Lecture I: A primer on power markets

Fred Espen Benth

Centre of Mathematics for Applications (CMA)
University of Oslo, Norway

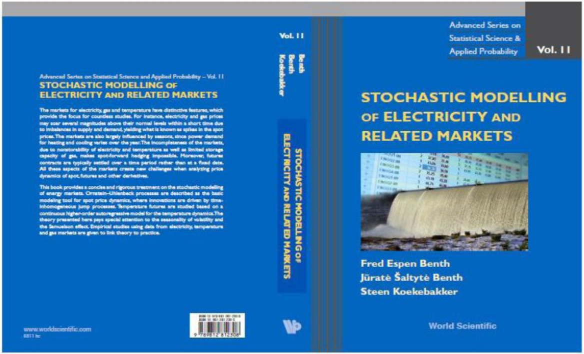
Fields Institute, 19-23 August, 2013

Overview of lectures

- Goal: give an introduction to the basics of modelling and pricing in energy
  - Focus on power, gas and weather:
  - spot, forwards and options

1. A primer on power
   - the case of NordPool
2. Stochastic volatility modelling in energy markets
   - Gas markets as the case
3. Stationary stochastic models
   - Spot power and weather markets
4. Option pricing in energy markets
   - Asian, spread and quanto options
5. Heath-Jarrow-Morton modelling of energy markets
   - Case study on NordPool data

For background and some of the theory in the course....



The NordPool Market

- The NordPool market organizes trade in
  - Hourly spot electricity, next-day delivery
  - Forward and futures contracts on the spot
  - European options on forwards
- Covers the Nordic region
  - Norway, Sweden, Denmark and Finland
  - Estonia and Lithuania
- Power production
  - Hydro, nuclear, coal, gas, wind, bio

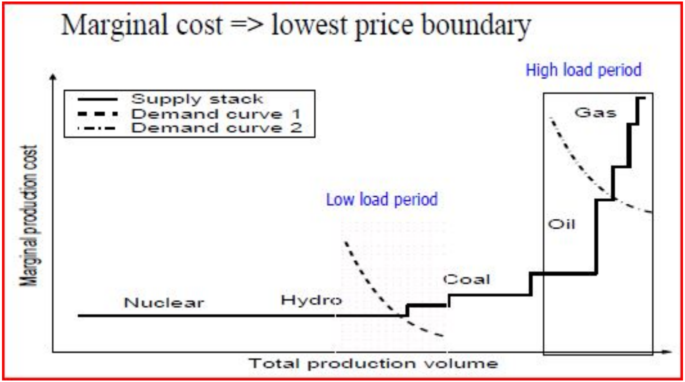
- Power generation in the NordPool area in 2011: 377.4 TWh

Energy source	Denmark	Finland	Norway	Sweden	Sum	Share	Class:
Wind power	8,9	0,5	1,3	6,1	16,7	4,4 %	2
Other renewable	2,4	10,5	0,0	11,2	24,1	6,4 %	2
Fossil fules	21,8	24,2	4,8	5,4	56,1	14,9 %	1
Nuclear power	0,0	22,3	0,0	58,0	80,3	21,3 %	3
Hydropower	0,0	12,3	121,4	65,8	199,4	52,9 %	2
Non-identifiable	0,0	0,7	0,0	0,0	0,7	0,2 %	4
Production	33,1	70,4	127,4	146,4	377,4	100,0 %	

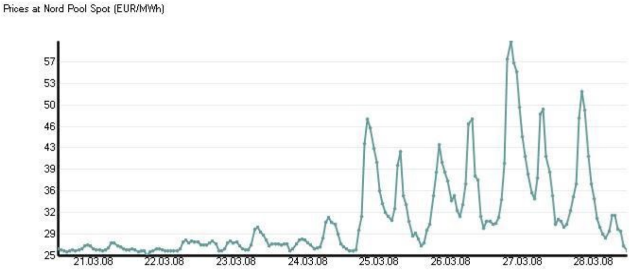
Elspot: the spot market

- A (non-mandatory) hourly market with physical delivery of electricity
  - About 70% of total production traded through NordPool (2010)
- Participants hand in bids before noon *the day ahead*
  - Volume and price for each of the 24 hours next day
  - Maximum of 64 bids within technical volume and price limits
- NordPool creates demand and production curves for each hour of the next day

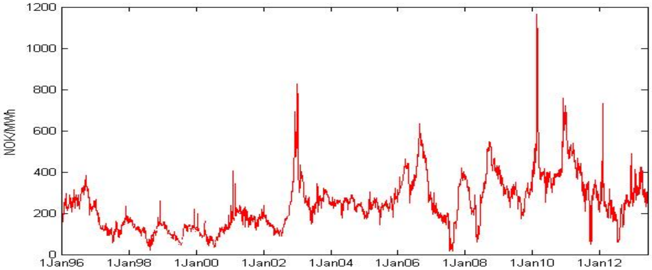
- Graphical illustration of the power generation stack



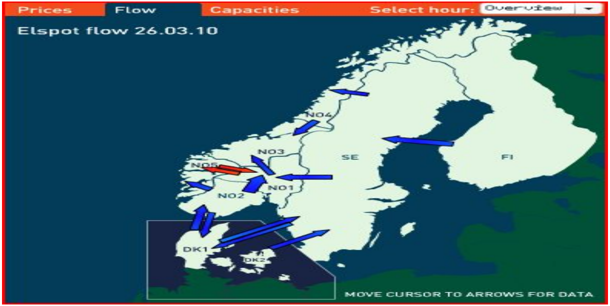
- The system price is the equilibrium
  - Price for delivery of electricity at a specific hour next day
  - The *daily* system price is the average of the 24 hourly
- Reference price for the forward market
- A series of hourly prices from Friday 21–Friday 28 March, 2008



- Historical daily system price from 1998 until May 2013 (NOK/MWh)

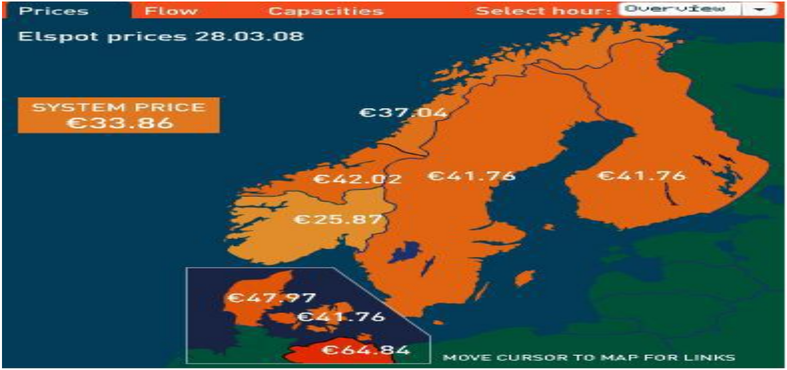


- Local imbalances in production and demand
- Example: In winter Norway may be in deficit of electricity due to cold weather
    - Importing from Sweden mostly



- Due to congestion (non-perfect transmission lines), *area prices* are derived
  - Finland separate area
  - Sweden and Denmark split into two
  - Norway may be split into several areas (up to 5)
- The area prices are the actual prices for the consumers/producers in the area in question

- Areas and area prices on March 28, 2008

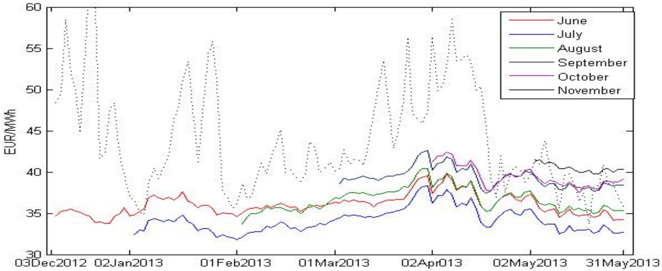


The forward and futures market

- Contracts with "delivery" of electricity over a period
  - Financially settled: The money-equivalent of receiving electricity is paid to the buyer
  - The reference is the hourly system price in the delivery period
- Delivery periods
  - Next day or week (futures-style)
  - Monthly
  - Quarterly (earlier seasons)
  - Yearly
- Overlapping delivery periods (!)

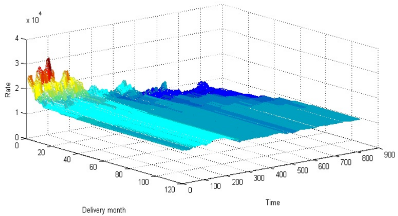
- Base load contracts
  - Delivery over all hours in the period
- Peak load contracts
  - Delivery over peak hours only
  - Peak hours are from 8 to 20 every day
  - Weekends and holidays are excluded
- Also here the futures-style contracts have short delivery period
- Contracts frequently called *swaps*
  - Fixed for floating spot price

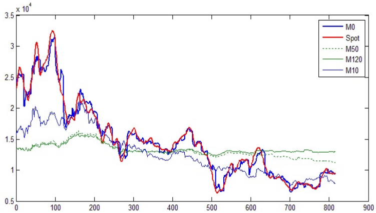
- Monthly (base-load) forward prices up to June 1, 2013
- Dotted line is the system price



The case of freight rates forwards

- Supramax rates at Baltic Exchange



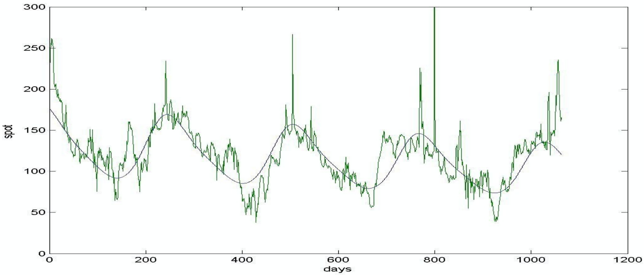


The option market

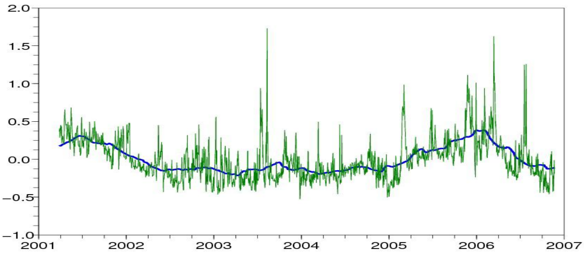
- European call and put options on electricity forwards
  - Quarterly and yearly delivery periods
- Low activity on the exchange
- OTC market for electricity derivatives huge
  - Average-type (Asian) options, quanto options, CfD's, spread options
  - Flexible load contracts, other swing options....

Stylized facts of power spot prices

- Seasonality on different time scales
  - Yearly
  - Weekly
  - Intra-daily
- Plot of NordPool system (spot) price



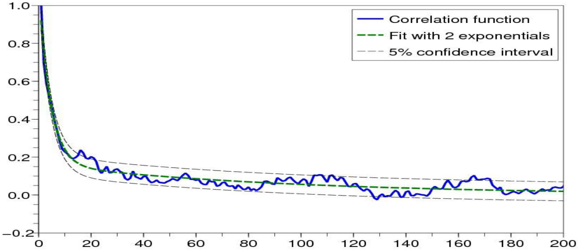
- Mean-reversion of spot prices
  - Energy prices driven by supply and demand
  - Prices will revert towards an equilibrium level
- However, to what level?
  - A fixed long-term level?
  - A stochastic level?
- Plot of UK PX log-spot prices with running mean



- Mean reversion shows up in the autocorrelation function (ACF)
  - Assuming stationarity in prices

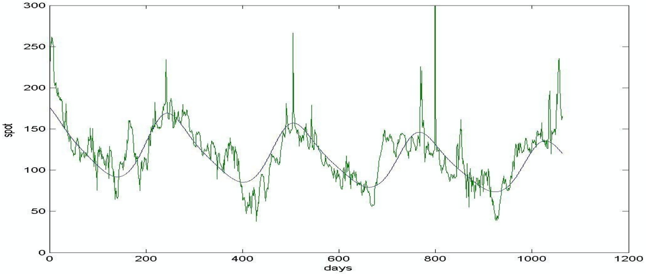
$$\rho(\tau) = \mathrm{corr}\left(S(t+\tau), S(t)\right)$$

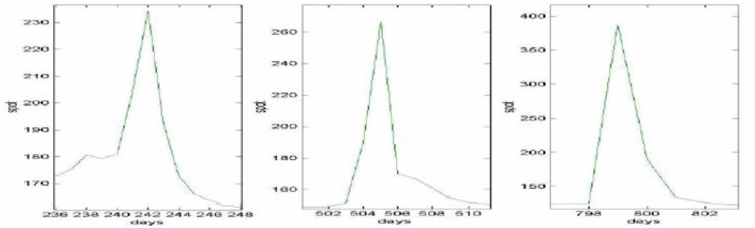
- Empirically, ACF's are often representable as sums of exponentials,
- This means that we have several scales of mean-reversion
  - Fast due to spikes
  - Medium and slow due to "normal" price variations
- Points towards several mean-reversion factors in dynamics



- Empirical ACF of EEX spot prices
  - Fitted with a sum of two exponentials
  - Multi-scale mean-reversion

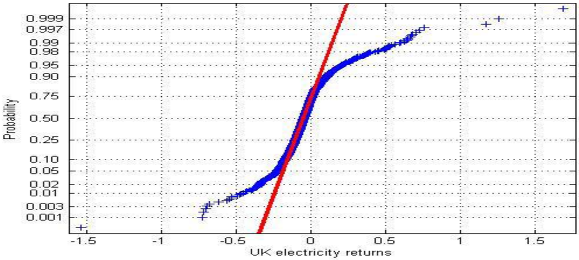
- Spikes in spot electricity
- Spike: A large price increase followed by a rapid reversion back to normal levels
  - Happens within 2-3 days
  - May be of several magnitudes
- Nord Pool price series



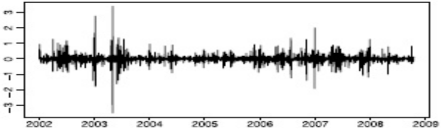


- Zoom-in of the three biggest spikes in NordPool series
- Note the rapid reversion, and magnitude of the increase
- Spikes occur in winter at Nord Pool
  - Other markets may not have seasonality in spike occurence
  - e.g. the German energy exchange EEX

- Spikes lead to highly leptokurtic spot price returns
- Example with UK electricity returns
  - Seasonality removed
  - Daily returns
  - Normal probability plot



- Returns are distinctively heavy tailed
  - Extreme events have much higher probability than the normal distribution can explain
- Small variations have higher probability than normal
- The effect of spikes....
  - ...but maybe also stochastic volatility?
  - Deseasonalized EEX logarithmic returns:



Stylized facts of power forward prices

Empirics of power forwards

- Every day at NordPool: Available forward contracts with different delivery periods
    - Given weeks, months, quarters, and years
    - Earlier: blocks (4 weeks) and seasons
    - Changed delivery periods over the years
- Desirable to have a *structured set* of power forward prices for each day
- Why?
    - To facilitate a study of empirical properties of returns

Outline of procedure

- Find a *smooth* curve of forward prices for each day t

$$\widehat{F}(t, T_1, T_2) \longrightarrow F(t, T)$$

- T will be the "delivery time", $t \leq T$
- Compute structured set of forward prices for each day t

$$F(t, T) \longrightarrow \sum_{T=T_1}^{T_2} F(t, T)/(T_2 - T_1)$$

Constructing forward prices

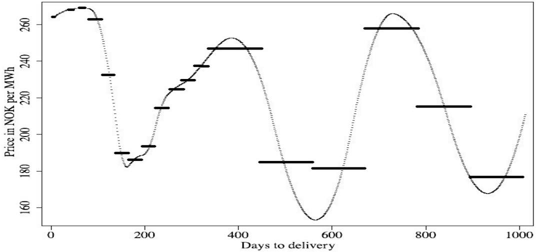
- Idea is taken from interest-rate theory (forward rate/yield curve)
  - Adams and van Deventer (1994)
  - Benth, Koekebakker, Ollmar (2007)
- Suppose

$$F(t, T) = \Lambda(T) + \epsilon(T)$$

- Λ seasonal structure, ϵ a polynomial spline, such that...
  1. ...implied forward prices match observed ones (or is in bid-ask spread when price does not exist)
  2. ...has "least variability" in the sense minimizing

$$\int_0^{\tau} [\epsilon''(t, T)]^2 \, dT$$

- Fourth order polynomial spline gives the solution
- Example: Data from Jan 2, 1997, seasonal structure given by a spot prognosis



Structured data set

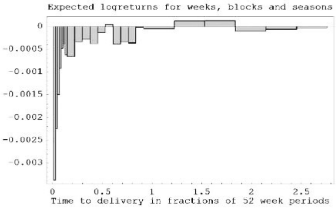
- Available daily prices for the different power forward contracts
    - Collected from Nordic power exchange NordPool
    - Ranging from 1997 to 2005
- 1,750 daily forward curves (weekends and holidays excluded)
- Structured data
    - 7 weekly delivery contracts
    - 10 block contracts (4 weeks length of delivery)
    - 6 seasonal contracts (4 blocks, ie 16 weeks)
- Delivery starts sequentially, first week immediately
- All prices converted to NOK (most data in NOK, a few in Euro)

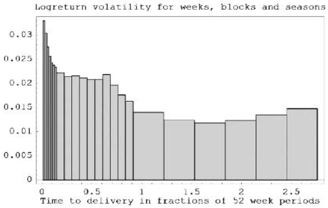
- Compute logreturns from forward prices

$$r_i(t) = \ln \frac{F(t, T_{1i}, T_{2i})}{F(t-1, T_{1i}, T_{2i})} \; i = 1, \ldots, 23$$

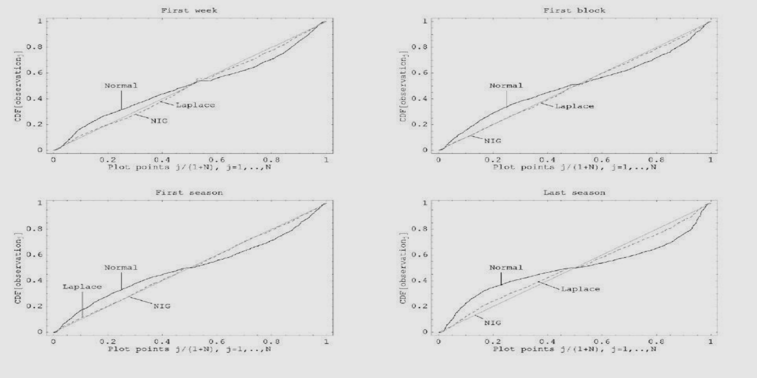
- General findings are:
  1. Distinct heavy tails across all segments
  2. No significant skewness
  3. Volatilities are in general falling with time to delivery (Samuelson effect)
  4. Significant correlation between different segments (idiosyncratic risk)

- Expected logreturn (left) and volatility (right)





- PP-plot to illustrate the heavy tails
  - NIG=normal inverse Gaussian distribution
- 45° line will be a perfect fit



The normal inverse Gaussian (NIG) distribution

- A normal mean-variance mixture model:
  - Let Z be inverse Gaussian distributed

$$f_{\mathrm{IG}}(z) = \frac{\delta}{\sqrt{2\pi}} z^{-3/2} \exp\left(\delta\gamma - \frac{1}{2}\left(\delta^2 z^{-1} + \gamma^2 z\right)\right), z > 0$$

  - Conditional distribution of X is normal:

$$X|Z \sim \mathcal{N}\left(\mu + \beta Z, Z\right)$$

- X is NIG with parameters α, β, μ and δ, where

$$\alpha = \sqrt{\gamma^2 + \beta^2}$$

- Density function

$$f_{\mathrm{NIG}}(x) = c\exp\left(\beta(x-\mu)\right)\frac{K_1\left(\alpha\sqrt{\delta^2+(x-\mu)^2}\right)}{\sqrt{\delta^2+(x-\mu)^2}}$$

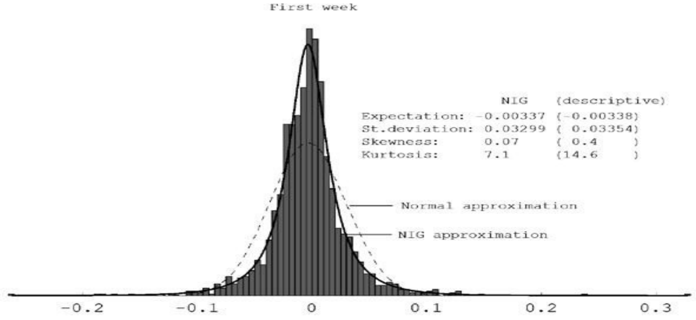
- $K_1(x)$ modified Bessel function of the third kind with index one. Normalizing constant c is known
- μ location, β asymmetry, δ scale ("volatility"), α steepness
  - smaller α yields steeper distribution, and thus fatter tails

$$\delta > 0\,, 0 \leq |\beta| < \alpha$$

- Moment generating function (MGF): for $-\alpha-\beta \leq \theta \leq \alpha-\beta$

$$M_{\mathrm{NIG}}(\theta) = \exp\left(\theta\mu + \delta\sqrt{\alpha^2-\beta^2} - \sqrt{\alpha^2-(\beta+\theta)^2}\right)$$

- Fitting NIG and normal to logreturns of forwards by maximum likelihood
- Example: first week

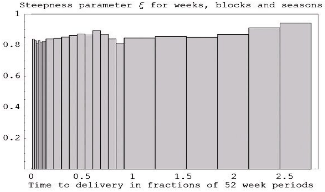


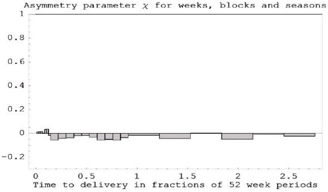
- *Shape triangle*: measure the "distance" from normality

$$\chi = \beta/\alpha\,, \quad \xi = (1 + \delta\sqrt{\alpha^2 - \beta^2})^{-1/2}$$

- Domain for (χ, ξ): $|\chi| < \xi < 1$
- χ measures the *asymmetry*
  - $\chi = 0$ symmetric NIG distribution
- ξ measures the *steepness* (or, *tail heaviness*)
  - $\xi \to 1$ gives Cauchy distribution, $\xi \to 0$ gives normal distribution

- Shape triangle parameters for the forwards
  - Cannot reject symmetric NIG distribution at 5%significant level in any segment
  - The "steepness" ξ larger than 0.8 in all segments

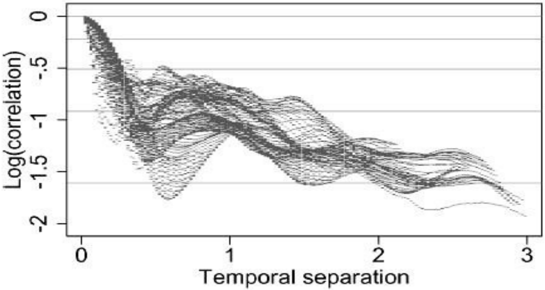




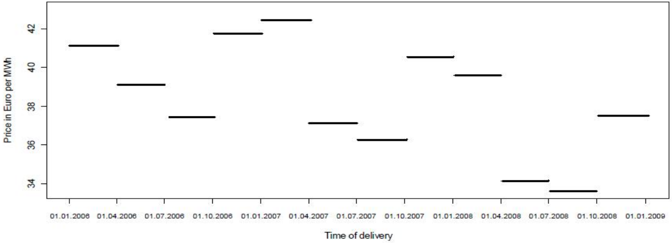
Correlation between returns

- Correlation between returns for different contracts
  - Various *length of delivery*
  - Various *time to delivery*
- Specialize to weekly contracts, over three years
  - Reconstruct weekly-delivery prices from smooth forward curve
  - Total 156 weekly-delivery forward prices every day
- Calculate the emprical correlation as a function of *distance between delivery*
  - Naturally measured in terms of weeks
  - Study by Frestad (2009)

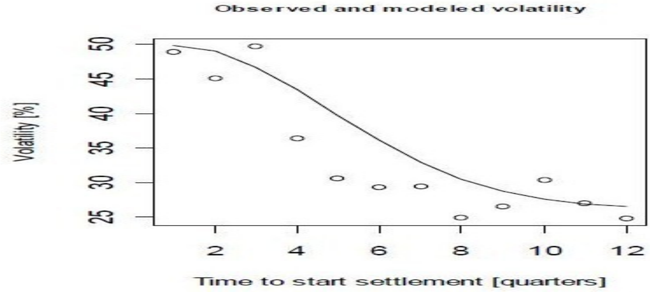
- Plot of log-correlation as a function of years between delivery
- Correlation decreases in general with distance between delivery
    - ...but in a highly complex way



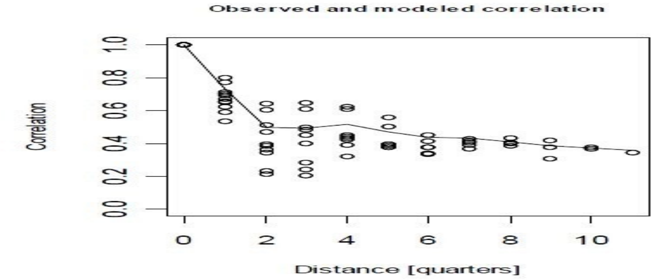
- A more recent study by Andresen, Koekebakker and Westgaard (2010)
- Analysis of base load quarter contracts constructed from NordPool data
  - The "forward curve" 1 January, 2006



- Observed Samuelson effect
  - Volatility of forwards decrease with time to maturity
  - Reflection of the mean-reversion of the forward price
  - The influence becomes insignificant in the long end of the market
- Plot of Nordpool quarterly contracts, empirical volatility



- Study of the correlation structure of quarterly contracts in NordPool



Summary so far

- Forward curve moves as a stochastic process parametrized by time-to-delivery x
  - Also by length-of-delivery
- Increasing volatility as we approach delivery
  - Samuelson effect
- Strong dependencies between maturity times
  - High degree of idiosyncratic risk in the market
- Non-Gaussian distributed logreturns
  - NIG seems to fit nicely

Some remarks on modelling and pricing of electricity

- Situation similar to that of fixed-income markets
- Spot price $\leftrightarrow$ short rate of interest
    - Power spot is non-storable!
    - Cannot create portfolios in the spot
- Forward contracts $\leftrightarrow$ forward rates
    - ... or at least zero-coupon bonds
- Modelling problem:
    - Spot modeling, to price forwards
    - What is the link between spot and forwards (Lectures II and III)?
    - HJM-approach, that is, direct modeling of forward prices (Lecture V)

Home work...

Ex. 1 Let $X(t)$ be the solution to an Ornstein-Uhlenbeck SDE

$$dX(t) = (\mu - \alpha X(t))\,dt + \sigma\,dB(t)$$

Find $X(s)$ given $X(t)$, $s \geq t \geq 0$.

Ex. 2 Compute the conditional expectation

$$\mathbb{E}[\exp(X(T))\,|\,\mathcal{F}_t]$$

with $T \geq t$.

References

- Adams and van Deventer (1994). Fitting yield curves and forward rate curves with maximum smoothness. J. Fixed Income 4
- Andresen, Koekebakker and Westgaard (2010). Modeling electricity forward prices using the multivariate normal inverse Gaussian distribution. J. Energy Markets 3(3)
- Benth, Koekebakker and Ollmar (2007). Extracting and applying smooth forward curves from average based commodity contracts with seasonal variation. J. Derivatives 15(1).
- Frestad (2009). Correlations among forward returns in the Nordic electricity market. Intern. J. Theor. Applied Finance, 12(5),
- Frestad, Benth and Koekebakker (2010). Modeling term structure dynamics in the Nordic electricity swap market. Energy Journal, 31(2)

Coordinates:

- fredb@math.uio.no
- folk.uio.no/fredb/
- www.cma.uio.no