Co-integrated Commodities, Proxy-Hedges and Structured Cash-Flows

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2. Forward Model
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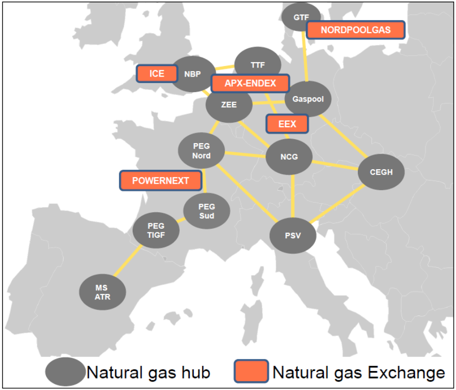
The European Market

Trading *hubs* in Europe:

- the National Balacing Point (**NBP**) in UK
- Zeebruegge in Belgium (**ZEE**)
- Title Transfer Facility (**TTF**) in the Netherlands
- **NCG** and **GASPOOL** in Germany
- **PEGn**, **PEGs** in France

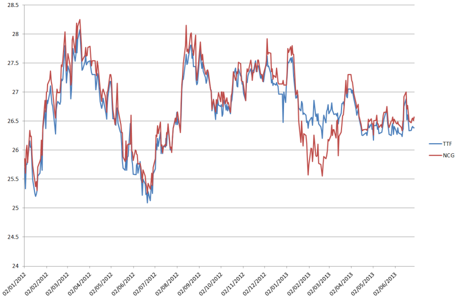
Hubs are connected:

- UK market and continental Europe are connected by the **interconnector**
- TTF and Zeebruegge are connected by a network of pipelines



Gas Futures Market

- monthly, quarterly, seasonally, yearly contracts
- seasonal contracts are *summer*(Apr-Sep) and *winter*(Oct-Mar)
- **cascading** of fwd contracts: on their last day of trading these futures are replaced with equivalent futures with shorter delivery periods
- day-ahead forwards, weekend ahead, ...



TTF and NCG year ahead prices

Gas Futures Market

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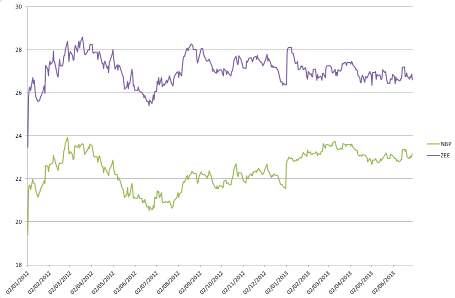


NBP and ZEE year ahead prices

Co-Movement in Commodity Markets

Commodity markets are (instantaneously) *correlated* due to

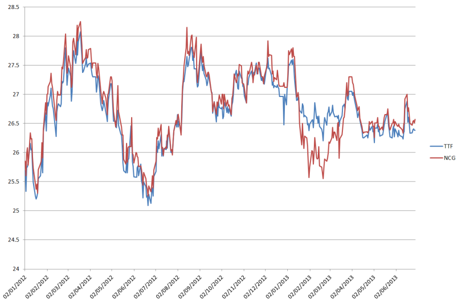
- market effects, e.g. pipeline outages
- weather conditions (temperature, rainfall, wind, ...)
- political situations (European decisions, conflicts, ...)
- (macro)-economic factors, e.g. stock crashes, financial crisis

Co-Movement in Commodity Markets

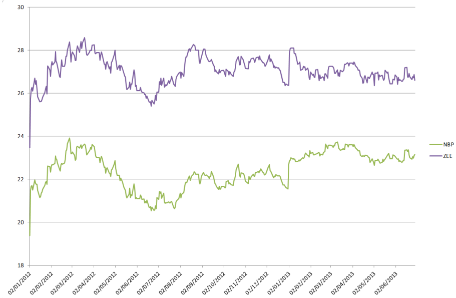
Commodity markets are fundamentally *related*,

- physically by transport pipeline, e.g. substitute one gas for the other
- by indexation, e.g. some gas indices are connected to oil indices like Brent, gasoil, fueloil
- by production, e.g. power is generated by burning coal or gas
- by refinement, e.g. crude oil is refined to get gasoil, fueloil, jet fuel, ...

Co-Movement in Commodity Markets



historical Y1 prices TTF, NCG in Euro/MWh



historical Y1 prices NBP, ZEE in pence/therm

- Are commodities only correlated?
- Does there exists a fundamental relationship, which drives the markets?

Correlation And Co-Movement

- A large portion of energy companys's risk profile is due to exposure to changing cross-commodity spreads.
- In order to understand this risk, to optimize a company's portfolio against this exposure and to montearize the spreads by trading, a multi market model is necessary which captures the **key value drivers** of the combined commodity dynamics.

- We would like to understand what impact a fundamental relationship between commodity markets has onto key value drivers, as volatility and correlation.

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Forward Model

Basic assumptions:

- We assume a two commodities market, G drives the market, P is driven. *For example, consider TTF as driving market and NCG as driven market.*
- Price indices for G and P can be observed at all times t. *For example, we can consider settlement forward prices (M1, Y1).*
- First, consider a forward model (say in historical measure), later in talk extension to spot.
- We assume a fundamental relationship between the commodities. *For example, gas hubs are connected, gas can be transported, ...*
- We assume an (instantaneous) correlation relationship between the commodities. *For example, instantaneous events (like cold weather across market borders, outages, ...) effect markets in a correlated way.*

Forward Model

$$
\begin{aligned}
dG &= \mu_G G dt + \sigma_G G dW_G \\
dP &= \kappa(c + b \log G - \log P) \cdot P dt + \sigma_P P dW_P
\end{aligned}
$$

Note:

- Dynamics of driving market is GBM.
- Dynamics of P inspired by one-factor Lucia-Schwartz model with mean-reversion to a stochastic level defined by G.
- P is mean-reverting to the stationary level

$$P = e^c \cdot G^b.$$

- Parameter b is called *characteristic exponent* of P, G.
- Markets are instantaneous *correlated* by $dW_G dW_P = \rho dt$.
- Markets are fundamentally related by

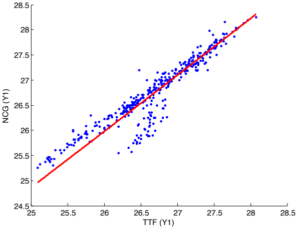
$$c + b \log G - \log P = 0,$$

  the driven market P is *mean-reverting* to this market relationship with mean-reversion speed $\kappa > 0$.

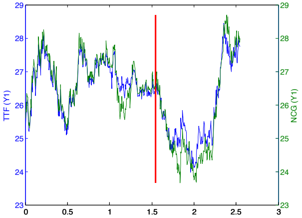
Examples

- commodities: TTF in Euro/MWh (driving), NCG in Euro/MWh (Y1 ahead) between 1/1/2012 and 30/06/2013.
- parameter estimation with maximum likelihood.

σ_G	σ_P	ρ	κ	b	c
10.92%	11.94%	0.78	18.37	1.12	-0.39



data points and equilibrium relation

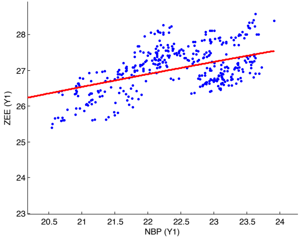


historical prices and (one) simulation

Examples

- commodities: NBP in pence/therm (driving), ZEE pence/therm (Y1 ahead) between 1/1/2012 and 30/06/2013.
- parameter estimation with maximum likelihood.

σ_G	σ_P	ρ	κ	b	c
14.74%	15.61%	0.76	13.61	0.29	2.41



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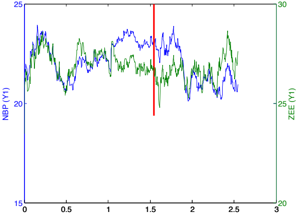
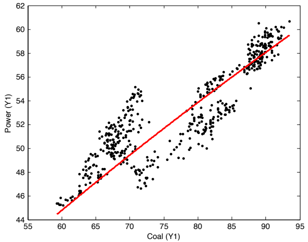


historical prices and (one) simulation

Examples

- commodities: API#2 coal in Euro (driving), German power (Y1 ahead) between 1/1/2010 and 31/12/2011.
- parameter estimation with maximum likelihood.

σ_G	σ_P	ρ	κ	b	c
17.20%	14.99%	0.69	4.20	0.64	1.17



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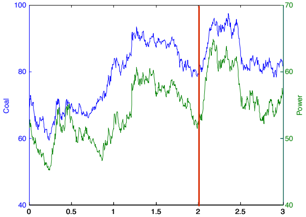


historical prices and (one) simulation

Examples

- *Location spreads* have large mean-reversion rate.
- *Dark spread* has medium mean-reversion rate.
- Spreads are typically high correlated.
- TTF and NCG are nearly proportional (in the considered time period).

Analytical Results - Terminal Variances

Let $\tilde{G}(t) := \log G(t)$, $\tilde{P}(t) := \log P(t)$ be log-prices and $v_{\tilde{G}}(s,t)$, $v_{\tilde{P}}(s,t)$ be their terminal variances at time $s < t$. It is,

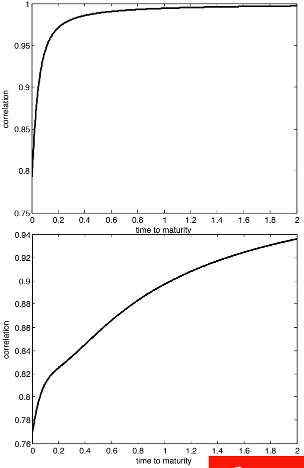
$$v_{\tilde{G}}(s,t) := \mathrm{Var}(\tilde{G}(t)|\mathcal{F}(s)) = \sigma_G^2 \cdot (t-s)$$
$$v_{\tilde{P}}(s,t) := \mathrm{Var}(\tilde{P}(t)|\mathcal{F}(s)) = \mathcal{I}_{s,t}^{(P)} + \mathcal{I}_{s,t}^{(G)} + \mathcal{I}_{s,t}^{(GP)}$$

with integral functions given by

$$\mathcal{I}_{s,t}^{(P)} := \frac{\sigma_P^2}{2\kappa}(1 - e^{-2\kappa(t-s)})$$
$$\mathcal{I}_{s,t}^{(G)} := b^2\sigma_G^2 \cdot \left[(t-s) - \frac{2(1-e^{-\kappa(t-s)})}{\kappa} + \frac{1-e^{-2\kappa(t-s)}}{2\kappa}\right]$$
$$\mathcal{I}_{s,t}^{(GP)} := 2b\rho\sigma_G\sigma_P \cdot \left[\frac{1-e^{-\kappa(t-s)}}{\kappa} - \frac{1-e^{-2\kappa(t-s)}}{2\kappa}\right].$$

Analytical Results - Terminal Correlation

- Terminal covariance and variance determine terminal correlation.
- For short maturities the terminal correlation is determined by the instantanteous correlation of the system, as in a GBM model.
- For long maturities the terminal correlation approaches 1.
- Although in each *discrete* time step (e.g. Euler scheme) an instantaneous correlation is driving the direction of the commodities, the (long term) termnial correlation depends on the fundamental relationship.
- The instantaneous correlation is not enough to describe the complete dynamics of the system.
- Co-movement in commodities results in a correlation term structure.



Spread and Spread Options

Let the spread between P and G be defined by

$$S(t) := P(t) - \alpha G(t)$$

with *conversion rate* $\alpha > 0$.

- Find *merit figure* to summarize most prominent behavior of the spread into a single number.

Use an *exchange option* and take the Black-Scholes implied volatility to describe the dynamics of the spread. Payoff is

$$\Lambda(G(T), P(T)) := (P(T) - \alpha \cdot G(T))^+ = (S(T))^+.$$

- *Actuarial valuation* for option value $\Pi_{t,T}$ yields,

$$\Pi_{t,T} := e^{-r(T-t)}\mathbb{E}^{\mathbb{P}}(\Lambda(G(T), P(T))|\mathcal{F}(t)).$$

Spread and Spread Options

Proposition

The Actuarial value at time t is

$$\Pi_{t,T} = e^{-r(T-t)} \cdot \left(H^G_{t,T} N(d_1) - \alpha H^P_{t,T} N(d_2) \right)$$

with

$$d_1 = \frac{\log(H_{t,T}/\alpha) + v(t,T)/2}{\sqrt{v}}, \qquad d_2 = d_1 - \sqrt{v}$$

$$H^G_{t,T} := \mathbb{E}^{\mathbb{P}}_t(G(T)), \qquad H^P_{t,T} := \mathbb{E}^{\mathbb{P}}_t(P(T))$$

$$H_{t,T} := H^P_{t,T} / H^G_{t,T}$$

and

$$v(t,T) = v_{\tilde{G}}(t,T) + v_{\tilde{P}}(t,T) - 2\mathrm{cov}_{\tilde{G}\tilde{P}}(t,T)$$

Spread Volatility

$$v(0,t) = (b-1)^2\sigma_G^2 \cdot t + 2(1-b)\frac{1-e^{-\kappa t}}{\kappa}(b\sigma_G^2 - \rho\sigma_G\sigma_P) + \frac{1-e^{-2\kappa t}}{2\kappa}(\sigma_P^2 + b^2\sigma_G^2 - 2b\rho\sigma_G\sigma_P)$$

- Standard model for commodities is Lucia-Schwartz two factor model,

$$S(t) = f(t) + X(t) + Y(t)$$
$$dX = -\kappa_X X dt + \sigma_X dW_1$$
$$dY = \sigma_Y dW_2$$

- S is price index, f is a deterministic function, $dW_1 dW_2 = Rdt$.
- Terminal variance in LS is given by

$$v^{LS}(0,t) = \frac{\sigma_X^2}{2\kappa_X}\left(1 - e^{-2\kappa_X t}\right) + \sigma_Y^2 t + \frac{2R\sigma_X\sigma_Y}{\kappa_X}\left(1 - e^{-\kappa_X t}\right).$$

Spread Volatility

$$v(0,t) = (b-1)^2\sigma_G^2 \cdot t + 2(1-b)\frac{1-e^{-\kappa t}}{\kappa}(b\sigma_G^2 - \rho\sigma_G\sigma_P) + \frac{1-e^{-2\kappa t}}{2\kappa}(\sigma_P^2 + b^2\sigma_G^2 - 2b\rho\sigma_G\sigma_P)$$

- Hence, spread variance can bee seen as terminal variance in LS by setting

$$\sigma_Y = |1-b|\sigma_G, \qquad \sigma_X = \sqrt{\sigma_P^2 + b^2\sigma_G^2 - 2b\rho\sigma_G\sigma_P}$$

$$R = \frac{|\rho\sigma_P - b\sigma_G|}{\sqrt{\sigma_P^2 + b^2\sigma_G^2 - 2b\rho\sigma_G\sigma_P}}, \qquad \kappa_X = \kappa$$

Thus, the two factor Lucia-Schwartz model is well suited to model **directly** the spread dynamics of a co-integrated commodity pair.

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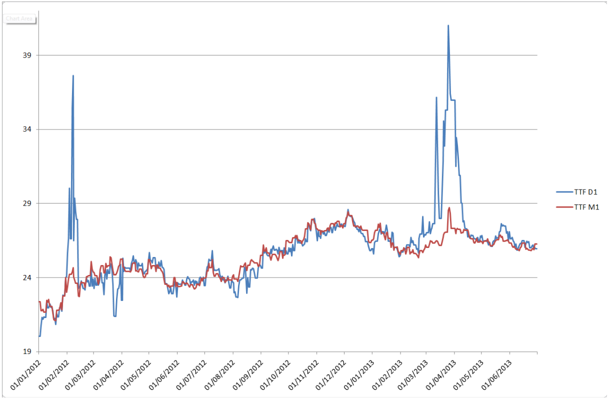
Toy (Spot) Model



Toy (Spot) Model

Basic assumptions:

- We assume a two commodities market, G drives the market, P is driven.
- We assume a forward and a spot market.
- We assume that each log-spot index *mean-reverts* to the log - month-ahead forward price. Hence, we consider the M1 price as the best information for future spot prices.
- We assume a fundamental relationship between the commodities.
- We assume an (instantaneous) correlation relationship between the commodities in the forward market.
- We assume an (instantaneous) correlation relationship between spot markets.

Toy (Spot) Model

$$S_G = \exp\left(g(t) + X_G(t) + \log G(t)\right)$$
$$S_P = \exp\left(p(t) + X_P(t) + \log P(t)\right)$$

with Ornstein-Uhlenbeck processes

$$dX_G = -\kappa_G^S X_G dt + \sigma_G^S dW_G^S$$
$$dX_P = -\kappa_P^S X_P dt + \sigma_P^S dW_P^S$$

and $G(t)$, $P(t)$ given by the forward model, $X_G(0) = X_P(0) = 0$.

Toy (Spot) Model

$$S_G = \exp\left(g(t) + X_G(t) + \log G(t)\right)$$
$$S_P = \exp\left(p(t) + X_P(t) + \log P(t)\right)$$

- $g(t), p(t)$ are deterministic functions to account for seasonality.
- G, P are correlated and have co-movement as shown above.

$$dX_G = -\kappa_G X_G dt + \sigma_G^S dW_G^S$$
$$dX_P = -\kappa_P X_P dt + \sigma_P^S dW_P^S$$

- κ^S is spot mean-reversion, σ^S is spot volatility.
- spot is correlated, $dW_G^S dW_P^S = \rho^S dt$.
- (Remark: there are spot products, like strips of daily options which are traded with an additional spread vs the underlying forward option. This spread is defined here by κ^S, σ^S).

Parameter Estimation

- First, estimate model parameters for forward markets (on M1 products).
- Second, estimate spot parameters by maximum likelihood. If necessary, filter data.
- Third, estimate instantaneous spot correlation on historical time series.

Example: driving market (G) is **TTF**, driven market (P) is **NCG**

forward estimation (M1)

σ_G	σ_P	ρ	κ	b	c
19.44%	19.87%	0.88	21.23	0.98	0.06

Parameter Estimation

- First, estimate model parameters for forward markets (on M1 products).
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Example: driving market (G) is **TTF**, driven market (P) is **NCG**

forward estimation (M1, adjusted)

σ_G	σ_P	ρ	κ	b	c
19.44%	19.87%	0.88	18.37	1.12	-0.39

Parameter Estimation

- First, estimate model parameters for forward markets (on M1 products).
- Second, estimate spot parameters by maximum likelihood. If necessary, filter data.
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Example: driving market (G) is **TTF**, driven market (P) is **NCG**

forward estimation (M1, adjusted)

σ_G	σ_P	ρ	κ	b	c
19.44%	19.87%	0.88	18.37	1.12	-0.39

spot estimation (including cold Feb12 and Mar13)

σ_G^S	σ_P^S	ρ^S	κ_G^S	κ_P^S
58.18%	60.63%	0.90	35.56	41.35

Analytical Resullts

Forward and spot dynamics are **not correlated**, hence

- terminal variances of OU and forward dynamic can be added,

$$v_{\tilde{S}_G}(0,t) = \frac{\sigma_G^2}{2\kappa_G^S}\left(1 - e^{-2\kappa_G^S t}\right) + v_{\tilde{G}}(0,t)$$

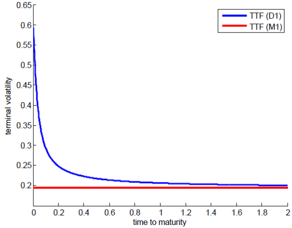
$$v_{\tilde{S}_P}(0,t) = \frac{\sigma_P^2}{2\kappa_P^S}\left(1 - e^{-2\kappa_P^S t}\right) + v_{\tilde{P}}(0,t)$$

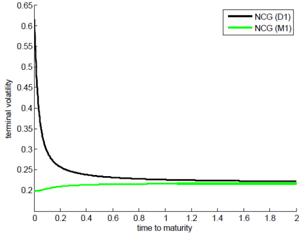
- terminal covariance

$$\mathrm{cov}_{\tilde{S}_G,\tilde{S}_P}(0,t) = \frac{\rho^S \sigma_G^S \sigma_P^S}{\kappa_G^S + \kappa_P^S}\left(1 - e^{-(\kappa_G^S+\kappa_P^S)t}\right) + \mathrm{cov}_{\tilde{G},\tilde{P}}(0,t)$$

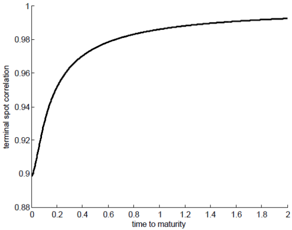
- from these formulas one can derive expression for the terminal (total) spot correlation as well.

Analytical Resullts





- Spot volatility is decreasing. Short term spot volatility is high.
- Long term spot volatility approaches long term forward volatility.
- Terminal spot correlation is increasing.



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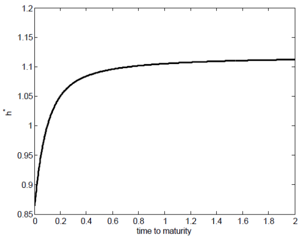
Proxy Hedges - Hedge Ratio

Proxy Hedge

- Typically, one has risk exposure to illiquid forward product (P).
- Idea is to hedge this risk exposure by an off-setting position in more liquid forwards (G) or off-setting position with forwards which are already in the portfolio.
- Adopting standard argumentation, the optimal hedge ratio is given by

$$h^*(T) = \rho(T) \cdot \sqrt{\frac{v_{\tilde{P}}(T)}{v_{\tilde{G}}(T)}}$$



- hedge ratio for NCG forwards hedged by TTF products.
- the hedge ratio depends on time to maturity and is different to ratio of a purely GBM model.

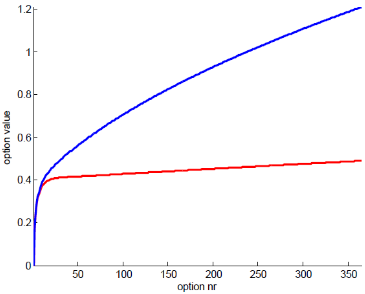
Location Spread Options

- The optionality of shipping gas from one hub to another can be valued by using a spread option.
- Typically, one considers a strip of daily spread options, *the right but not the obligation to ship gas by a pipeline.*

- Study the impact of co-movement to option valuation - toy example (not actual pricing!)
- The terminal log spot prices are normally distributed in the above toy spot model, hence we can use standard machinery to value spread options (e.g. approximation formulas or direct quadrature, ...)

Location Spread Options

- Consider a strip of 365 daily spread options on TTF and NCG. Assume $g(t) = p(t) = \log(30)$ and strike $K = 0$, hence ATM. The first option of the strip shall expiry the next day.
- Compute a valuation with co-movement and the above parameters, and a valuation with switched off co-movement (i.e. forward $\kappa = 0$).



- Valuation in co-movement model:

  0.44 Euro / MWh

- Valuation in purely correlated GBM model:

  0.86 Euro / MWh

Conclusion

- Simple, analytical traceable co-integrated model for two commodities (forward and spot) to study spread dynamics.
- Model suggests that direct modeling of co-integrated spread dynamics is admissible.
- Fundamental relationships on commodity markets have impact on long term terminal correlation.
- Co-movement has impact on risk exposure of energy companys and should be considered in risk models.