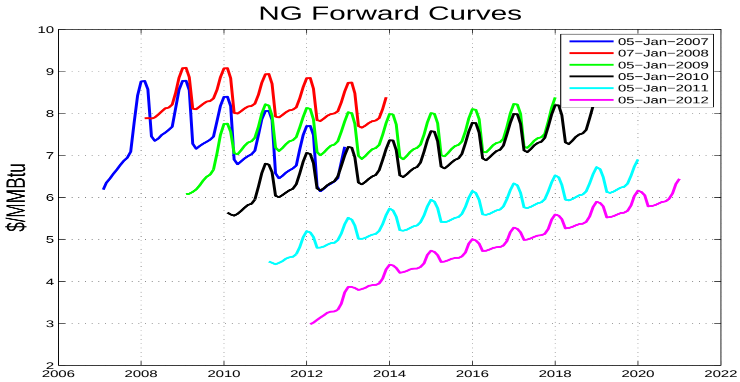
- WTI forward curve at a variety of dates.
  - Note the range of prices as well as the changes in the monotonicity



Basic Concepts and Notation

Forward Curves

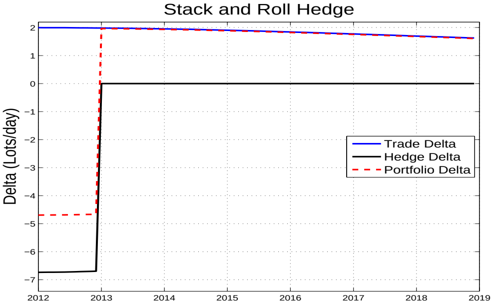
- NG forward curve at a variety of dates.
  - Seasonality is superimposed on macro structure.
  - Note the breakdown from the WTI price levels in recent years.



Temporal Correlation

Stack-and-Roll Hedging

- Consider a long position established on 31May2011 of 2/day (2 lots per calendar day) of Henry Hub natural gas for financial settlement over the term Jan12 to Dec18.
- The delta profile below is (arguably) the optimal Cal12 hedge.



Temporal Correlation

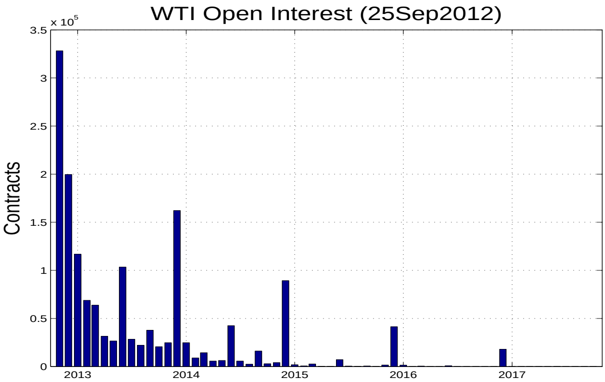
Stack-and-Roll Hedging

- Hedging long-tenor risk with short-tenor positions is referred to as "stack-and-roll" hedging.
  - The short-tenor hedge is "stacked" against the long-tenor positions.
  - As circumstances permit the hedge is "rolled" forward by unwinding the short-tenor position when long-tenor hedges can be established.
- The situation in commodities is fundamentally different than in rates.
  - Rates: Liquidity spans tenors with highly liquid swaps or bonds in 2's, 5's, 10's and 30's.
  - Commodities: Liquidity is always concentrated at short tenors.
- It is hard to neutralize multiple-factors.

Temporal Correlation

Stack-and-Roll Hedging

- WTI open-interest by contract:



Temporal Correlation

Stack-and-Roll Hedging

- Why did we choose to sell -6.75/day of the Cal12 strip?
- Notation:
  - Denote the Cal12 strip by weights $\vec{w}$ corresponding to the exact volumes per month of the calendar strip (1/day).
  - Denote the original 2/day purchase over the seven-year strip by $\vec{W}$.
  - Let $\vec{w}_*$ and $\vec{W}_*$ denote these respective weights multiplied component-wise by the discount factors to the contract settlement dates.
- The minimum variance problem becomes:

$$\min_{\alpha} \operatorname{var}\left[\alpha\left(\vec{w}_*^{\dagger} d\vec{F}\right) + \vec{W}_*^{\dagger} d\vec{F}\right] \tag{1}$$

- The unknown α is the optimal quantity of the Cal12 hedge.

Temporal Correlation

Stack-and-Roll Hedging

- Defining the vectors obtained from component-wise multiplication:

$$\vec{\eta} \equiv \vec{w}_* \vec{F} \qquad \text{and} \qquad \vec{\zeta} \equiv \vec{W}_* \vec{F}$$

the solution is:

$$\alpha = -\frac{\vec{\eta}^\dagger A \vec{\zeta}}{\vec{\eta}^\dagger A \vec{\eta}} \tag{2}$$

where:

- A is the matrix of the returns covariance between the set of contract months spanning the problem.
- The numerator is the covariance between the risk that we want to hedge $\vec{W}_*^\dagger d\vec{F}$ and the risk of the strip that we will use to effect the hedge $\vec{w}_*^\dagger d\vec{F}$.
- The denominator is the variance of the hedge value.

Temporal Correlation

Stack-and-Roll Hedging

- Returns PCA:

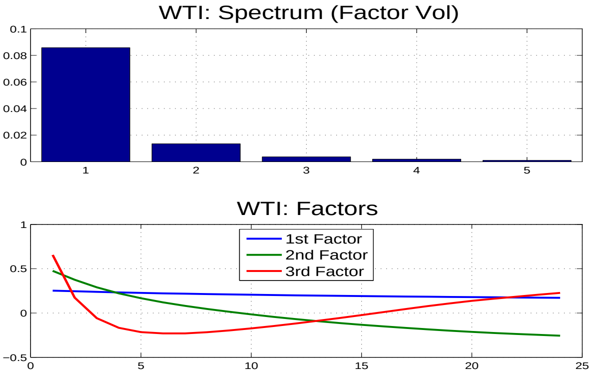
$$r(t, \bar{T}_n) = \sum_j \lambda_j^{\frac{1}{2}} \Phi_j \left(\bar{T}_n - t \right) Z_j(t)$$

  where $\{\lambda_j, \Phi_j\}$ are the eigenvalues/eigenvectors of the covariance matrix of the returns series.

- The next figure shows the results from PCA analysis of returns of the first 36 nearby series 2002 through 2012.
  - The top plot shows $\sqrt{\lambda_j}$ to put things into factor standard deviations.
  - The first two factors comprise over 99% of the variance.
  - The first factor decays with tenor as expected; the second factor also exhibits the standard structure of having one sign change, this occuring at a tenor of approximately 1 year.

Temporal Correlation

Stack-and-Roll Hedging



Temporal Correlation

Stack-and-Roll Hedging

- If we assume that the first PCA factor is the *only* driver, the minimum-variance problem reduces to:

$$\alpha = -\frac{\vec{\zeta}^{\dagger} \vec{\Phi}_1}{\vec{\eta}^{\dagger} \vec{\Phi}_1} \tag{3}$$

  where $\vec{\Phi}_1$ is the first PCA factor.

- Equivalently:

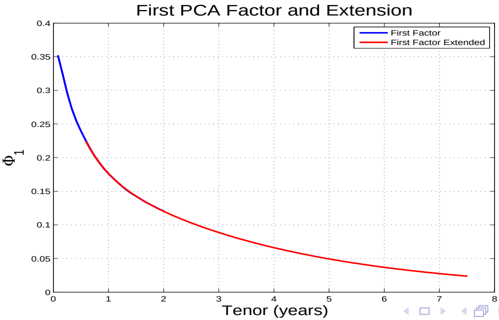
$$0 = \left[\alpha \vec{\eta} + \vec{\zeta}\right]^{\dagger} \vec{\Phi}_1$$

- The result is a portfolio that is orthogonal to the first factor.
- This was the approach that we used to calculate the hedge shown earlier.

Temporal Correlation

Stack-and-Roll Hedging

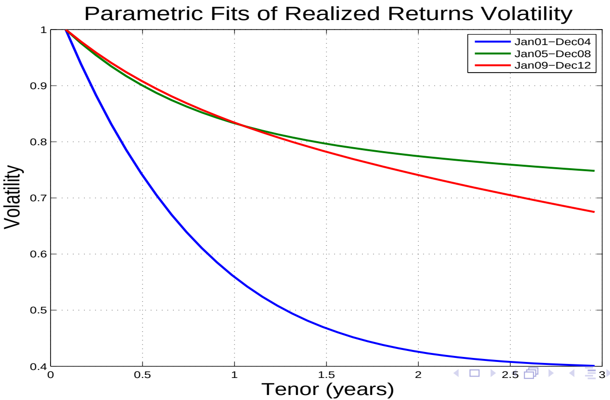
- The PCA results used forwards with tenor of three years (liquid).
- The stack-and-roll problem required extrapolation.
  - Here we used a two-factor exponential fit with the results shown below
  - The extrapolated values were then used for Φ_1
- Key questions:
  - How much risk are we sustaining due to the unhedged higher factors?
  - How stable are the covariance statistics?



Temporal Correlation

Forward Yields, Inventory and Forward Dynamics

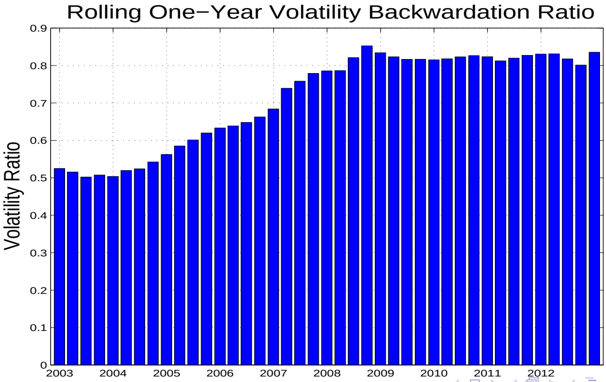
- This fiture shows the L^2 error of a double-exponential fit of empirical returns variance using 10-day returns for three different non-overlapping 4 year intervals with starting dates: Jan2001, Jan2005 and Jan2009.



Temporal Correlation

Forward Yields, Inventory and Forward Dynamics

- The following shows rolling 4 year estimate of the 1 year volatility normalized by the 0 year vol, indexed by end date of estimation interval.
- As this and the previous figures show, the variation in estimated backwardation is significant.



Temporal Correlation

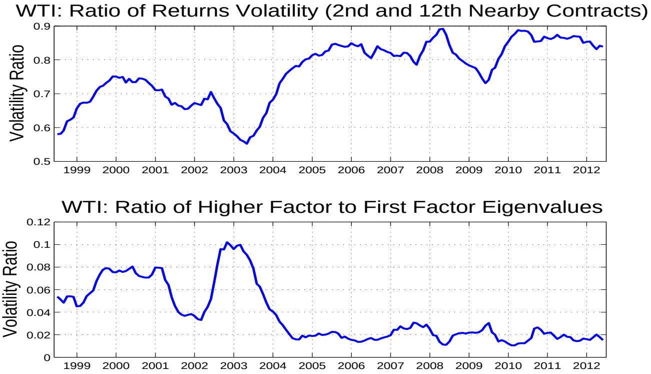
Forward Yields, Inventory and Forward Dynamics

- The dynamics of the WTI forward curve has become increasingly one-dimensional in nature.
- In the next figure:
  - The top plot displays the ratio of the rolling one-year realized volatility for the 12th nearby contract returns to that of the 2nd nearby, with the results plotted versus the mid-point of the calendar averaging window.
  - The lower plot shows the ratio of higher order total volatility to the that of the first factor: $\frac{\left(\sum_{j=2}^{24} \lambda_j\right)^{\frac{1}{2}}}{\lambda_1^{\frac{1}{2}}}$ from PCA analysis of the first 24 nearby contracts, applied over the same rolling one-year windows.

Temporal Correlation

Forward Yields, Inventory and Forward Dynamics

- These phenomena have signficant consequences for hedging strategy construction and model development.



Temporal Correlation

Forward Yields, Inventory and Forward Dynamics

- Forward curves can be viewed as yield curves.
- Forward yield:

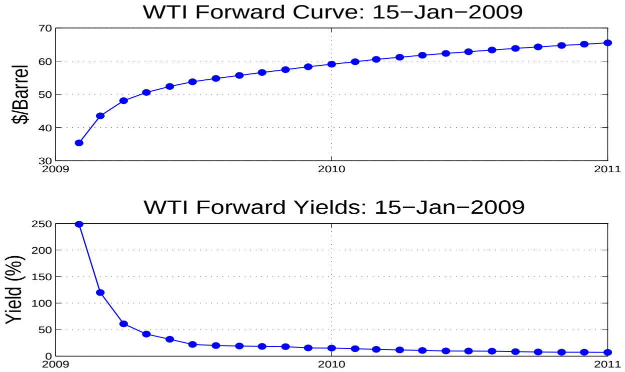
$$y(t, T, T+S) = \frac{1}{S} \log \left[\frac{F(t, T+S)}{F(t, T)} \right]$$

- The forward yield annualized rate implied by borrowing to buy the commodity at time T and sell it at time $T + S$.
- Negative forward yields imply that market participants are willing to pay a premium for earlier delivery
  - This is effectively lending at negative rates.
  - This happens when supply is contrained.

Temporal Correlation

Forward Yields, Inventory and Forward Dynamics

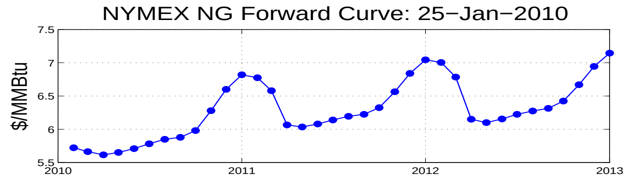
- Yields often exhibit extreme values.
  - The following is the WTI forward curve and forward yield for $S =$ one month in early Jan2009.

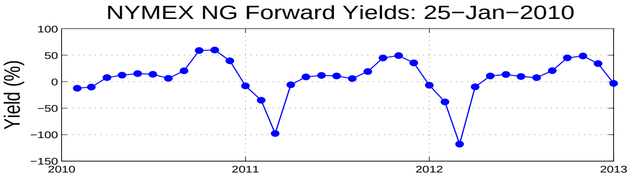


Temporal Correlation

Forward Yields, Inventory and Forward Dynamics

- Seasonality yields negative forward yields consistently for seasonal commodities.





Temporal Correlation

Forward Yields, Inventory and Forward Dynamics

- For a consumption commodity arbitrage arguments yield an inequality:

$$F(t, T) \leq F(t, t)e^{[r(t,T)+q(t,T)](T-t)}$$

  - r and q are funding and storage rates.
  - One can always buy at the spot price and store to delivery at T.
  - The convenience yield provides the comfort of an seeing an equality:

$$F(t, T) = F(t, t)e^{[r(t,T)+q(t,T)-\eta(t,T)](T-t)}.$$

- Key Points:
  - All that can be ascertained from market data is $q - \eta$.
  - The cost of storage is not exogenous.
    - Storage owners will charge what the market will bear.
    - The cost of storage is in reality a function of forwards and vols as opposed to an input.

Temporal Correlation

Forward Yields, Inventory and Forward Dynamics

- Incentives: The huge credit-crisis contango resulted in a massive increase in the use of VLCCs store oil and refined products.
- The figure shows the result outside of the Port of Singapore during Jan2009. (Source: Google Maps)

Temporal Correlation

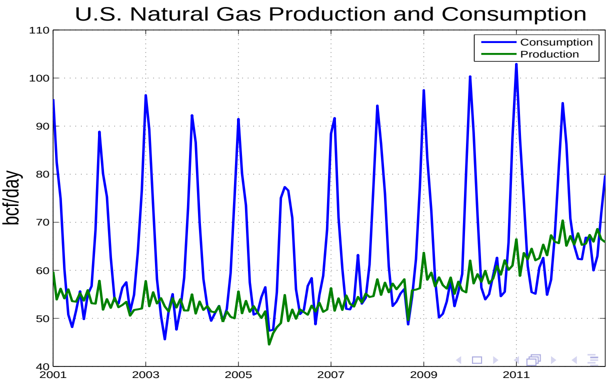
Forward Yields, Inventory and Forward Dynamics

- Facilities exist to dampen the effects of anticipated (seasonal) and unanticipated demand fluctuations.
- Natural gas is particularly interesting.
- U.S. Natural Gas Markets
  - Annual gas consumption is roughly 25 Tcf with roughly 4 Tcf of imports.
  - Gas consumption is highly seasonal due to winter heating requirements.
  - Approximately 4 Tcf of natural gas storage facilitates accommodation of winter demand.

Temporal Correlation

Forward Yields, Inventory and Forward Dynamics

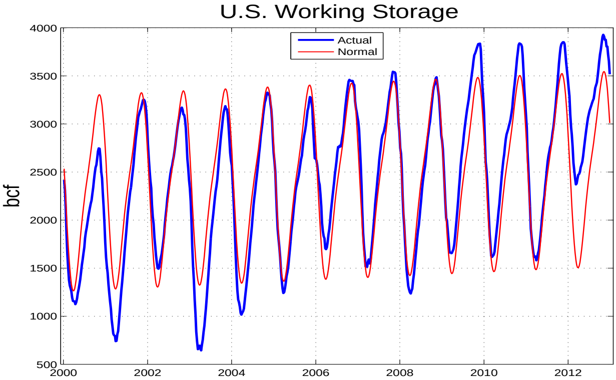
- North American demand is highly seasonal.
  - High winter peak demand and the relatively mild summer peaks (air conditioning power demand met with CC generation)
  - Non-seasonal production profile.
  - Recent increase in domestic production—shale gas glut.



Temporal Correlation

Forward Yields, Inventory and Forward Dynamics

- Roughly 4.2bcf of storage capacity resolves the production versus consumption mismatch.
- Compare historical inventory levels versus "normal" .
  - "Normal" is a Fourier fit with the number of modes used determined by an out-of-sample selection method with estimates of working capacity.

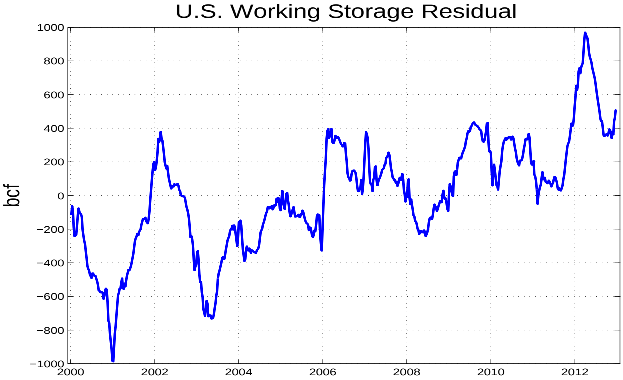


Temporal Correlation

Forward Yields, Inventory and Forward Dynamics

- Storage Residual: $R(t) \equiv S(t) - \bar{S}(t)$

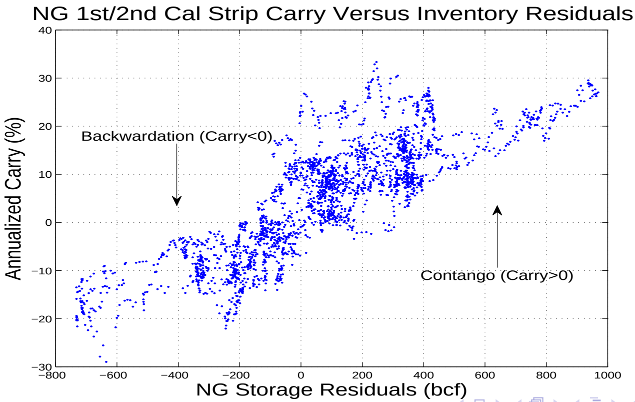
$$\bar{S}(t) = \alpha + \beta t + \sum_{k=1}^{K} [\gamma_k \sin(2\pi kt) + \delta_k \cos(2\pi kt)] \qquad (4)$$



Temporal Correlation

Forward Yields, Inventory and Forward Dynamics

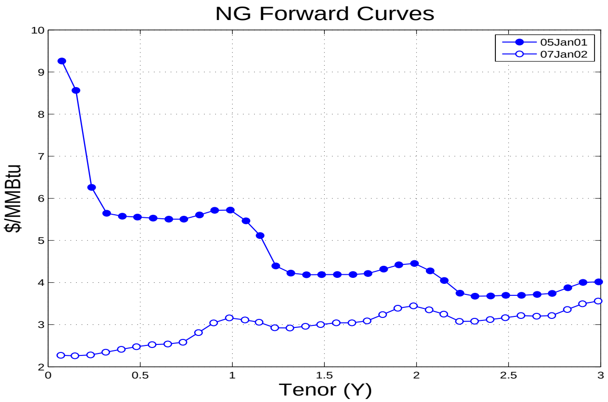
- Forward yields are related to the storage residual $R(t)$.
- The figure shows salendar strip forward yields versus storage residual.
  - Calendar strips are used to "strip out" seasonal effects.



Temporal Correlation

Forward Yields, Inventory and Forward Dynamics

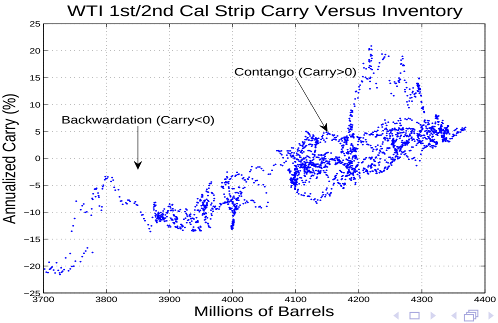
- A static view: Forward curves on January 2001 and January 2002.
  - Note the higher prices, backwardation and greater seasonality in 2001.



Temporal Correlation

Forward Yields, Inventory and Forward Dynamics

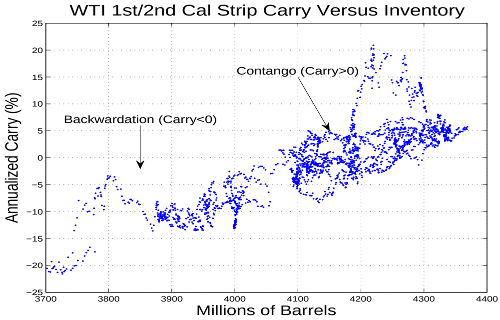
- This is a commonly observed phenomenon:
  - High forward yields (contango) incentivizes owners of storage to inject—this occurs when there is a surplus.
  - Negative forward yields (backwardation) encourages withdrawals—during times of scarcity.
  - The figure shows the forward yield between the first two cal strips of the WTI forward curve yield versus OECD crude oil stocks.



Temporal Correlation

Forward Yields, Inventory and Forward Dynamics

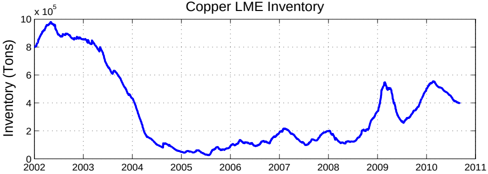
- This is a commonly observed phenomenon:
  - High forward yields (contango) incentivizes owners of storage to inject—this occurs when there is a surplus.
  - Negative forward yields (backwardation) encourages withdrawals—during times of scarcity.
  - The figure shows the forward yield between the first two cal strips of the WTI forward curve yield versus OECD crude oil stocks.

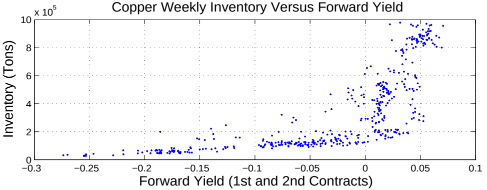


Temporal Correlation

Forward Yields, Inventory and Forward Dynamics

- Lessons from copper:
  - A non-seasonal consumption commodity with credible inventory time series (LME).
  - The upper plot shows that copper inventories have spanned a considerable range over the past decade.
  - The lower is weekly averages of forward yields versus inventory.

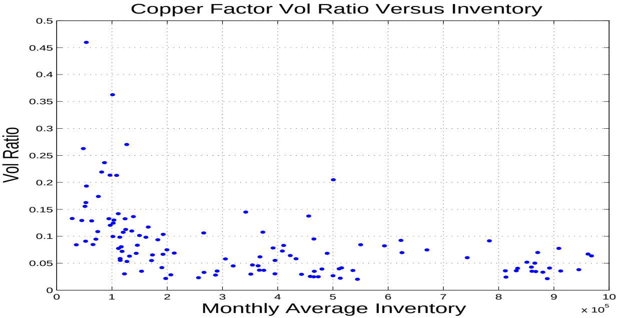




Temporal Correlation

Forward Yields, Inventory and Forward Dynamics

- PCA on copper daily forward returns yields the usual factors.
  - The figure shows the volatility of factors 2 and above over the volatility of the first factor by month versus average inventory.
  - Note the apparent increase in higher factor contributions at low inventory.
  - This is one origin of the (apparent) non-stationarity of returns covariances.



Common Modeling Approaches

Comment on Spread Options—Examples

- Calendar Spread Options
  - Example: A CSO straddle payoff takes the form:

$$[F(\tau, T_2) - F(\tau, T_1)]^+$$

    where τ denotes option expiry.
  - In general the payoff is a function of the spread between two contracts at expiry τ:

$$F(\tau, T_2) - F(\tau, T_1) - K.$$

  - CSO are closely related to physical storage and hedges thereof.

- Swaptions
  - The option payoff references a strip of contract prices:
  - For example a call swaption has the payoff:

$$\max\left[\sum_m d(\tau, T_m)\left(F(\tau, T_m) - K\right), 0\right]$$

Common Modeling Approaches

Comment on Spread Options—Examples

- Tolling Deals/Heatrate Options
  - A spread-option between power and a fuel, typically natural gas with payoff:
    $$\max\left[F_B(\tau,\tau) - H_* G(\tau,\tau) - V, 0\right]$$
    for a sequence of days indexed by τ, where:
    - F_B and G denote the prices of power (delivery bucket B) and natural gas respectively.
    - H_* is the heatrate and V is as strike (unit cost of running).

- Crack-Spread Options:
  - Options on the spread between refined product and a reference crude oil prices:
    $$\max\left[F_{\mathrm{Product}}(\tau, T) - F_{\mathrm{crude}}(\tau, T), 0\right]$$
    - Product is usually heating oil or gasoline.
    - Both forwards in common units (e.g. \$/Barrel).

Common Modeling Approaches

Comment on Spread Options—Margrabe

- The spot prices of two assets X and Y under the money-market EMM are modeled as two standard GBMs:

$$\begin{aligned} dX_t &= rX_t dt + \sigma_X X_t dB_t^{(X)} \\ dY_t &= rY_t dt + \sigma_Y Y_t dB_t^{(Y)} \end{aligned}$$

  where the correlation between the two BMs is ρ

- Consider the following spread option with value:

$$d(T)\tilde{E}\left[\max\left(X_T - Y_T, 0\right)\right]$$

  - $d(T)$ is the discount factor (assume that interest rates are deterministic).
  - Note that all of the options just mentioned are of this form for zero strikes.

Common Modeling Approaches

Comment on Spread Options—Margrabe

- The standard valuation approach is via change of numeraire.[1]
  - In the Y-measure $\tilde{E}_Y$ in which Y_t is the numeraire, all assets discounted by Y must be martingales.
  - Denote the value of the option by $V(t, X_t, Y_t)$.
  - V must be an $\tilde{E}_Y$ martingale:

$$\frac{V(0, X_0, Y_0)}{Y_0} = \tilde{E}_Y\left[\frac{V(T, X_T, Y_T)}{Y_T}\right]$$

  - This implies:

$$V(0, X_0, Y_0) = Y_0 \tilde{E}_Y\left[\max\left(\frac{X_T}{Y_T} - 1, 0\right)\right]$$

[1] See Carmon and Durrleman, Pricing and Hedging Spread Options, 2003

Common Modeling Approaches

Comment on Spread Options—Margrabe

- We know that the ratio:

$$R_T \equiv \frac{X_T}{Y_T} = \frac{X_0}{Y_0} e^{\sigma_X B_X(T) - \sigma_Y B_Y(T) + \text{"} Drift Terms \text{"}}$$

- Also: $\sigma_X B_X(T) - \sigma_Y B_Y(T)$ is a normal random variable with:

$$\hat{\sigma}^2 \equiv \text{var}\left[\sigma_X B_X(T) - \sigma_Y B_Y(T)\right] = T\left[\sigma_X^2 + \sigma_Y^2 - 2\rho\sigma_X\sigma_Y\right]$$

.

- Finally, a fact about log-normals
  - If R is log-normal with variance $\hat{\sigma}^2$ then:

$$E\left[\max(R - K, 0)\right] = E(R)N(d_1) - KN(d_2)$$

  - where:

$$d_{1,2} = \frac{\log\left[\frac{E(R)}{K} \pm \frac{1}{2}\hat{\sigma}^2\right]}{\hat{\sigma}}$$

Common Modeling Approaches

Comment on Spread Options—Margrabe

- Assembling the facts we have:

$$\begin{aligned} V(0, X_0, Y_0) &= Y_0 \left[\tilde{E}_Y \left(\frac{X_T}{Y_T} \right) N(d_1) - N(d_2) \right] \\ &= X_0 N(d_1) - Y_0 N(d_2) \end{aligned}$$

  where:

$$d_{1,2} = \frac{\log\left(\frac{X_0}{Y_0}\right) + \pm\frac{1}{2}\hat{\sigma}^2}{\hat{\sigma}}$$

- Note: This is (arguably) intuitive:
  - The value of the option is as if we used Black with X as the underlying and strike K replaced with Y_0.
  - There is no discounting as funding is embedded in the Y asset.
  - The implied vol $\hat{\sigma}$ is obtained from the returns variance of $X - Y$.

Common Modeling Approaches

Comment on Spread Options—Margrabe

- In our context forwards are not spot assets and the result must be discounted.
  - Let $X_t = F(t, T_1)$ and $Y_t = F(t, T_2)$.
  - The previous valuation formula must include discounting:

$$V(0, F(0, T_1), F(0, T_2)) = d(T)\left[F(0, T_1)N(d_1) - F(0, T_2)N(d_2)\right]$$

  - This can be verified by direct integration (more later).
  - Note that if all vols are set to zero then the value of the option is intrinsic:

$$V(0, F(0, T_1), F(0, T_2)) = d(T)\max\left[F(0, T_1) - F(0, T_2), 0\right]$$

- Analogous formulas hold for other standard European option payoffs.

Common Modeling Approaches

Comment on Spread Options—Margrabe

- Greeks have similarly analogous forms:

$$\frac{\partial V}{\partial X} = N(d_1) \qquad \frac{\partial V}{\partial Y} = -N(d_2)$$

and

$$\Gamma = \frac{N'(d_1)}{\hat{\sigma}} \begin{pmatrix} \frac{1}{X} & -\frac{1}{Y} \\ -\frac{1}{Y} & \frac{X}{Y^2} \end{pmatrix}$$

- Γ is positive-definite because:

$$\bar{\alpha}^t \Gamma \bar{\alpha} = \frac{1}{X} \left[\alpha_1 - \frac{X}{Y} \alpha_2 \right]^2$$

.

- The spread option payoff is an option:

  When Δ-hedged, all directions point up.

Common Modeling Approaches

Reduced Form Models

- The term “reduced-form” refers to modeling frameworks which posit price dynamics in the absence of “fundamental” considerations.
- In commodities there are two meta-classes of reduced form models:
  - Spot/Convenience yield models: Model the joint behavior of spot price and convenience yields (and perhaps other variables).
  - Factor models: The HJM framework applied to commodities.
- These approaches are effectively functionally identical.

Common Modeling Approaches

Reduced Form Models—Spot/Convenience Yield Models

- "Schwartz-type" models and descendants.[2]
- Explicit modeling of spot price dynamics with additional processes added.
  - These additional processes are typically convenience yields or long term price levels.
  - The original two-factor incarnation (Gibson and Schwartz):

$$\begin{aligned} dS_t &= (r_t - \delta_t)\, S_t dt + \sigma S_t dB_t^{(1)} \\ d\delta_t &= \kappa\,(\theta - \delta_t)\, dt + \gamma dB_t^{(2)} \end{aligned}$$

  - Here we let $S_t \equiv F(t,t)$.
  - δ_t is the instantaneous convenience yield. When $\delta_t > 0$ the spot price has a negative drift (net of financial carry r_t).
- Calibration to forwards requires finding time-varying drifts.

[2] See Carmona and Ludkovski: "Spot Convenience Yield Models for the Energy Markets."

Common Modeling Approaches

Reduced Form Models—Gaussian Exponential Models

- Gaussian exponential framework:

$$dF(t,T) = F(t,T)\sum_{j=1}^{J}\sigma_j(T)e^{-\beta_j(T-t)}dB_t^{(j)}$$

  - We have for now the form $\sigma_j(T)$.
  - Often the BMs are assumed independent for simplicity.

- Intuition (2-factor):

  - If $\sigma_2 \equiv 0$, this is a one-factor model identical to that described in the first section:

$$\sigma(T-t) = \alpha e^{-\beta(T-t)}$$

  - The second factor will typically have $\beta_2 \ggg \beta_1$ and is intended to represent shorter time-scale forward returns.

Common Modeling Approaches

Reduced Form Models—Gaussian Exponential Models

- This is "HJM" for commodities introduced by Clewlow-Strickland.
- Some useful facts:
  - The integral of the returns for factor j on contract T over $[0, t]$ is:

$$
\begin{aligned}
\sigma_j(T)\int_0^t e^{-\beta_j(T-s)}dB_s^{(j)} &= \sigma_j(T)e^{-\beta_j(T-t)}\int_0^t e^{-\beta_j(t-s)}dB_s^{(j)} \\
&= \sigma_j(T)e^{-\beta_j(T-t)}Y_j(s)
\end{aligned}
$$

where we have defined: $Y_j(t) = \int_0^t e^{-\beta_j(t-s)}dB_s^{(j)}$.

*****This means that the distribution of returns for all tenors are simultaneously described by the processes** $Y_j(t)$.

*****This means that the dynamics of the entire forward curve are prescribe by a J-dimensional stochastic process.**

Common Modeling Approaches

Reduced Form Models—Gaussian Exponential Models

- Some useful facts: (cont)
  - **Returns are normally distributed** since any integral of the form $\int_0^t \phi(u)dB_u$ is normally distributed with mean zero.

    The variance is obtained by the Ito isometry:

    $$E\left[\left(\int_0^t \phi(s)dB_s\right)^2\right] = \int_0^t \phi^2(s)ds.$$

    Therefore, the returns variance for $F(t,T)$ is:

    $$V(t,T) \equiv \sum_{j=1}^{J} \sigma_j^2(T) \int_0^t e^{-2\beta_j(T-s)} ds$$

Common Modeling Approaches

Reduced Form Models—Gaussian Exponential Models

- Some useful facts: (cont)
  - Recalling that $Y_j(t) = \int_0^t e^{-\beta_j(t-s)} dB_s^{(j)}$, differentiating with respect to t yields:

$$dY_j = -\beta_j \left[\int_0^t e^{-\beta_j(t-s)} dB_s^{(j)} \right] dt + dB_t^{(j)}$$

or

$$dY_j = -\beta_j Y_j dt + dB_t^{(j)}$$

The Y's mean-revert toward a mean of zero with mean-reversion rates specified by the β's.

Diffusions of this form are called Ornstein-Uhlenbeck processes.

Properties:

- $E[Y_t | Y_0] = Y_0 e^{-\beta t}$
- $\mathrm{var}[Y_t | Y_0] = \frac{1-e^{-2\beta t}}{2\beta}$
- $\mathrm{var}[Y_\infty] = \frac{1}{2\beta}$

Common Modeling Approaches

Reduced Form Models—Gaussian Exponential Models

- Some useful facts: (cont)
  - The resulting form for $F(t, T)$ is:

$$F(t,T) = F(0,T)e^{\sum_{j=1}^{J}\left[\sigma_j(T)e^{-\beta_j(T-t)}Y_j(t)\right] - \frac{1}{2}V(t,T)}$$

  - To see this note that $F(t, T)$ is a martingale and that for any normal random variable Z: $E\left[e^{Z}\right] = e^{\frac{1}{2}\sigma_Z^2}$.
  - Note that the previous calculation is nothing more than the exponential equivalent of the previous GBM integration:

$$F_t = F_0\ e^{-\frac{1}{2}\sigma^2 t + \sigma B_t}$$

with $\sigma^2 t$ replaced by the appropriate exponential integrals.

Common Modeling Approaches

Reduced Form Models—A Caricature Model

- A simple caricature of spot (daily) prices processes is for the returns of the daily spot prices to be i.i.d. normal:

$$F(t,t) = F_m(T_m)e^{\zeta Z_t - \frac{1}{2}\zeta^2}$$

where

- F_m denotes the contract month containing day t
- Z_t is standard normal.
- ζ is the spot volatility.

Common Modeling Approaches

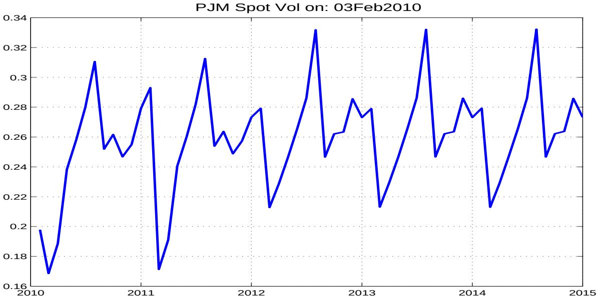
Reduced Form Models—A Caricature Model

- Spot volatility is implied by the monthly and daily vols

$$\zeta^2 = \bar{\sigma}_D^2 T_d - \bar{\sigma}_M^2 T_m$$

where:

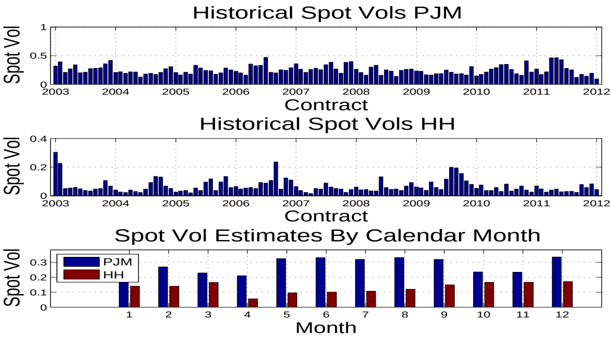
- $T_d \approx T_m + \frac{1}{24}$.
- $\bar{\sigma}_M$ and $\bar{\sigma}_D$ are implied vols for monthly and daily options.
- The following figure shows the ATM spot vol by contract month for PJM.



Common Modeling Approaches

Reduced Form Models—A Caricature Model

- For comparison the following figure shows historical spot volatility for PJM and TETM3.
  - The reference price is the BOM contract fixing so that spot returns are defined as $\log \left[\frac{p_d}{F_m(T_e)} \right]$.



Common Modeling Approaches

Reduced Form Models—Advantages

- A "good" reduced form model is tractable by construction-and captures some features of forward dynamics.
- Models such as the GEM models above can match observed covariance structures reasonably well.
  - Allowing for the BMs to be correlated the two factor model implies a correlation surface:

$$\rho(T,S) = \frac{\sigma_1^2 e^{-\beta_1(T+S)} + \sigma_1\sigma_2\rho\left[e^{-(\beta_1 T+\beta_2 S)} + e^{-(\beta_2 T+\beta_1 S)}\right] + \sigma_2^2 e^{-\beta_2(T+S)}}{\sigma(T)\sigma(S)}$$

  - Since the returns of the T contract can be written as:

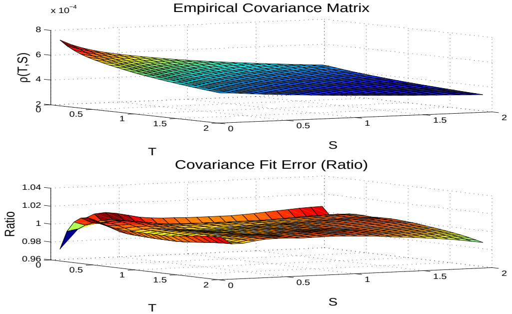
$$\sigma_1 e^{-\beta_1 T}\left[dB_t^{(1)} + \frac{\sigma_2}{\sigma_1} e^{(\beta_1-\beta_2)T} dB_t^{(2)}\right]$$

  the free parameters in $\rho(T,S)$ are:
    - The difference in the decay rates $\beta_2 - \beta_1$.
    - The volatility ratio $\lambda = \frac{\sigma_2}{\sigma_1}$
    - The correlation ρ

Common Modeling Approaches

Reduced Form Models—Advantages

- Example: WTI returns from Jan2007 to Dec2010:
    - Minimizing the Frobenius norm of the difference of the empirical and model covariance matrices yields:
        - The optimal decay rates are: $\bar{\beta} = [0.106, 1.528]$
        - The estimated correlation between the factors is $\rho = 0.119$



Common Modeling Approaches

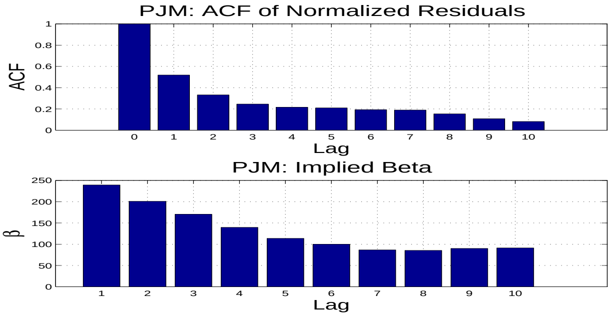
Reduced Form Models—Advantages

- Separation of time-scales:
  - In natural gas and power common tradeables reference annual, monthly and spot prices at the daily or hourly level daily.
  - This is commonly handled by a large range of mean-reversion rates (β's) in models.
  - Typically $\beta_1 \in [.1, .5]$ and $\beta_2 \ggg 10$ for a two-factor model.
  - Spot returns statistics give us some guidance to β_2.
  - A useful definition for spot returns: $\log\left[\frac{p_d}{F_m(T_e)}\right]$ where $F_m(T_e)$ denotes the forward price for the contract month at expiration.

Common Modeling Approaches

Reduced Form Models—Advantages

- What are spot returns for a non-storable commodity?
  - The plot below shows both the ACF as well as the implied β by lag.
  - The implication is that very high mean-reversion rates are required for some factors.
  - This is easily handled in the reduced-form framework.



Common Modeling Approaches

Reduced Form Models—Disadvantages

- Production implementations are almost always predicated on constant correlation parameters.
  - Covariances may evolve with local time t due to term structure of volalitiy but underlying correlation parameters between the BMS are usually assumed to be constant.
- Correlation parameters are not static.
  - Inventory affects covariance structure and forward yield affect inventory.
  - In the case of power many input fuels effect market clearing prices and ultimately should effect forward correlation structure.
- There is rarely enough liquidity in vol and correlation tradables to facilitate a "robust" calibration of reduced-form models.
  - Statistical estimation is used to set many parameters.
  - Models are designed with tractability as a dominant consideration diminishing empirical relevance and limiting their utility as valuation extrapolators.

Common Modeling Approaches

Econometric and Structural Models

- **Major Theme: Severe limitations in liquidity and spectrum of market tradables will render rote application of reduced form models problematic in many circumstances.**

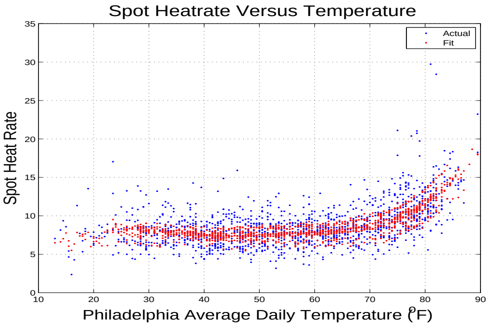
- Econometric and structural models are intended to produce more realistic price processes which in theory should function better for pricing and hedging given limited market data.

- Structural Models:
  - In the case of some energy commodities, a great deal is known about supply and demand and market mechanics.
    - Generator stacks and load dynamics in power.
    - Weather-driven demand dynamics and inventory in natural gas.
  - Models that start with a caricature of the underlying market mechanics, are referred to as structural models.
    - Stack models: In which power prices are set via load (often weather driven) clearing through a generator stack.
    - Inventory models: In which price clearing occurs in the presence of an inventory process.

Common Modeling Approaches

Econometric and Structural Models

- Econometric Models:
  - Craft regressions tailored to individual markets, often using similar stylized facts as for structural models.
  - The following figure shows:
    - Historical peak spot heatrates (the ratio of power to natural gas prices) for PJM (more later) versus KPHL temperature.
    - A regression relating expected heatrates versus temperature.



Common Modeling Approaches

Econometric and Structural Models

- Econometric Models:
  - Based on regressions of historical behavior of relevant underlying variables.
  - The results yield simulation methods to generate the joint distribution of future realizations of these variables.
  - These realizations yield physical measure distributions of:
    - The payoff Π of whatever the structure is that you are valuing.
    - Available hedges $\vec{\mathcal{H}}$ which trade at market prices $\vec{p}_{\mathcal{H}}$.
  - Standard portfolio analysis method can then be applied—for example, construction of minimum variance hedges:

$$\min \operatorname{var} \left[\Pi + \vec{w}^{\dagger} \left(\vec{\mathcal{H}} - \vec{p}_{\mathcal{H}} \right) \right]$$

On Deck

- Tolling Deals
- Variable Quantity Swaps
- CSOs and Natural Gas Storage