Game-theory Approach for Electricity and Carbon Allowances

A study of markets coupling

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Issues and goals

Carbon markets as part of instruments to foster reduction of carbon dioxide emissions in order to respond to climate changes issues.

- To what extent are carbon market truly efficient to mitigate CO_2 emissions ?
- How to establish a good market design enabling mitigation ?

Our two approaches$^{(*)}$:

- Qualitative : A game theory approach for the penalty/auctioning design in a cap and trade scheme.
- Quantitative : An indifference price from a producer view point in European Union Emission Trading Scheme (EU ETS).

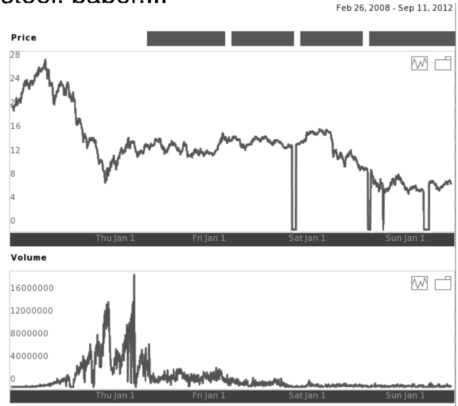
(*) This work is funded by the French Environment and Energy Management Agency ADEME.

The European Union Emission Trading Scheme

EU ETS : Exchange for allowances involving specific industrial sectors power generation, cement, iron and steel, paper....

The Kyoto phase (2008-2012)
covers almost the half of the overall GHG emissions in Europe

(source : BlueNext)



Third phase (Strengthening) : 2013-2020

- setting an overall EU cap : a 20% cut in EU economy-wide emissions relative to 1990 levels
- a move from allowances for free to auctioning

Our two approaches

Focus on the electricity producers

- Game Theory approach

  A non cooperative game between J electricity producers that face

  - an electricity market that aggregates demands and producers offers
  - an auction market for carbon allowances with a penalty system.

  Analysis of the overall behavior of the system with the help of a Nash equilibrium state (if there exists one).

  Static model with observed exogenous demand curve.

- Stochastic control approach

  Individual point of view of a producer that want to evaluate his position in the carbon allowance market.
  We compute the indifference price for a given production portfolio, in the EU ETS context.

  Stochastic model (based on the dynamics of the electricity spot price).

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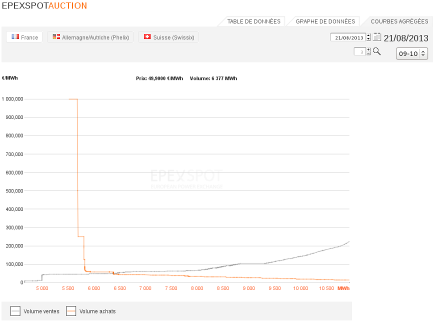
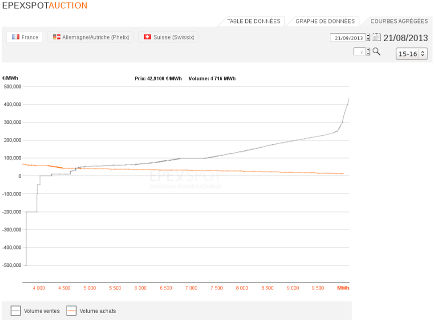
The game model

On the electricity market, J electricity producers



(a) delivery 9-10 am



(b) delivery 3-4 pm

The demand function $p \mapsto D(p)$ is positive, decreasing and left continuous, defined for $p \in [0 + \infty)$.

The offers : each producer j is characterized with

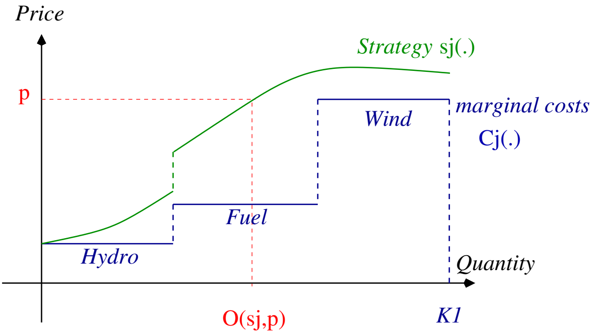
- κ_j, the electricity production capacity
- $q \mapsto \mathcal{C}_j(q) > 0$, the marginal production cost bounded and increasing.

The selling price strategy $q \mapsto s_j(q)$ gives the unit price at which the producer is ready to sell the quantity q.

- $s_j(\cdot) : [0, \kappa_j] \to [0 + \infty)$ bounded
- sell at a loss is forbidden. The producer cannot set a unit price lower than the marginal production cost

Admissibility

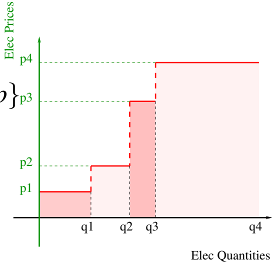
$$s_j(q) \geq \mathcal{C}_j(q),$$
$$\forall q \in [0, \kappa_j].$$



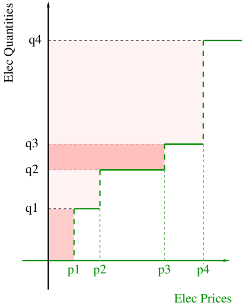
For a strategy $q \mapsto s_j(q)$,
the associated offer is

$$p \mapsto \mathcal{O}(s_j; p) := \sup\{q,\ s_j(q) < p\}$$

maximal quantity of electricity
at unit price p brought by producer j.



Strategy



Offer function

Aggregation of the J offers

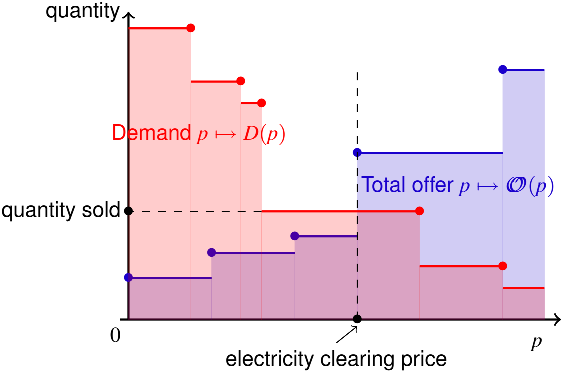
for a strategy profile $\mathbf{s} = (s_1(\cdot), \ldots, s_J(\cdot))$,

$$p \mapsto \mathbb{O}(\mathbf{s}; p) := \sum_{j=1}^{J} \mathcal{O}(s_j; p)$$

the quantity of electricity that can be sold on the market at unit price p.

$p \mapsto \mathbb{O}(\mathbf{s}; p)$ is an increasing surjection defined from $[0, +\infty)$ to $[0, \sum_{j=1}^{J} \kappa_j]$, and such that $\mathbb{O}(\mathbf{s}; 0) = 0$.

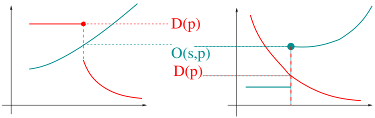
Electricity market clearing



Clearing price

$$\overline{p}(\mathbf{s}) = \sup\{p;\ \mathcal{O}(\mathbf{s};p) < D(p)\}$$

$$\overline{q}_j(\mathbf{s}) = \begin{cases} \mathcal{O}(\mathbf{s}_j;\overline{p}), & \text{if } D(\overline{p}) \geq \mathcal{O}(\mathbf{s};\overline{p}), \\ \mathcal{O}(\mathbf{s}_j;\overline{p}_-) + \Delta^-\mathcal{O}(\mathbf{s}_j;\overline{p})\dfrac{D(\overline{p}) - \mathcal{O}(\mathbf{s};\overline{p}_-)}{\Delta^-\mathcal{O}(\mathbf{s};\overline{p})}, & \text{if } D(\overline{p}) < \mathcal{O}(\mathbf{s};\overline{p}) \end{cases}$$



The CO_2 emissions cost

$q \mapsto e_j(q)$ marginal emission rate.

- Producers must pay a penalty $\mathfrak{p}$ for the uncovered emission.

$$\mathcal{C}_j(q) \curvearrowright \mathcal{C}_j(q) + e_j\, \mathfrak{p}$$

- Producers can buy allowances on the auction market, by proposing a $w \mapsto A_j(w)$ price function to the market (the unit price Producer j is ready to pay for the quantity w of CO_2 quota)

$$\mathcal{C}_j(q) \curvearrowright \mathcal{C}_j(\mathbf{A}; q) + e_j\, \mathcal{P}^{\text{CO2}}(A_1, \ldots, A_J)$$

- Eligibility requirement : can not buy more that its total emissions capacity.

Admissible strategy profile

$$\Sigma = \left\{ ((s_1(\cdot), A_1(\cdot)), \cdots, (s_J(\cdot), A_J(\cdot))) \text{ such that } \forall j,\ s_j(\cdot) \geq \mathcal{C}_j(\mathbf{A}; \cdot) \right\},$$

1. Observation of the demand $p \mapsto D(p)$
2. Auction on the CO_2 market with A_j
3. Proposition of price function s_j on the electricity market.

The strategies evaluation functions

The J producers behave non cooperatively, and strive to maximize their market shares.

- the electricity sold is to be delivered at short term (hours)
- the production unit are operating
- there is a cost for stopping a production unit

Definition

A strategy profile $(\mathbf{s}^*, \mathbf{A}^*) \in \Sigma$ is a Nash equilibrium if

$$\forall j \in 1, \ldots, J, \ \forall \ (s_j(\cdot), A_j(\cdot)) \text{ such that } ((\mathbf{s}^*, \mathbf{A}^*)_{-j}); (s_j(\cdot), A_j) \in \Sigma,$$
$$\phi_j(\mathbf{s}^*, \mathbf{A}^*) \geq \phi\big((\mathbf{s}^*, \mathbf{A}^*)_{-j}; (s_j(\cdot), A_j)\big),$$

where we define the evaluation function ϕ by

$$\phi_j((\mathbf{s}, \mathbf{A})) := \overline{q}_j(\mathbf{s}).$$

Nash equilibrium on the electricity market

For $((\kappa_j, \mathcal{C}_j), j = 1, \ldots, J)$, $p \mapsto D(p)$ given.

Proposition

(i) For any strategy profile $\mathbf{s} = (s_1, \ldots, s_J)$, any producer $j \in \{1, \ldots, J\}$ cannot be penalized by deviating form strategy s_j to strategy $\mathcal{C}_j$, namely we have :

$$q_j(\mathbf{s}) \leq q_j(\mathbf{s}_{-j}, \mathcal{C}_j).$$

(ii) For any strategy profile $\mathbf{s} = (s_1, \ldots, s_J)$ such that $\overline{p}(\mathbf{s}) = \overline{p}(\mathbf{s}^*)$, and for any j such that $s_j = \mathcal{C}_j$ we have :

$$q_j(\mathbf{s}) \geq q_j(\mathbf{s}^*).$$

The strategy profile $\mathbf{s}^* = (\mathcal{C}_1, \ldots, \mathcal{C}_J)$ is a Nash equilibrium.

A unique electricity price and a unique quantity of electricity bought to each producer follow from any Nash equilibrium.

The CO_2 Auction market reaction

To price strategy A_j the market extract the offer (to buy) function $p \mapsto \theta(A_j; p)$, the maximal quantity the producer j is ready to buy a price p

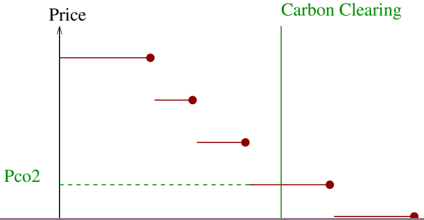
$$\theta(A_j; p) = \sup\{w,\ A_j(w) \geq p\}.$$

The number $\mathbb{W}$ of the allowances to buy on the auction market
The CO_2 market reacts by aggregating the J offers

$$\Theta(\mathbf{A}, p) = \sum_{j=1}^{J} \theta(A_j; p) := \sum_{j=1}^{J} \sup\{w,\ A_j(w) \geq p\} =$$

and set the market price as

$$\mathcal{P}^{\text{CO2}} = \inf\{p,\ \Theta(\mathbf{A}, p) < \mathbb{W}\}.$$



Nash equilibrium on coupled markets

Definition

A strategy profile $(\mathbf{s}^*, \mathbf{A}^*) \in \Sigma$ is a Nash equilibrium with no excess of purchase loss if

$$\forall j \in 1, \ldots, J, \forall\ (s_j(\cdot), A_j(\cdot)) \text{ such that } ((\mathbf{s}^*, \mathbf{A}^*)_{-j}); (s_j(\cdot), A_j) \in \Sigma,$$
$$\phi_j(\mathbf{s}^*, \mathbf{A}^*) \geq \phi\big((\mathbf{s}^*, \mathbf{A}^*)_{-j}; (s_j(\cdot), A_j)\big),$$

where we define the evaluation function ϕ by

$$\phi_j((\mathbf{s}, \mathbf{A})) := \overline{q}_j(\mathbf{s}).$$

Moreover

$$\overline{q}_j(\mathbf{s}^*) = 0 \quad \Longrightarrow \quad \delta_j(\mathbf{A}^*) = 0.$$

None of the producers would accept to bother to go on the CO_2 market if there is no reward to do that.

How many allowances a producer is willing to buy ? The answer is in the tax point of view

For simplicity :

One power plant by producer

$q \mapsto C_i(q) := \mathcal{C}_i \mathbf{1}_{\{q \in [0, \kappa_i]\}}$

For any tax level $\tau \in [0, \mathfrak{p}]$

$$\{\mathcal{C}_j(q), j = 1, \ldots J\} \curvearrowright \{\mathcal{C}_j(q) + e_j\, \tau, j = 1, \ldots J\} \rightarrow \begin{cases} p_{\text{elec}}(\tau) \\ \{\overline{q}_j(\tau), j = 1, \ldots J\} \end{cases}$$

Willing to buy ?

$$\overline{\mathcal{W}}(\tau) = \sum \frac{\overline{q}_j(\tau)}{e_j}$$

$$\underline{\mathcal{W}}(\tau) = \sum \frac{\kappa_j}{e_j} \mathbf{1}_{\{\overline{q}_j(\tau) > 0\}}$$

(H) Carbon market design hypothesis :

The number $\mathbb{W}$ of the allowances to buy on the auction market must satisfy

$$\overline{\mathcal{W}}(0) > \mathbb{W} > \underline{\mathcal{W}}(\mathfrak{p})$$

A Nash equilibrium with no excess of purchase loss

$$\tau^{\text{guess}} := \inf\{\tau \in [0,T] \text{ such that } \sum_j \mathcal{W}_j(\tau) \leq \mathbb{W}\},$$
$$\underline{\tau^{\text{guess}}} := \inf\{\tau \in [0,T] \text{ such that } \sum_j \underline{\mathcal{W}}_j(\tau) \leq \mathbb{W}\}.$$

Assume that the $(\mathcal{C}_i, e_i)$ are all different.

Proposition

For all players k, we define $w \mapsto A_k^*(w) = \dfrac{p_{\text{elec}}(\underline{\tau^{\text{guess}}}) - \mathcal{C}_k}{e_k} \mathbf{1}_{\{w \geq \kappa_k/e_k\}}$

For the **unique** player i such that $\frac{p_{\text{elec}}(\underline{\tau^{\text{guess}}}) - \mathcal{C}_i}{e_i} = \underline{\tau^{\text{guess}}}$ and $\frac{p_{\text{elec}}(\tau^{\text{guess}}) - \mathcal{C}_i}{e_i} = \tau^{\text{guess}}$, we define

$$w \mapsto A_i^*(w) = \begin{cases} \underline{\tau^{\text{guess}}} + \delta \mathbf{1}_{\{w < \bar{q}_i(\underline{\tau^{\text{guess}}})/e_i - \epsilon\}} \\ \tau^{\text{guess}} \mathbf{1}_{\{\bar{q}_i(\underline{\tau^{\text{guess}}})/e_i - \epsilon \geq \kappa_i/e_i\}} \end{cases}$$

We define also $s_j^*(q) = (\mathcal{C}_i + \tau^{\text{guess}} e_i)\mathbf{1}_{\{q \leq \kappa_i\}}$.
Then $(\mathbf{s}^*, \mathbf{A}^*)$ is a Nash equilibrium with no excess of purchase loss on the carbon auction market.

At this equilibrium

- The carbon price is τ^{guess}
- The electricity price is $p_{\text{elec}}(\tau^{\text{guess}})$
- We can also compute the quantities and the corresponding CO_2 emissions
- We can replace κ_j/e_j by any arbitrary maximal quantity to buy on the auction market.

To go further

- Uniqueness ?
- From carbon auction market to bid/ask market ?

The indifference price approach

- Individual optimization problem for an agent (producer) with respect to the CO_2 market
- Captures the value beyond/below which the agent is interested in selling/buying allowances
- Deals with the risk aversion of the agent
- Prospect of understanding the sensitivity of the production behavior with respect to allowances allocation and tax design
- Not a model for market prices dynamics

EU ETS market rules

- Each phase : divided in yearly compliance periods
- At the beginning of the period : the state/EU decides how it distributes allowances to producers.
- At the end of the period : each agent must own as much allowances as its yearly CO_2 (eq) emission quotas

  If excess of emissions : the agent pays a penalty : 100€ per ton CO_2 (eq).
- In between : agent may sell/buy allowances on organized exchanges (ECX,eeX,...) or over the counter.

CO_2 allowance indifference price : general settings

- The agent control is the production strategy : $(\pi_t)_{0\leq t\leq T}$
  $\pi_t = (\pi_t^1, \ldots, \pi_t^n) \in \mathbb{A} := \{\eta \in \mathbb{R}^n; 0 \leq \eta_i \leq p^i_{\text{max}}\}$
- The processes under control are :
  - $\mathcal{E}_t^\pi$: CO_2 emissions at time t
  - W_t^π : wealth at time t
- The market parameters :
  - Θ_0 : allocated allowances at time $t = 0$
  - $\mathfrak{p}(\cdot)$: the penalty function, increasing and vanishing on $\mathbb{R}_-$

Criterion for optimal control process Π^* :

$$\mathbb{E}\left[\mathcal{U}(W_T^{\Pi^*} - \mathfrak{p}(\mathcal{E}_T^{\Pi^*} - \Theta_0))\right] = \sup_{\pi\in Adm} \mathbb{E}\left[\mathcal{U}(W_T^\pi - \mathfrak{p}(\mathcal{E}_T^\pi - \Theta_0))\right]$$

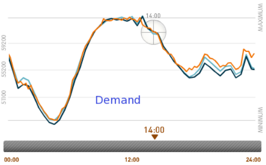
$\mathcal{U}$ is a strictly increasing and concave utility function.
Indifference price $\mathcal{P}^{\text{CO2}}(q)$ for buying/selling q allowances, at $t = 0$:

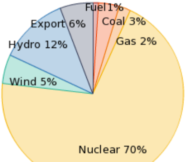
$$\mathcal{P}^{\text{CO2}}(q) = \inf\{p \in \mathbb{R}; \sup_\pi \mathbb{E}\left[\mathcal{U}(W_T^\pi - qp - \mathfrak{p}(\mathcal{E}_T^\pi - \Theta_0 - q))\right] - \sup_\pi \mathbb{E}\left[\mathcal{U}(W_T^\pi - \mathfrak{p}(\mathcal{E}_T^\pi - \Theta_0))\right] > 0\}$$

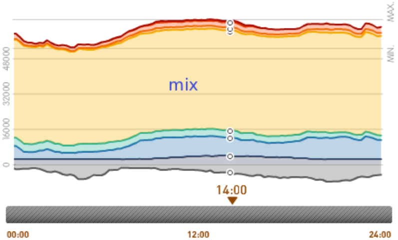
For simplicity : interest rate is set to zero

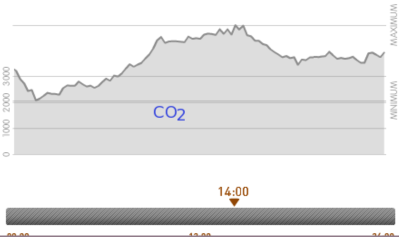
We focus on the electricity production sector

Huge part of the market : 70% of the allocation plan (phase 1).
More flexibility for emissions : fuel switching
Inelastic demand : the spot market price captures the stochastic demand









Global demand, production (mix), and emission on 2012-26-04, (from RTE France)

The wealth dynamics for the electricity producer

- Stochastic input : the electricity spot price S.
- A 3D state space (w, e, s) for $(W^{\pi}_{\cdot}, \mathcal{E}^{\pi}_{\cdot}, S_{\cdot})$
- The control $\pi_{\cdot} = (pi^{1}_{\cdot}, \ldots, \pi^{n}_{\cdot})$, a progressively measurable process valued in $\mathbb{A}$: generated power, plants portfolio (coal, gas, oil,hydro, wind, PV, ...)
- The dynamics of the wealth process : for all $t \geq \theta$

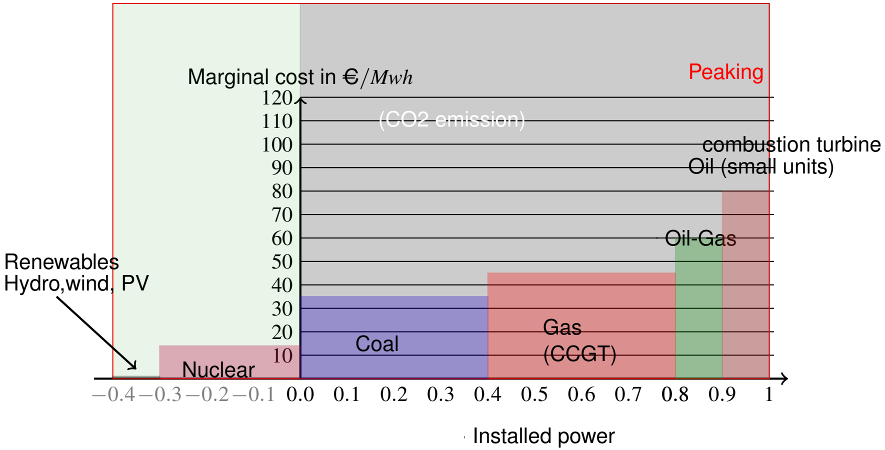
$$dW_t^{\pi;\theta,w,s} = \underbrace{\left\{(\pi_t \cdot \mathbf{1} - Q_t^{OTC})S_t^{\theta,s} + Q_t^{OTC}\mathcal{P}(t) - C(t,\pi_t)\right\}}_{h(t,S_t^{\theta,s};\pi_t)} dt,$$

$$W_\theta = w,$$

- Deterministic production costs : $C(t,\pi) = \sum_{i=1}^{n} \int_0^{\pi^i} C_m^i(t,p)dp$,
- Deterministic quantity and price for contractual production : Q_t^{OTC} and $\mathcal{P}(t)$, bounded

Merit order of the marginal production cost

$$C(t,\pi) = \sum_{i=1}^{n} \int_0^{\pi^i} C_m^i(t,p)\,dp$$

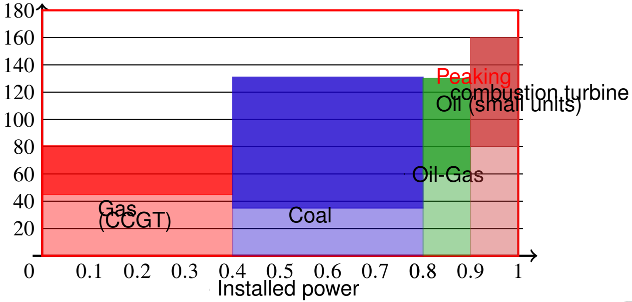


CO_2 emissions dynamics

$$d\mathcal{E}^{\pi}_t = \underbrace{\sum_{1=1}^{n}\left(\int_0^{\pi^i_t} \alpha^i_m(p)dp\right)}_{\alpha(\pi_t)} dt, \quad \mathcal{E}_0 = e, \alpha(p) \text{ bounded on } \mathbb{A};$$

$\alpha_m^{\text{coal}} = 0.96, \quad \alpha_m^{\text{gas}} = 0.36, \quad \alpha_m^{\text{oil-gas}} = 0.60, \quad \alpha_m^{\text{oil}} = 0.80.$

Marginal cost in €$/Mwh$, with the CO2 penalty of 100 €

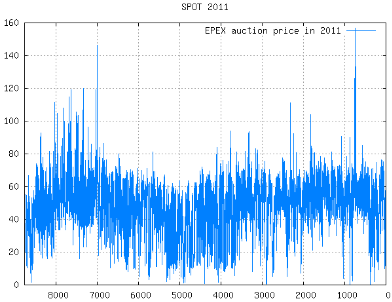


Electricity spot price and emissions dynamics

A diffusion process for electricity spot market price $(S_t, t \geq 0)$

$$\begin{cases} dS_t^{\theta,s} = b(t, S_t^{\theta,s})dt + \sigma(t, S_t^{\theta,s})dB_t, \ \forall t \geq \theta \\ S_\theta^{\theta,s} = s \end{cases}$$

- b and σ Lipschitz in s uniformly in t
- $(B_t, t \geq 0)$ a Brownian motion, $|b(t,s)| + |\sigma(t,s)| \leq \kappa_t + K|s|$, with $\int_0^T \kappa_s^2 ds < \infty$



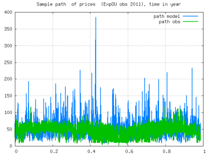
Epex auction prices in 2011

The spot price calibration $S_t = s(\exp(X_t) - a)$

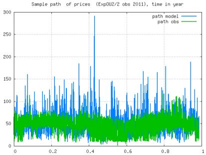
Calibration of X_t as

- an Ornstein Uhlenbeck (OU) process : $dX_t = \theta(\mu - X_t)dt + \sigma dB_t$,
- an OU square process : $X_t = Y_t^2$ and $dY_t = \theta(\mu - Y_t)dt + \sigma dB_t$,
- a CIR process : $dX_t = \theta(\mu - X_t)dt + \sigma\sqrt{X_t}dB_t$,
- an OU-Variance Gamma process : $dX_t = \alpha(\mu - X_t)dt + dZ_t$ with $Z_t = mt + \theta G_t + \sigma B_{G_t(\kappa)}$

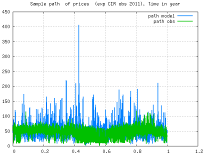
on the Epex spot market data



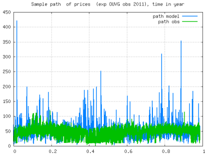
(c) Exp OU



(d) Exp OU^2



(e) Exp CIR



(f) Exp OUVG

The Hamilton-Jacobi-Bellman PDE

$$\text{for } x=(w,e,s), \qquad v(t,x)=\sup_{\pi\in Adm}\mathbb{E}_{t,x}\left\{\mathcal{U}(W_T^{\pi;t,w,s}-\mathfrak{p}(\mathcal{E}_T^{\pi;t,e}-\Theta_0))\right\}, \tag{1}$$

$$\begin{cases} \dfrac{\partial v}{\partial t}+b(t,s)\dfrac{\partial v}{\partial s}+\dfrac{\sigma^2(t,s)}{2}\dfrac{\partial^2 v}{\partial s^2}+\sup_{\pi\in\mathbb{A}}\left\{h(t,s;\pi)\dfrac{\partial v}{\partial w}+\alpha(\pi)\dfrac{\partial v}{\partial e}\right\}=0 \\ v(T,w,e,s)=\mathcal{U}(w-\mathfrak{p}(e-\Theta_0)) \end{cases}$$

By considering the operator $\mathcal{H}$

$$\mathcal{H}(t,x,p,M)=\frac{1}{2}Tr(\Sigma\Sigma^t M)(t,x)+\sup_{\pi\in\mathbb{A}}\{\lambda(t,x,\pi)\cdot p\}$$

$$\Sigma(t,x)=\begin{pmatrix}0&0&0\\0&0&0\\0&0&\sigma(t,x_3)\end{pmatrix} \qquad \lambda(t,x,\pi)=\begin{pmatrix}h(t,x_3;\pi)\\ \alpha(\pi)\\ b(t,x_3)\end{pmatrix}$$

$$\boxed{\partial_t v+\mathcal{H}(t,x,D_x v,D_x^2 v)=0,\ \forall(t,x)\in[0,T[\times\mathbb{R}\times\mathbb{R}\times\mathbb{R}_+^*}$$

Well-posedness of the HJB equation

b and σ Lipschitz uniformly in time.
$\mathfrak{p}$ with linear growth
$v(t, w, e, s) = o\left(\exp(C(2 + |w| + |e|)\right)$.

Theorem

• With above assumptions the value function of the stochastic control problem is a continuous (viscosity) solution to the HJB equation.

• Assume further

$$\exists \kappa, \ |b(t,s)| \leq \kappa(1+s)$$
$$|\sigma(t,s)| \leq \kappa(1+\sqrt{s}), \ \forall s > 0.$$

Then the HJB equation no more than one viscosity solution.

Proof : following Barles, Buckdahn and Pardoux 96, Crandall Ishii and Lions 92.
Pham 2007.

CO_2 indifference price

Buying/selling q allowances at time 0 at price p :

$$v(0, w - qp, e - q, s) = \sup_{\pi \in Adm} \mathbb{E}\left\{\mathcal{U}\left(W_T^{\pi;t,w,s} - qp - \mathfrak{p}(\mathcal{E}_T^{\pi;t,e} - q - \Theta_0)\right)\right\}$$

As v is continuous in w, e, the indifference price for q allowances is $\mathcal{P}^{\text{CO2}}(q)$ such that

$$\boxed{v(0, w - q\mathcal{P}^{\text{CO2}}(q), e - q, s) = v(0, w, e, s)}$$

- $\mathcal{P}^{\text{CO2}}(q; T, w, e, s) = \lim_{t \to T} \mathcal{P}^{\text{CO2}}(q; t, w, e, s) = \dfrac{\mathfrak{p}(e - \Theta_0) - \mathfrak{p}(e - q - \Theta_0)}{q}$

Solve the HJB equation

- Numerical scheme for fully non linear PDE
  - Implicit-Explicit scheme
  - Optimal control computation algorithm
  - Consistency, Stability, Monotonicity, Convergence (see Barles and Souganidis 91, Barles and Jakobsen 07, Forsyth and Labahn 08)

- Artificial boundary condition
  - Restrict to a compact the computational domain

Input

- Data for the producer model
- Calibration of the spot price

HJB : dimension reduction with exponential utility

$\mathcal{U}_{\exp}(w) = \dfrac{1-\exp(-\rho w)}{\rho}$, for $\rho > 0$

$W_r^{\pi;t,w,s} = w + W_r^{\pi;t,0,s}$, $r \geq t$

$$v(t,w,e,s) = \mathcal{U}(w)g(t,e,s)$$

where g solve the following HJB, $z = (e,s)$:

$$\begin{cases} g_t + \mathcal{G}(t,z,g,D_z g, D_z^2 g) = 0 \\ g(T,(e,s)) = \exp(\rho\, \mathfrak{p}(e - \Theta_0)) \end{cases}$$

with $\mathcal{G}(t,z,a,p,M) = \frac{1}{2} Tr\left\{\Sigma\Sigma^t M\right\}(t,z) + \inf_{\pi\in A}\left\{B(t,z,\pi)\cdot p - m(t,z,\pi)a\right\}$

$\tilde{\Sigma}(t,z) = \begin{pmatrix} 0 & 0 \\ 0 & \sigma(t,z_2) \end{pmatrix} \qquad B(t,z,\pi) = \begin{pmatrix} \sum_{i=1}^n \int_0^{\pi_i} \alpha_m^i(p)dp \\ b(t,z_2) \end{pmatrix}$

$m(t,z,\pi) = \rho h(t,z_2,\pi)$

$$g(t,(e,s)) = \inf_{\pi\in} \mathbb{E}\left\{\exp\left(-\rho\left(\int_t^T dW_u^{\pi;t,0,s} - \mathfrak{p}(\mathcal{E}_T^{\pi;t,e} - \Theta_0)\right)\right)\right\}$$

Numerical Example

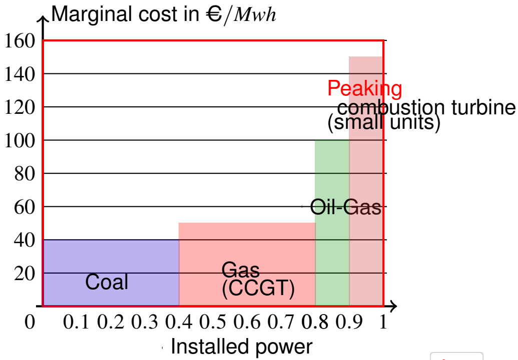
- $\mathcal{U}(x) = -\exp(-\rho x)$, then

$$\boxed{\mathcal{P}^{\text{CO2}}(q; 0, w, e, s) = \frac{1}{\rho q} \log\left(\frac{\upsilon(0, w, e-q, s)}{\upsilon(0, w, e, s)}\right)}$$

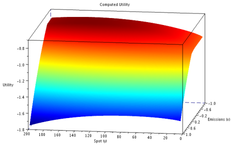
- Spot price of the form $S_t = s(\exp(X_t) - a)$ with X CIR.
- Penalty $\lambda = 100$
- Model data for the producer : $n = 4$ plants

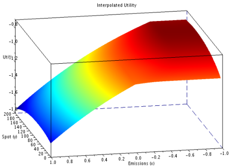
Marginal costs :
$C_m^{\text{coal}} = 40$, $C_m^{\text{gas}} = 50$,
$C_m^{\text{oil-gas}} = 100$, $C_m^{\text{oil}} = 150$

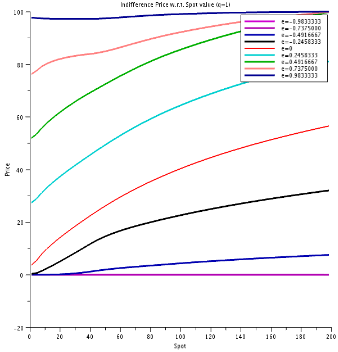
Marginal emission rates :
$\alpha_m^{\text{coal}} = 0.96$, $\alpha_m^{\text{gas}} = 0.36$,
$\alpha_m^{\text{oil-gas}} = 0.50$, $\alpha_m^{\text{oil}} = 0.80$



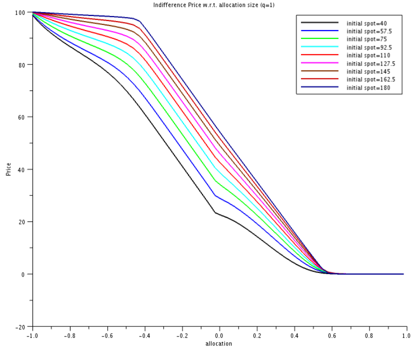
Value function and indifference prices (exp(CIR) case)







Indif. price in terms of the initial spot price



Indif. price in terms of the initial allocation

You can download CarbonQuant on the website carbonvalue.gforge.inria.fr

Collaboration work with Jacques Morice and Selim Karia (Inria Chile and Inria France).