

***EU ETS Futures Spread Analysis and Recommendations for Effective
Trading and Market Design***

Walid Mnif

Joint work with: Matt Davison
The University of Western Ontario,
London, Ontario, Canada.

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General Introduction

- CO_2 levels in the atmosphere have been linked with global warming
- Kyoto treaty agreed CO_2 emissions should be reduced.
- How to do this in a free market economy?
- Policy instruments include: Emission tax, cap-and-trade , or a hybrid of both.

Emission Tax

- Price that an emitter must pay per unit of greenhouse gas emission.
- Companies choose between paying the emission tax or reducing their pollution: Tax rate vs. Marginal cost of abatement.

Cap-and-Trade

- Regulator sets an absolute emissions limit or cap and issues equivalent tradable allowances.
- Market-based mechanism
- Emitters with expensive abatement cost can buy emissions rights from those who can abate more cheaply.

Hybrid Mechanism (aka safety valve system)

- When prices are high, companies may purchase allowances from the regulator rather than from the market.
- May have a floor price in addition to the ceiling price.

Earlier Work (Non-exhaustive)

- Equilibrium models:

Hitzemann and Uhrig-Homburg (2011), Borovkov et al. (2011), Carmona et al. (2010), Hinz and Novikov (2010), Kijima et al. (2010), Carmona et al. (2009), Seifert, Fehr and Henz (2009), and Maeda (2004),...

- Stochastic framework:

Carmona and Hinz (2011), Çetin and Verschuere (2009), Grüll and Kiesel (2009),...

The European Union Emissions Market Scheme (EU ETS)

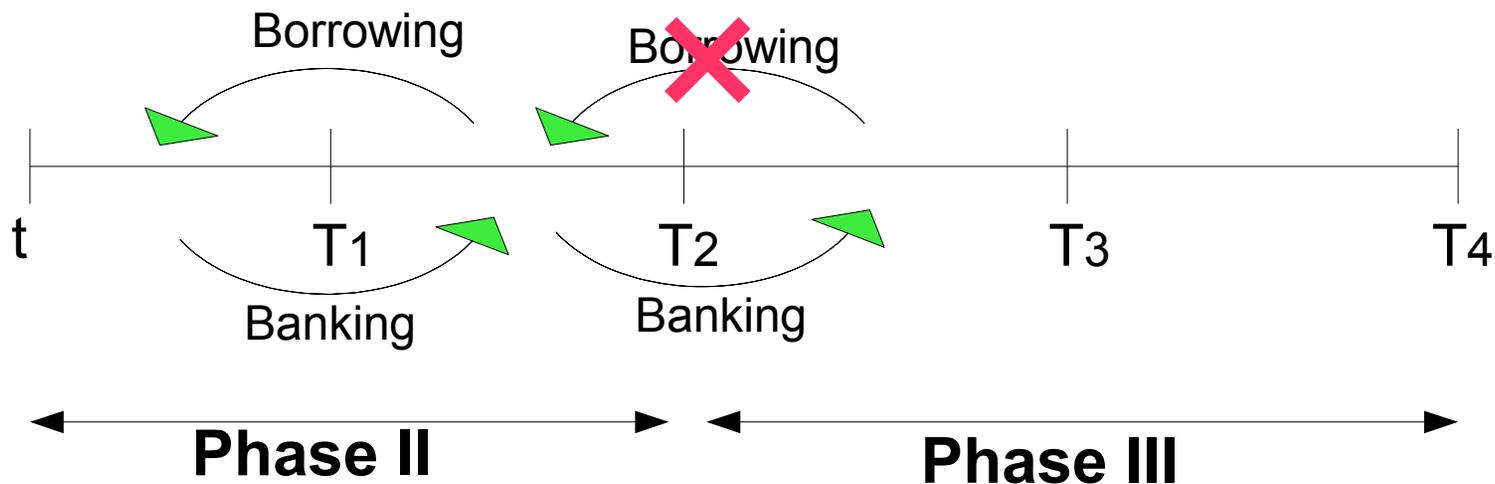
- Cap-and-trade scheme limited to European industrial installations.
- Progressively introduced until fully applied to all sectors by 2027.
- Companies not providing allowances to cover their total emissions must pay a penalty and deliver the missing allowances in the following year.
- Started in 2005, and separated into different phases: Pilot (I): 2005-2007, Kyoto (II): 2008-2012, Post Kyoto (III): 2013-2020, IV: 2021-2028.

- Phase I

- ▷ Excessive free allocation + No banking.
- ▷ Abated 3% of total emissions.

- Phases II and III:

- ▷ Number of Allowances: \searrow 1.74% annually.
- ▷ Intra/inter-phase Banking is allowed.
- ▷ Borrowing is possible between years falling within the same phase, but not from the next phase.



Spot vs Futures markets?

- Largest: spot Market: NYSE Euronext ; futures market: European Climate Exchange.
- Futures are more liquid than spot allowances:
 - ▷ Spot: Considered as a good → Subject to Value-Added Tax (VAT).
 - ▷ Futures: VAT exempt and treated as financial transactions.
- Spot allowances were stolen from national registries and traded → Temporary suspension on spot trading on several exchanges.
- Spot market value: ↘ from US\$7.5 billion in 2008 to US\$2.8 billion in 2011.
- Futures market value: Increasing steadily to reach US\$130.8 billion in 2011.

Prices high or low, not in between!!!

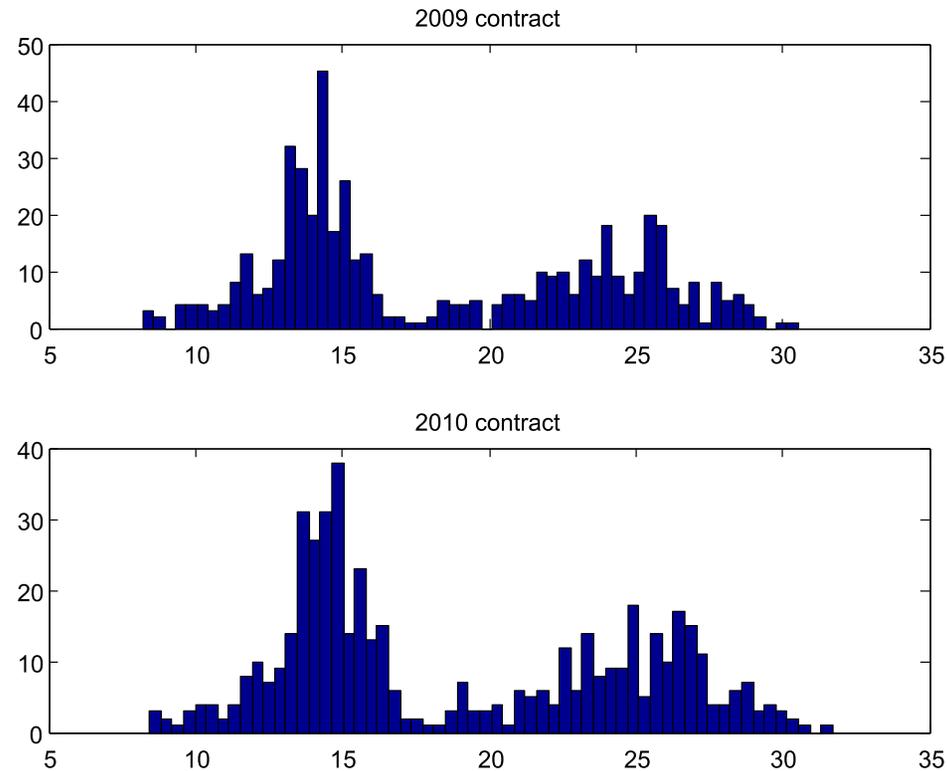


Figure 1: Price distribution over time of Dec-2009 and Dec-2010 contracts

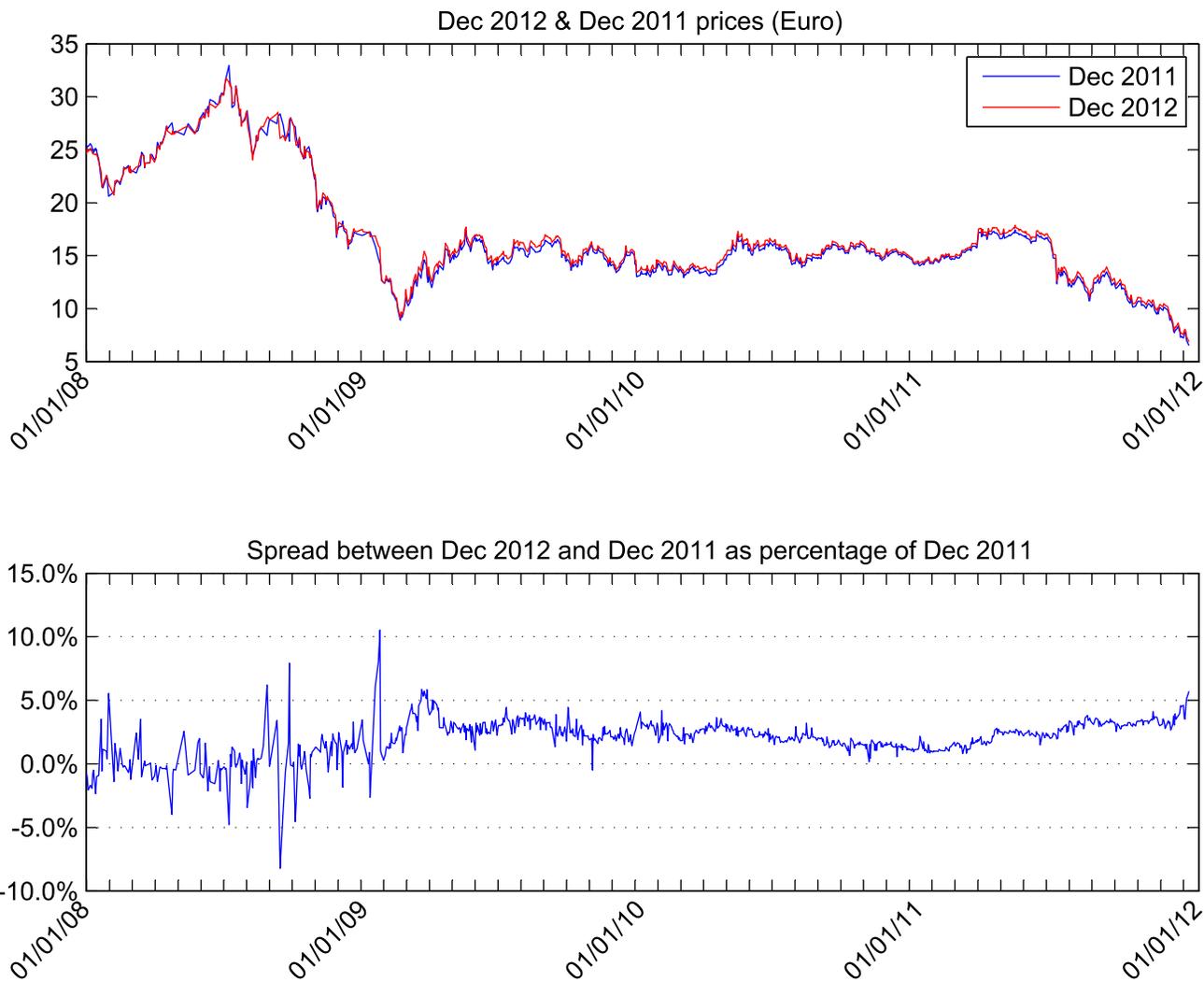


Figure 2: Spread between Dec-2012 and Dec-2011 contracts **discounted** to December 2011 money value using EURIBOR Futures.

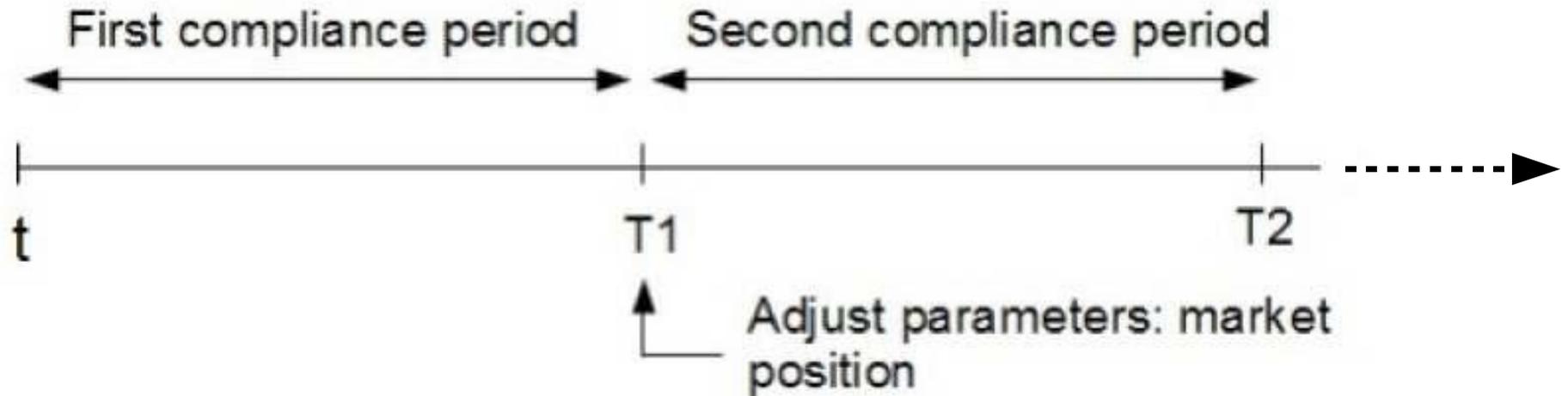
Objectives

- Study the spread.
- Investigate discrete and continuous time models and estimate their parameters.
- Provide pricing tools for different contingent claims that incorporate our empirical findings.
- Suggest recommendations for both participants and regulator.

Part I: Discrete-Time Model

Futures Allowance Dynamics

- S_t : d-dimensional vector of discounted futures allowance price, $S_t^i, t \leq T_i$ used for compliance at T_i



Futures Allowance Dynamics

- Discrete time framework with $(\Omega, (\mathcal{F}_t)_{t \geq 0}, \mathbb{P})$
- Define the \mathcal{F}_t -measurable process ξ_t^i such that:

$$S_t^i = \xi_t^i S_{t-1}^i, \forall t \geq 1,$$

under market completeness assumption.

- Assume:
 - ▷ Market is incomplete
 - ▷ Information set describing expected market position strongly affects the prices

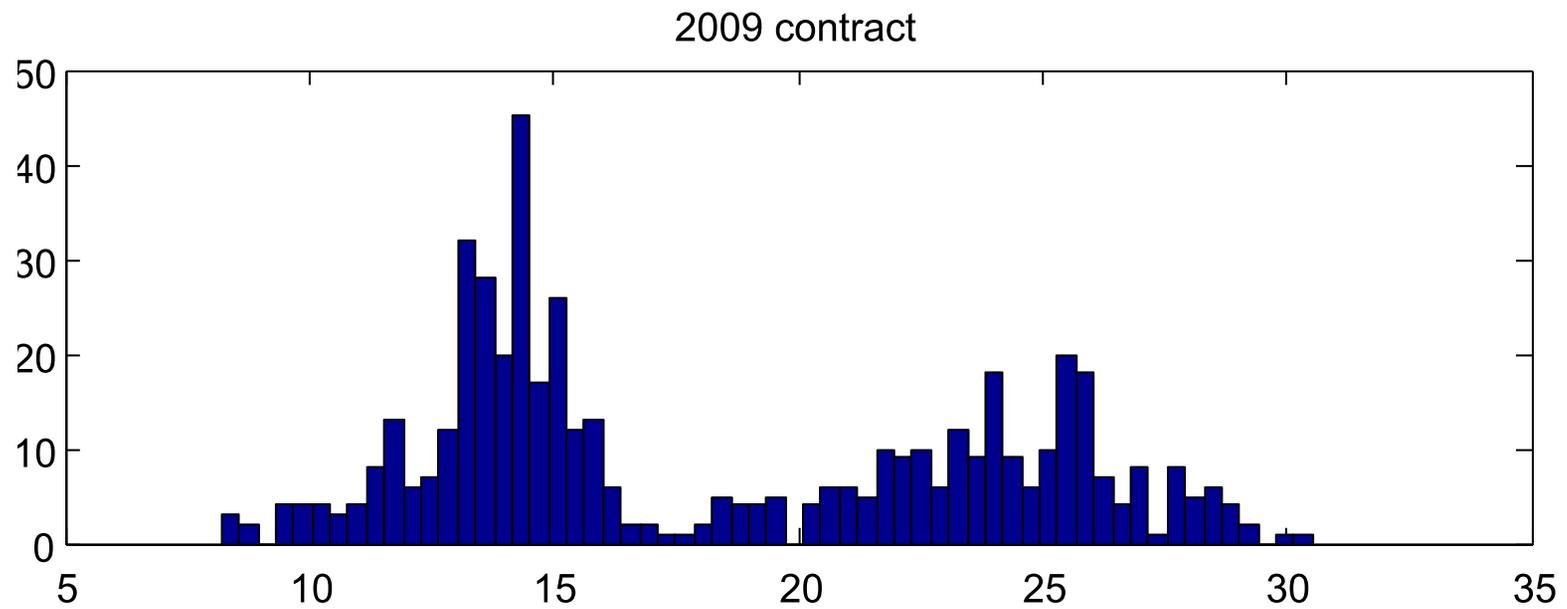


Figure 1: Price distribution over time of Dec-2009 and Dec-2010 contracts * (Upper part)

Part I Road Map

- Introduce futures allowance dynamics model based on expected market sentiment.
- Parameter estimation for Dec-2009 and Dec-2010 contracts.
- Show that a more appropriate hedging strategy include positions in futures that mature at subsequent compliance dates.
- Recommend policy makers to introduce a new tradeable security.

Futures Allowance Dynamics

- So we model:

$$S_t^i = f_{t-1}^i(\xi_t^i, Y_{t-1})S_{t-1}^i, \forall t \geq 1,$$

where

- ▷ f_{t-1}^i : \mathcal{F}_{t-1} -measurable growth function
- ▷ Y_t : non-observable process reflecting the implied investors market expectation position at time t for the subsequent compliance dates.

Futures Allowance Dynamics

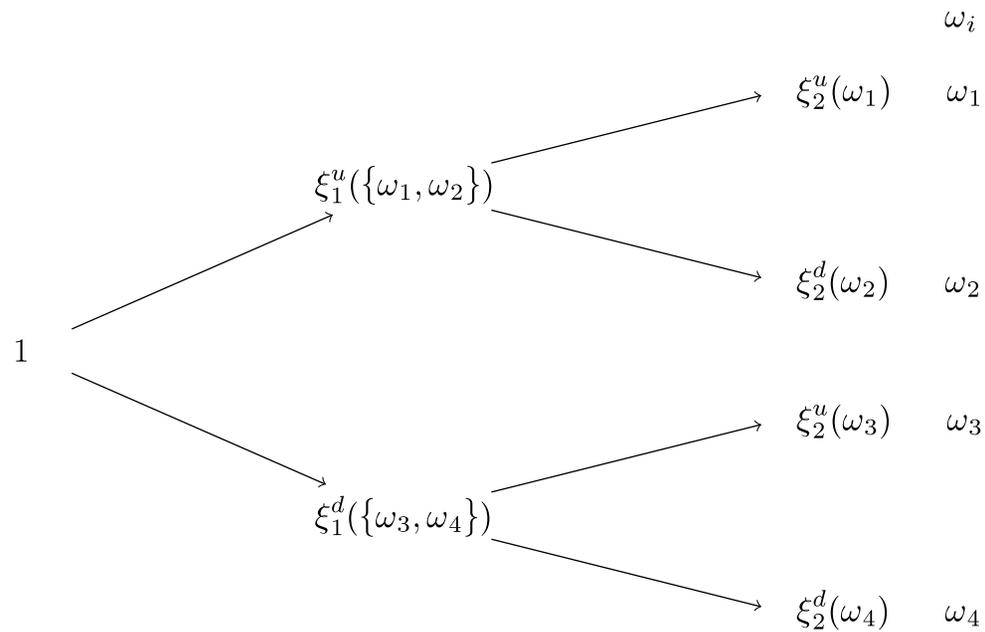


Figure 3: Traded asset growth per step tree in absence of Y_t .

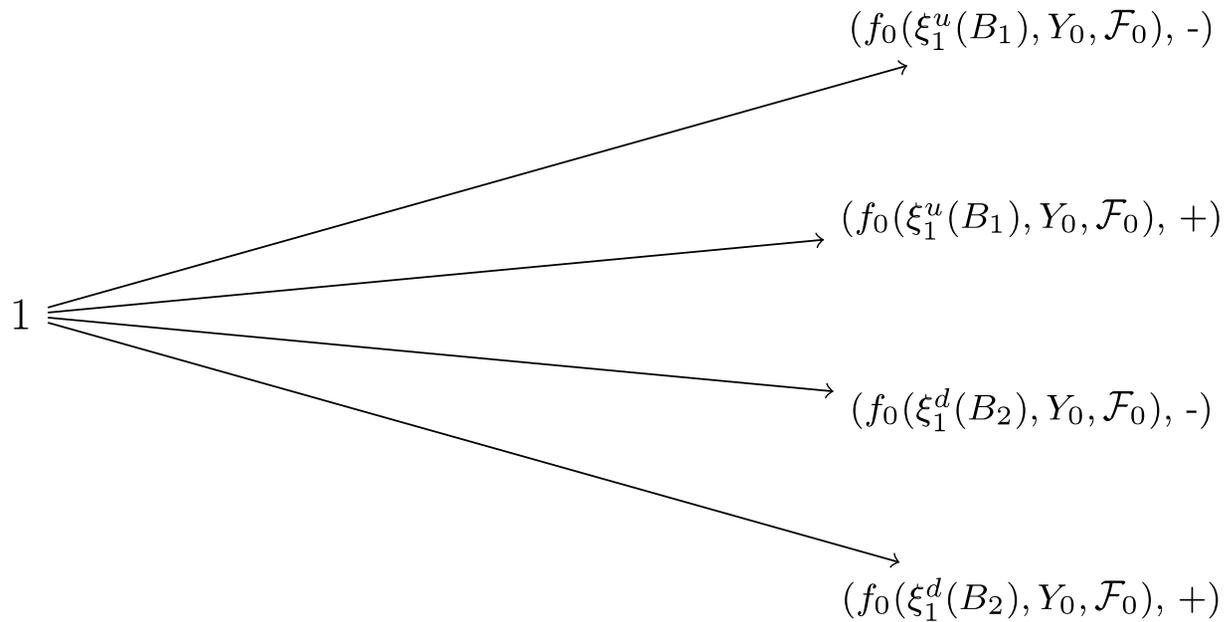


Figure 4: Growths in presence of Y_t at $t=1$, $B_1 = \{\omega_i, i = 1, \dots, 8\}$, and $B_2 = \{\omega_i, i = 9, \dots, 16\}$.

“-”: the market is expected to be short, “+”: the market is anticipated to be long.

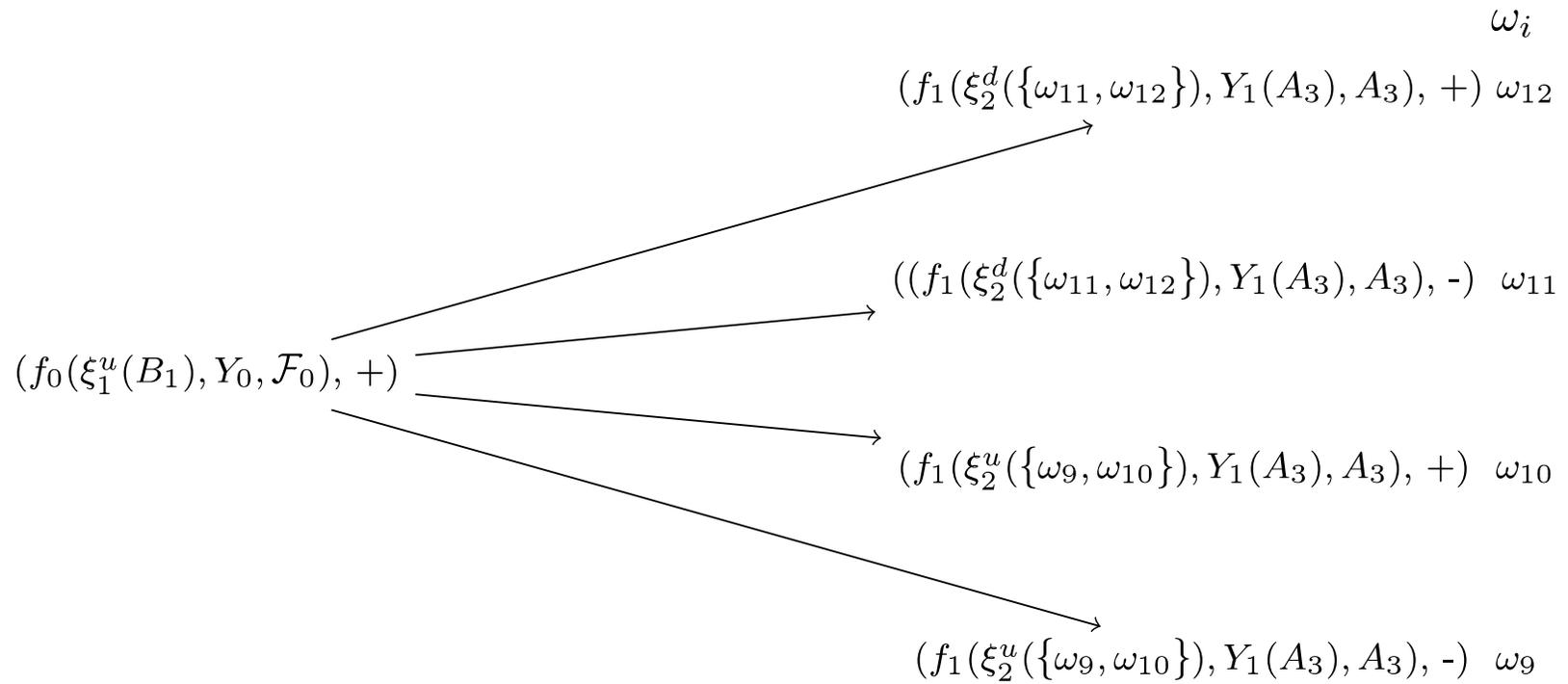


Figure 5: Returns tree in presence of Y_t at $t=2$, where $A_j = \{\omega_{(j-1)*4+i}, i = 1, 2, 3, 4.\}, j = 1, 2, 3, 4.$

Futures Allowance Dynamics

- Parameter estimation
 - ▷ Focus on Dec-2009 (S_t^1) and Dec 2010 (S_t^2) contracts
 - ▷ January 2008- December 2009
 - ▷ Dec-2009 provides information about current (2009) market expected position
 - ▷ Assume $\xi_t^i = \xi^i = \text{Empirical average}$
- We consider a special case for f_{t-1}^i
 - ▷ $f_{t-1}^1(\xi_t^1, Y_{t-1}) = \xi^1 + Y_{t-1}^1$
 - ▷ $f_{t-1}^2(\xi_t^2, Y_{t-1}) = \xi^2 + Y_{t-1}^2$

Futures Allowance Dynamics

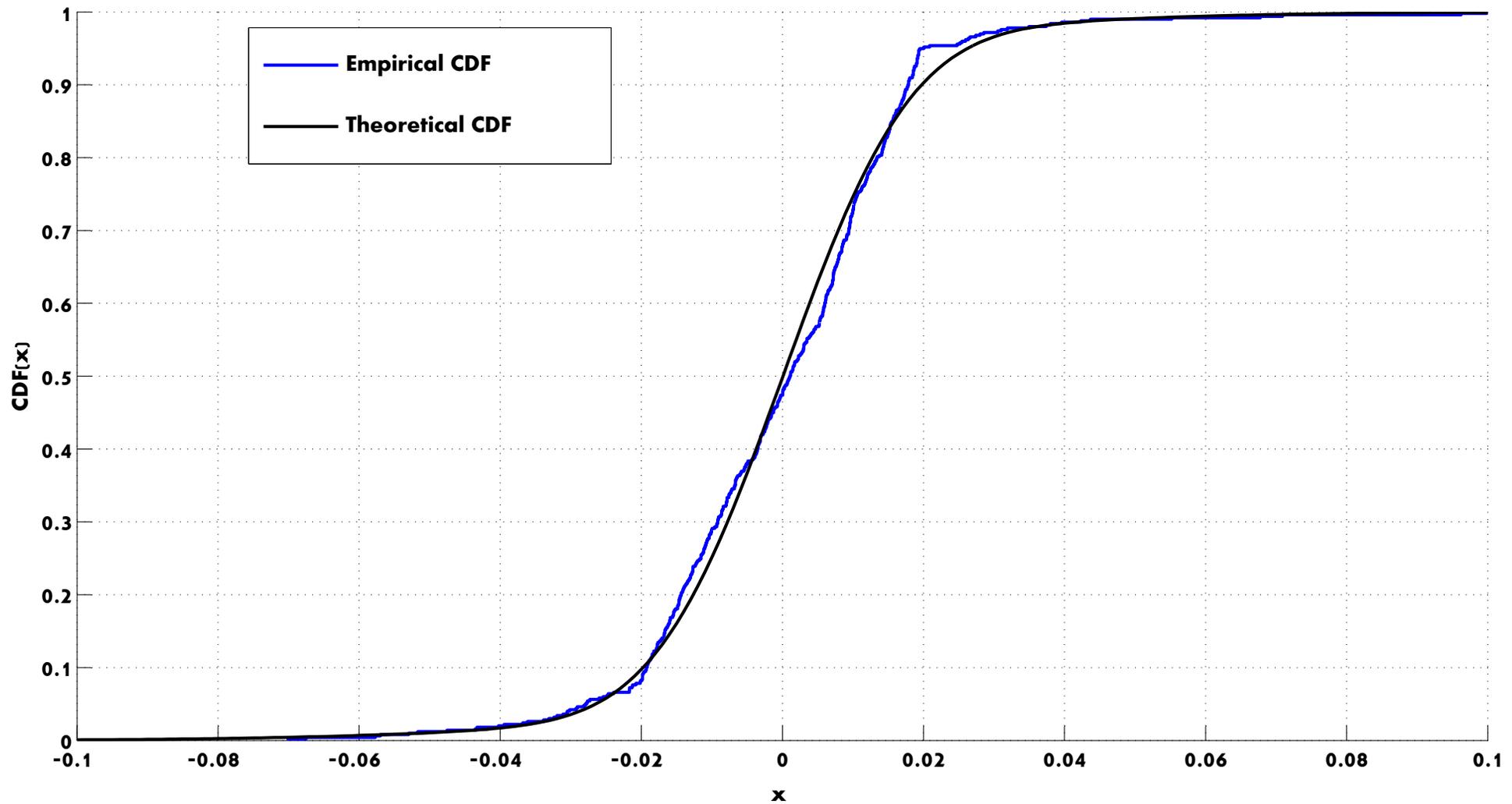
- Estimated growth conditioned on move being positive or negative:

$$\xi^{1u} = 1.021 \quad \xi^{1d} = 0.980$$

$$\xi^{2u} = 1.038 \quad \xi^{2d} = 0.982$$

- Y_t^1 i.i.d.:
 - Follows a Gaussian mixture distribution.
 - Kolmogorov-Smirnov test is accepted at a significance level of 99%.

Figure 6: Comparison of empirical CDF of Y_t^1 with a Gaussian mixture CDF.



Futures Allowance Dynamics

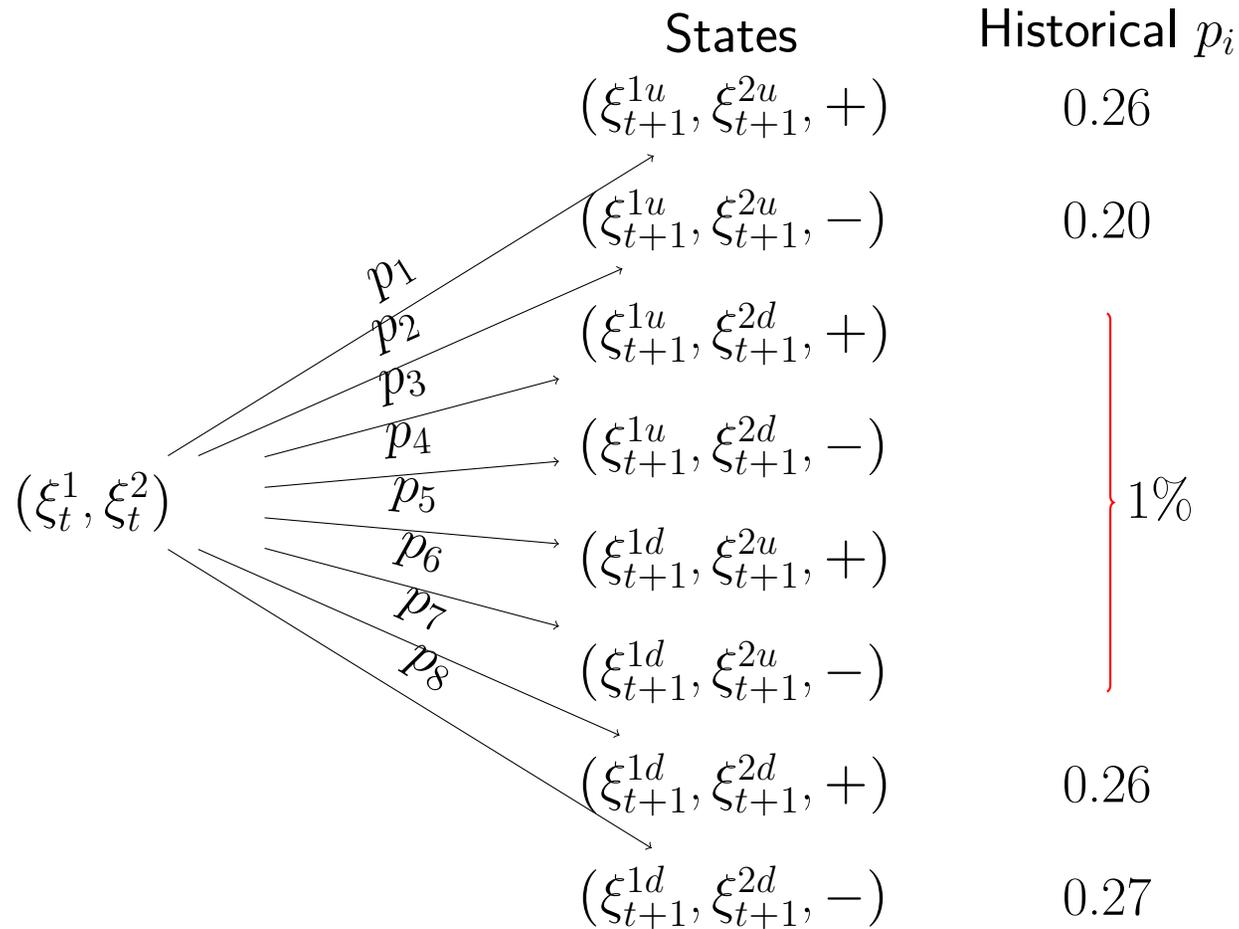
- $Y_t^2 = g(Y_t^1) + I_t + u_t$
 - ▷ u_t i.i.d such that $E[u_t | (Y_t^1, I_t)] = 0$
 - ▷ I_t represents the impact of the expected market position at time T_1 on Y_t^2 .
- I_t is not observed
 - ▷ $I_t = h_0 + h_1 MS_t + v_t$
 - ▷ $E[Y_t^2 | (Y_t^1, I_t, MS_t)] = E[Y_t^2 | (Y_t^1, I_t)]$
 - ▷ MS_t : Expected market position = $\text{Sign}(Y_t^1)$.
- Regression: $Y_t^2 = (h_0 + a_0) + \sum_{k=1}^p a_k (Y_t^1)^k + h_1 MS_t + \epsilon_t$

Futures Allowance Dynamics

	With MS_t		I_t omitted	
	R^2	a_1	R^2	a_1
p=0	68%	-	-	-
p=1	74%	0.5	60%	1
p=3	75%	0.3	63%	1
p=9	76%	0.5	70%	2
p=10	76%	0.5	70%	2

Table 1: Parameters resulting from the OLS estimator as function of the polynomial degree p : a) I_t is omitted or b) it is approximated by the proxy variable MS_t .

Figure 7: States of nature generated at time $t + 1$ by a knot at time t .



Pricing Framework: Investor Side

Random variable $H \in \mathcal{L}^2(\mathcal{F}_T, \mathbb{P})$ describes the payoff

$$(V_0, \zeta) = \arg \min_{(c, \vartheta) \in \mathbb{R} \times \Theta} E_P[(H - c - G_T(\vartheta))^2],$$

where

$$\Theta := \{\text{predictable processes } \vartheta \mid \vartheta'_k \Delta S_k \in \mathcal{L}^2(P)\},$$

$$G_T(\vartheta) := \sum_{j=1}^T \vartheta'_j \Delta S_j.$$

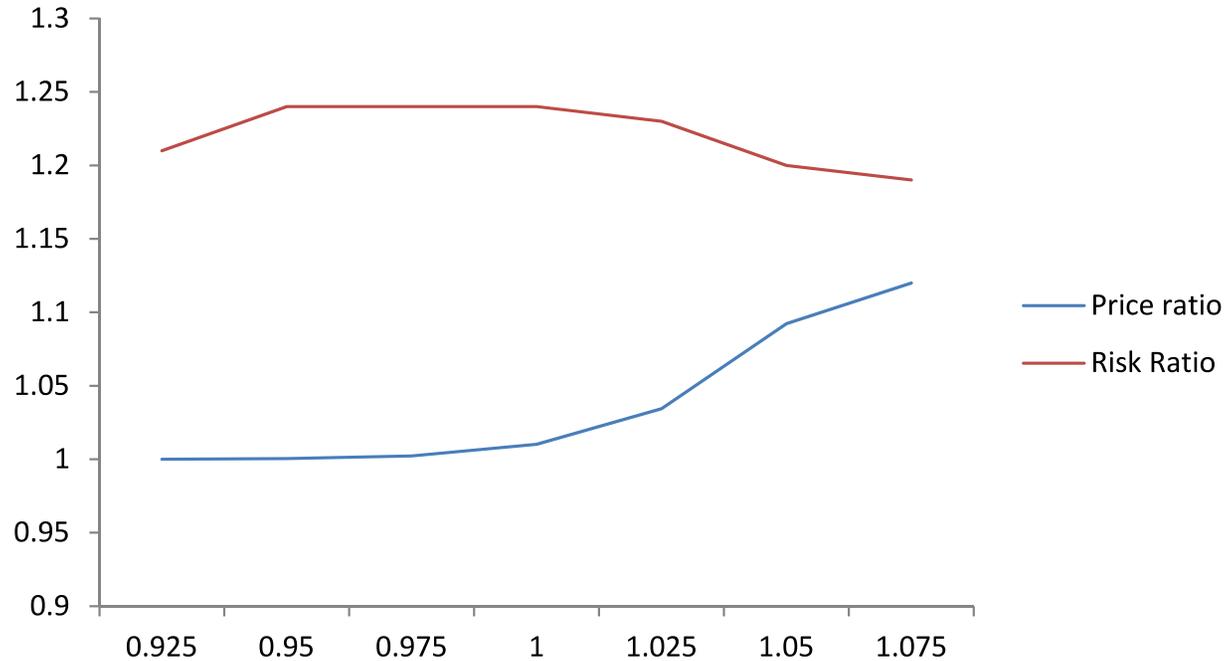
Pricing Framework: Investor Side

- Price a derivative written on S_t^1
 - ▷ Usual markets: Dynamic hedging position on S_t^1
 - ▷ Figure 7: Returns of S_t^1 and S_t^2 have same dynamic pattern
 - ▷ Spread between prices: Evaluates the uncertainty of the expected market position
- Consider portfolios:
 - ▷ I : trading on $S^1 \Rightarrow$ Uses only information provided by S^1
 - ▷ II : trading on both S^1 and $S^2 \Rightarrow$ Uses information provided by both S^1 and S^2
- Short position scenario: prices quoted on 4/4/2008:

$$S_0^1 = \text{€}23.96 \text{ and } S_0^2 = \text{€}24.61.$$

Figure 8: Comparison between strategies for pricing 5 day calls written on S_t^1 for different moneyness. Market is initially assumed to be short but anticipated to be long later on.

$$\text{Price ratio} = \frac{\text{Price I asset}}{\text{Price II assets}}, \text{Risk ratio} = \frac{\text{Unhedged risk I asset}}{\text{Unhedged risk II assets}}$$



Pricing Framework: Investor Side

Proposition 1. *Assume a probability space $(\Omega, \mathbb{F}, \mathbb{P})$, $H \in \mathcal{L}^2(\mathcal{F}_T, \mathbb{P})$, and stochastic process $(S'_t, \Xi_t)'_{t \in \mathcal{T}} \in \mathcal{L}^2_{d+1}(\mathbb{P})$ adapted to the filtration $\mathbb{F} = (\mathcal{F}_t)_{t \in \mathcal{T}}$ such that $E[\Delta S_k^2 | \mathcal{F}_{k-1}]$ and $E[(\Delta S'_k, \Delta \Xi_k)'^2 | \mathcal{F}_{k-1}]$ are \mathbb{P} -a.s. invertible and satisfy the non-degeneracy condition.*

If $P(E[(H - V_0 - G_T(\zeta, S))\Delta \Xi_T | \mathcal{F}_{T-1}] \neq 0) > 0$, then hedging with $(S'_t, \Xi_t)_{t \in \mathcal{T}}$ is more efficient than hedging with $(S_t)_{t \in \mathcal{T}}$.

- S_t^2 can partially explain the unhedgeable risk of strategy A.
- Special feature of the carbon market due to its banking and borrowing features.
- Multiperiod pricing framework more efficient than one period model.

Pricing Framework: Investor Side

- Interdependency between compliance periods:
 - ▷ Requires a multiperiod pricing framework
 - ▷ Allows emitters to reduce the risk in the natural short position
 - ▷ Might decrease market liquidity: non-emitters fearing long term regulatory change may exit the market
- An equivalent solution to encourage non-emitters to trade:
 - ▷ Requires the intervention of the regulator
 - ▷ Implies the introduction of a new tradeable asset in addition to emissions rights

Pricing Framework: Regulator Side

- New tradeable asset G
 - ▷ Allow some of the intrinsic market risk to be hedged
 - ▷ Exogenous to market participants
 - ▷ Considers the social wealth of market parameters Γ , initially set up by the regulator
 - ▷ Consistent with arbitrage free theory
- Indifference pricing
 - ▷ $U(X^{x,\alpha}, \Gamma)$: Utility function of the representative agent
 - ▷ x : Initial wealth; α : initial allowance allocation.
 - ▷ The price $\nu_t(G_{T_1})$ of G is given via:

$$\sup_{\alpha} E_{\mathbb{P}} [U(X^{x,\alpha}, \Gamma)] = \sup_{\alpha} E_{\mathbb{P}} [U(X^{x+\nu_t(G_{T_1}),\alpha} - G, \Gamma)]$$

Pricing Framework: Regulator Side

- Exponential utility

$$U(x) = -e^{-\gamma x}, \quad \forall x \in \mathbb{R} \text{ and } \gamma > 0,$$

“Risk aversion ” γ parameter selected by the regulator.

- Price obtained by moving backward

$$\nu_t(G_{T_1}) = \mathcal{E}_Q^{(t,t+1)}(\nu_{t+1}(G_{T_1})),$$

$$\mathcal{E}_Q^{(s,s+1)}(L_{s+1}) = E_Q \left(\frac{1}{\gamma_s} \log (E_{\mathbb{P}}(e^{\gamma_s L_{s+1}} | \mathcal{F}_s \vee \mathcal{F}_{s+1}^S)) | \mathcal{F}_s \right),$$

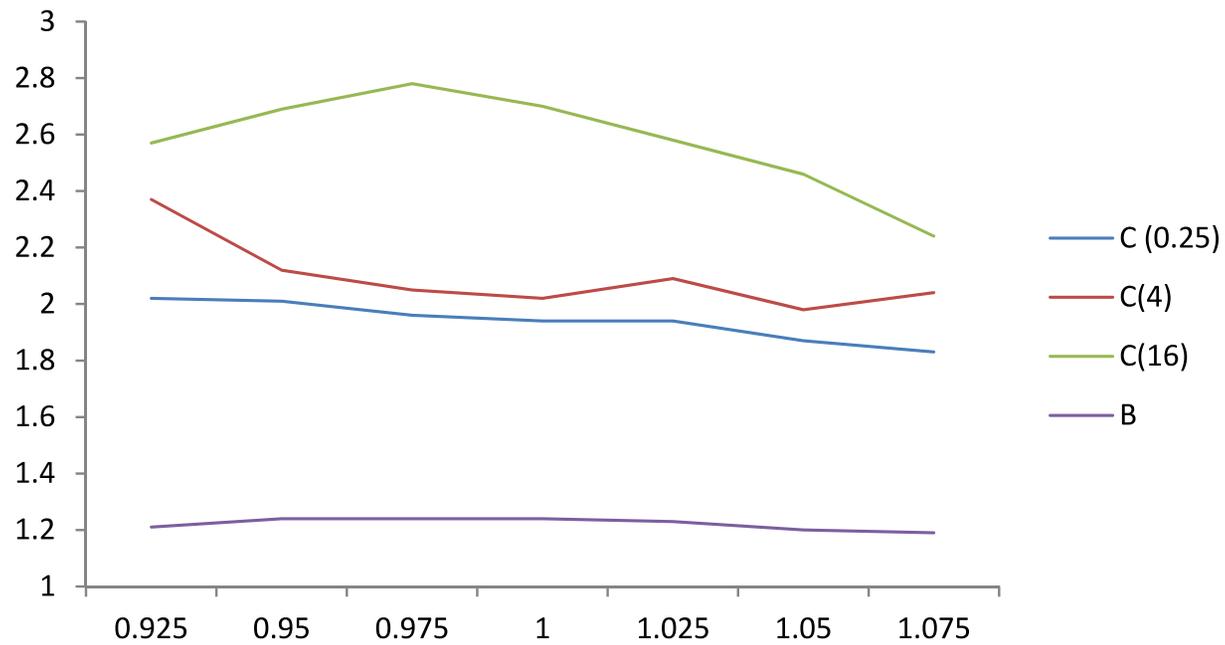
$$\mathcal{E}_Q^{s,s}(L_s) = L_s, \quad \nu_{T_1}(G_{T_1}) = G_{T_1}$$

$$\mathcal{F}^S = \sigma\{S^1\}, \quad Q \text{ is the } S^1 \text{ equivalent martingale measure.}$$

Pricing Framework: Regulator Side

- Example: Digital option
- Pays out a certain amount if a predefined event happens at future time T
- Regulator announcement about the market position at time t : set of Y_t values observable
- Example: Regulator pays 1 unit if he announces the market is short and expected to remain short at the next compliance date
- Strategy III: Investor holds position on S_t^1 and ν_t .

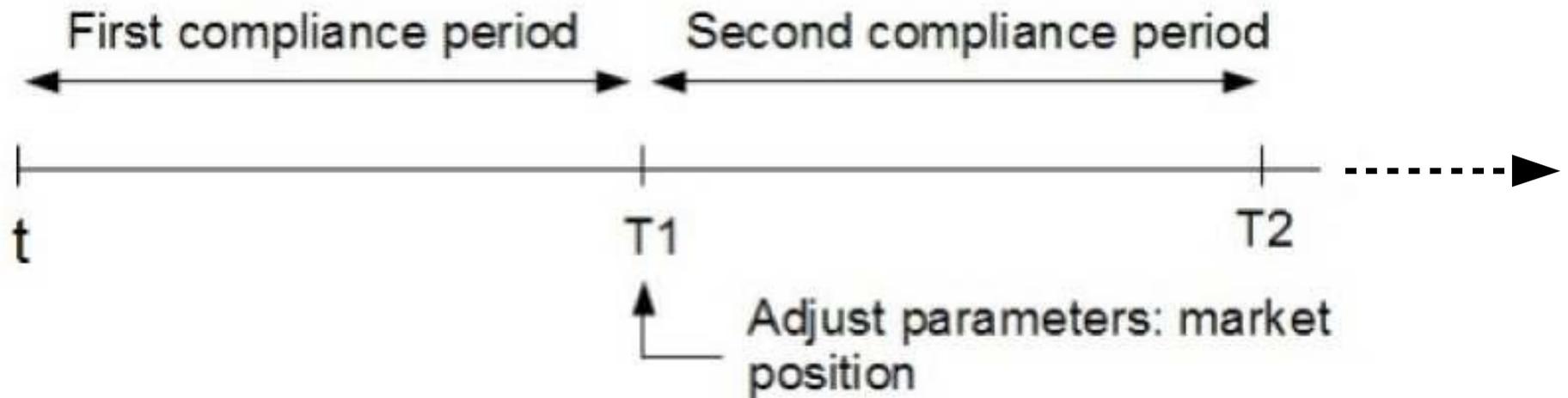
Figure 9: Risk ratio between strategy I and III for different values of $\gamma - C(\gamma)$ – to price 5 day calls for different moneyness.



Part II: Continuous-Time Model

Futures Dynamics

- F_t : d-dimensional vector of discounted futures allowance price, $F(t, T_i), t \leq T_i$ used for compliance at T_i



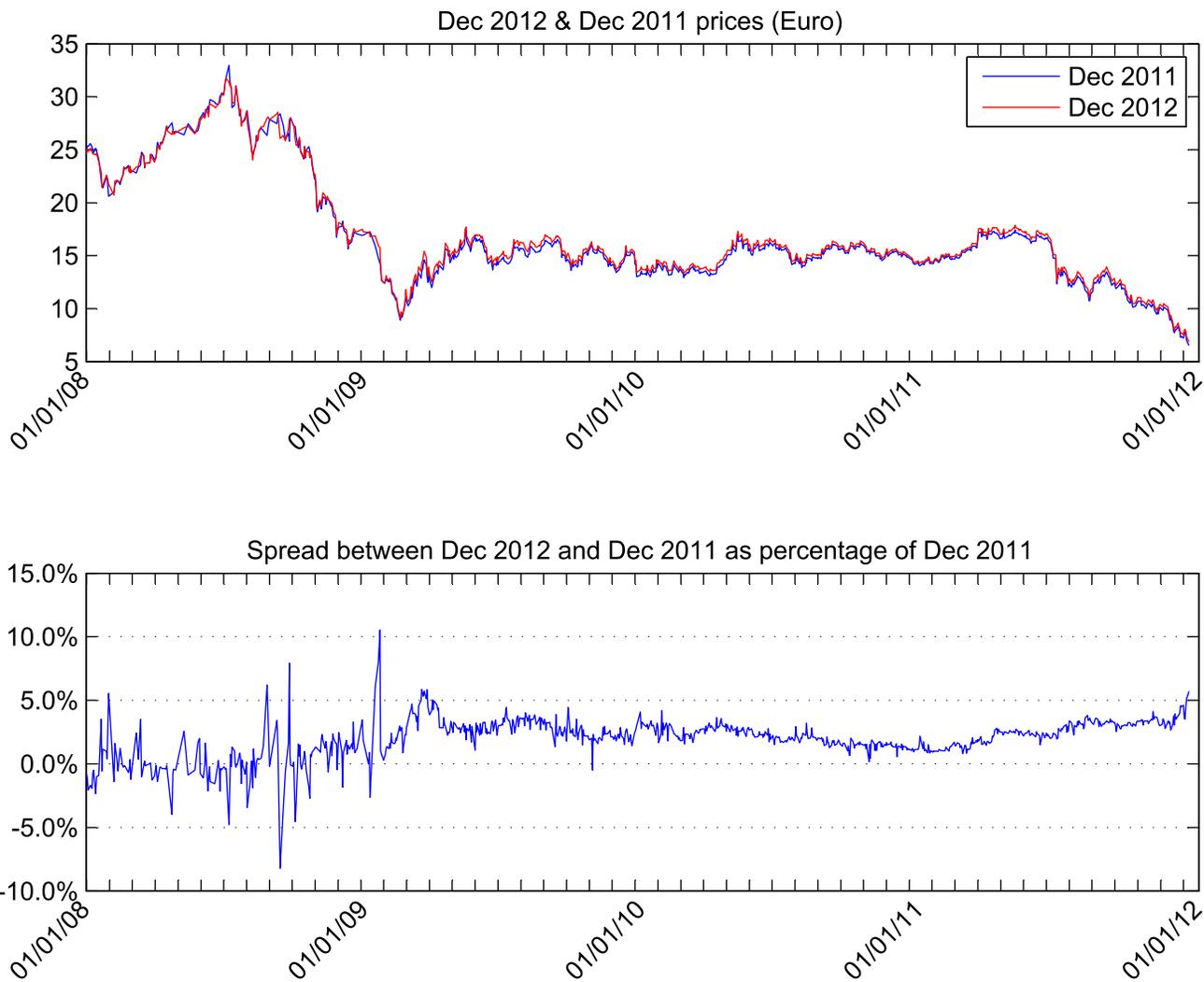


Figure 2: Spread between Dec-2012 and Dec-2011 contracts discounted to December 2011 money value using EURIBOR Futures.

Objectives

- Analyze the spread in order to understand the
 - ▷ relationship between subsequent contracts
 - ▷ impact of any unexpected release of information on returns
- Present a pricing tool for contingent claims under different market schemes where the
 - ▷ market is incomplete.
 - ▷ Black/Scholes framework is a special case.

Part II Road Map

- Futures Dynamics
- Parameter Estimation
- Pricing Framework

Futures Dynamics

Two futures $F(t, T_1)$ and $F(t, T_2)$ mature at subsequent compliance dates $T_1 < T_2$

$$\frac{dF(t, T_1)}{F(t^-, T_1)} = \mu_1 dt + \sigma_{11} dW_{1t} + \varphi_{11} dN_{1t} + \varphi_{12} dN_{2t}, \quad F(0, T_1) > 0,$$

$$\frac{dF(t, T_2)}{F(t^-, T_2)} = \mu_2 dt + \sigma_{21} dW_{1t} + \sigma_{22} dW_{2t} + \varphi_{21} dN_{1t} + \varphi_{22} dN_{2t}, \quad F(0, T_2) > 0.$$

- W_{it} , $i = 1, 2$ are independent Brownian motions.
- N_{it} , $i = 1, 2$ is a Poisson process with λ_i , $i = 1, 2$.

Parameter Estimation

The estimation procedure:

1. Determines return dynamics
2. Numerically maximizes the Log-Likelihood function of the returns
3. Uses step 2 output as an initial guess to a Generalized Expectation-Maximization algorithm
 - ▷ Iterative algorithm
 - ▷ Hypothetical experiment assumes that the total number of jumps that occurs at each time step is constant
 - ▷ Hidden information: Total number of jumps.

Parameter Estimation

Estimate	Dec 2010 - Dec 2011	Dec 2011 - Dec 2012
$\hat{\sigma}_{11}$	44%	47%
$\hat{\sigma}_{21}$	42%	46%
$\hat{\sigma}_{22}$	2%	8%
$\hat{\lambda}_1$	13	20
$\hat{\varphi}_{11}$	1.5%	0.7%
$\hat{\varphi}_{21}$	1.9%	-1.3%
$\hat{\lambda}_2$	20	27
$\hat{\varphi}_{12}$	-0.9%	-0.6%
$\hat{\varphi}_{22}$	-1.2%	0.9%

Table 2: Estimated parameters for Dec 2010 - Dec 2011 and Dec 2011 - Dec 2012 futures.

$$\frac{dF(t,T_1)}{F(t^-,T_1)} = \mu_1 dt + \sigma_{11} dW_{1t} + \varphi_{11} dN_{1t} + \varphi_{12} dN_{2t},$$

$$\frac{dF(t,T_2)}{F(t^-,T_2)} = \mu_2 dt + \sigma_{21} dW_{1t} + \sigma_{22} dW_{2t} + \varphi_{21} dN_{1t} + \varphi_{22} dN_{2t}.$$

Pricing Framework

- Market is incomplete: Delta hedging does not replicate the payoff
- An efficient hedging strategy must be defined with respect to a performance criteria
- We assume the price of a strategy is *fair* if it minimizes the mean square error of the cumulative cost process

Detour: Pricing Framework – Intuition from Discrete time

One period discrete time model, contingent claim H written on F_t :

- ξ_0 : Number of stocks to hold at time 0
- $\eta_i, i = 0, 1$: Bank account amount

Desire:

- have $H = \xi_0 F_1 + \eta_1$
- minimize the cost of the strategy.

Detour: Pricing Framework – Intuition from Discrete time

The cost process at time:

- 0: the initial portfolio value $C_0 = V_0 = \xi_0 F_0 + \eta_0$
- 1: such that : $C_1 - C_0 = \eta_1 - \eta_0$, therefore $C_1 = H - \xi_0(F_1 - F_0)$.

The performance measure is:

$$E [(C_1 - C_0)^2] = E [(H - V_0 - \xi_0(F_1 - F_0))^2]$$

The best strategy is the one that minimizes the performance measure.

End Detour: Pricing framework

The price V_0 of any contingent claim H is:

$$V_0 = E_{\widehat{P}}[H],$$

where \widehat{P} is an equivalent local martingale measure called the minimal martingale measure.

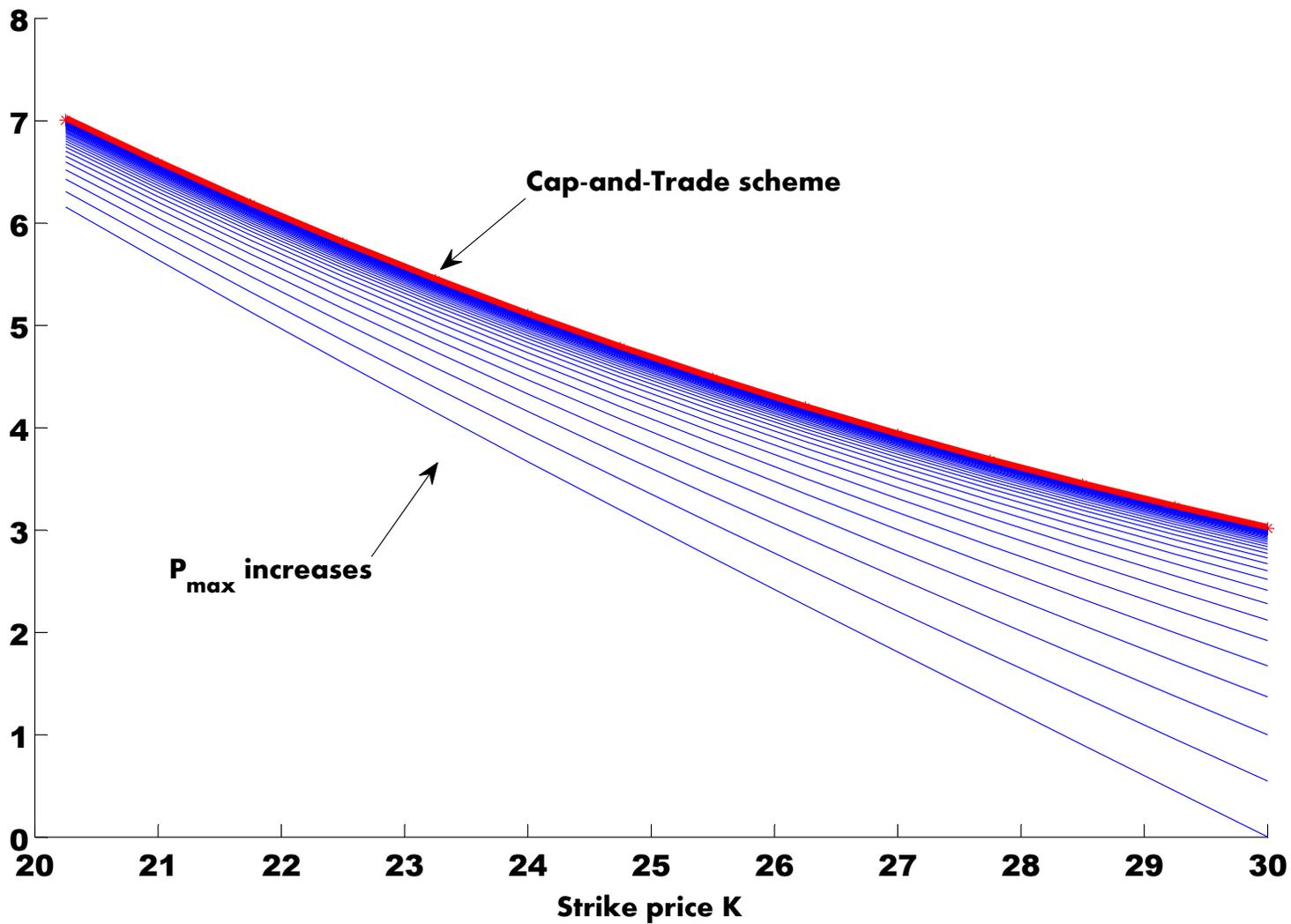
Pricing framework

- A hybrid market is a cap-and-trade market with a price cap P_{max} .
- $P_{max} = \infty \implies$ Cap-and-trade scheme.
- For a contingent claim H , its initial price under a
 - ▷ cap-and-trade scheme is: $E_{\widehat{P}}[H]$
 - ▷ hybrid scheme is : $E_{\widehat{P}}[\mathbb{I}_{\tau > T_1} h(F(T_1, T_1))] + \widehat{P}(\tau \leq T_1) h(P_{max})$],

where h is the payoff function of H and

$$\tau = \min\{t | F(t, T_1) \geq P_{max}\}$$

Figure 10: Call prices under a hybrid scheme converges to cap-and-trade scheme as P_{max} varies from 30 to 150 with unit increment.



Conclusions

- Propose mathematical models to describe futures dynamics under the assumption of market incompleteness.
- Empirical Investigation: Most market uncertainty can be explained by one factor
- Strategy involving all traded assets is more efficient than a strategy that includes only positions on the underlying futures contracts.

Conclusions

- Investigated incomplete market pricing techniques in which the optimal hedging strategy is chosen by minimizing either the quadratic risk or the mean conditional square error of the cumulative cost process.
- Regulator could improve market design and offer hedging tools against extreme scenarios.
- Regulator should actively participate in the market.

Thank you !

Figure 11: Empirical PDF of Y_t^1 .

