Enhanced Transfer of Wind Energy into Surface Waves

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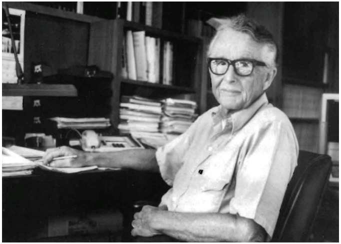
Workshop on Ocean Wave Dynamics

Dedication



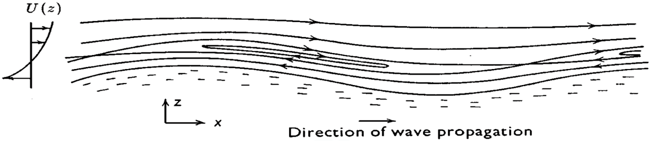
Dedicated to the Memory of
a Colleague and a Friend,
John Walter Miles (1920–2007) *requiescat in pace.*

Historical Introduction

- 56 years ago, Fritz Ursell stated in his famous review:
- "Wind blowing over water surface generates waves in the water by a physical process which can not be regarded as known."
- Despite tremendous amount of research and pioneering work of John Miles it is still difficult, even now, to answer the question,
- "Have we really clarified the physical process of wind wave generation and decay?"

Miles (or critical layer) Mechanism

- Quasi-inviscid model.
- Role of Reynolds stresses is to determine the unperturbed mean velocity profile.
- Air flows concurrently with the waves, there is a height, (z_c) where $U(z) = c$.
- Upward motion of air induces a sinusoidal pressure variation over waves.



Miles CL Mechanism

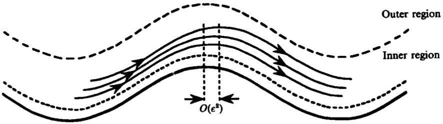
- Leads to a vortex sheet (periodically varying strength) forming at z_c.
- The vortex force on waves lead to an energy transfer.
- According to this mechanism, amplitude grows only if the wave is moving.
- For fixed undulation (where the critical layer is at the wave surface) there is no asymmetric pressure and hence no wave growth.

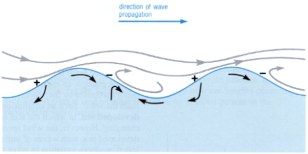
Non-Separated Sheltering (NSS)

- Need to include Reynolds stresses close to surface (inner region).
- The boundary layer thickens on the leeside of wave
- Leads to mean flow separation when the slope is large enough.
- Thickness of the inner region is asymmetric and thus inviscid flow in outer region is asymmetrically displaced about the wave.
- Leads to an out-of-phase component to the pressure perturbation.

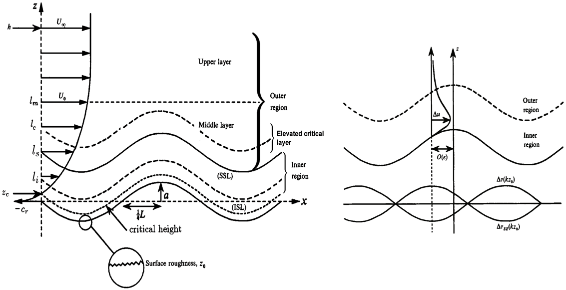
NSS Mechanism

- Related to Jeffreys' sheltering hypothesis.
- For separated flows over moving waves of large slope.
- Works only if $a \sim O(\lambda)$ – very restricted.



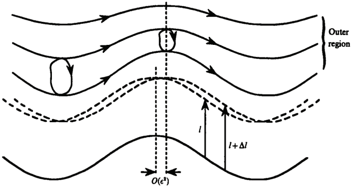


Inner Region



- Turbulence tend to local equilibrium structure.
- Asymmetry in inner region makes $\overline{u_i'^2}$ out of phase at surface.
- This also contributes to the energy flux to the wave motion.

Outer Region

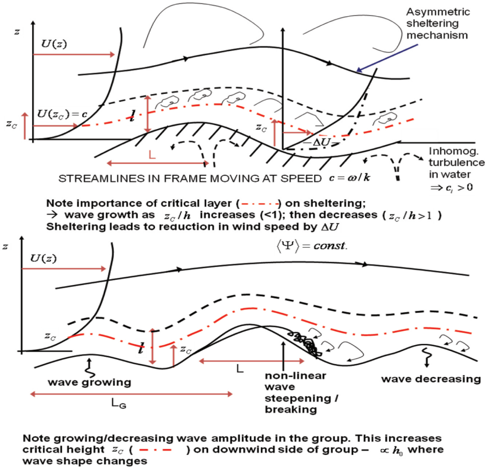


- NSS in inner region change $\Delta\ell$ in the displacement of the largely inviscid outer-region flow.
- RDT of the Reynolds stresses in the outer region is displaced downwind of the crest.
- This contributes to the energy flux.

Modelling Methodology

- Physical mechanisms involved for wind over surface of unsteady and groups of waves.
- Multi-deck theory for turbulent shear flows (Eddy-viscosity in inner and RDT in outer regions).
- Combining NSS and unsteady critical layer mechanisms.
- Explain why groups are most efficient mechanism for air-sea energy exchange.

Model Schematics



Governing Equations

- Linearized, Reynolds-averaged equations for u and w and kinematic perturbation pressure p

$$
\begin{aligned}
&u_x + w_z = 0, \\
&(U - c)u_x + U'w = -\wp_x + \sigma_x + \tau_z, \\
&(U - c)w_x = \wp_z + \tau_x,
\end{aligned}
$$

- Reynolds stresses

$$\wp \equiv \mathrm{p} + \overline{w'^2} - (\overline{w'^2})_0, \;\; \sigma \equiv -(\overline{u'^2} - \overline{w'^2}) - \sigma_0, \;\; \tau \equiv -\overline{u'w'} - \tau_0$$

- $(\overline{w'^2})_0, \sigma_0$ and τ_0 are the unperturbed values of $\overline{w'^2}, -(\overline{u'^2} - \overline{w'^2})$ and $-\overline{u'w'}$.

Turbulence Closure

- Transport equation for turbulent kinetic energy $\frac{1}{2}\overline{q^2}$

$$(U - c)\partial_x \left(\tfrac{1}{2}\overline{q^2}\right) = D + G - \varepsilon'$$

- Diffusion term

$$D = \varrho\kappa\tau_0^{1/2}\partial_z \left[z\partial_z \left(\tfrac{1}{2}\overline{q^2}\right)\right]$$

- Generation term

$$\begin{aligned} G &= -\overline{u'^2}u_x - \overline{w'^2}w_z - \overline{u'w'}(U' + u_z + w_x) - \tau U' \\ &= \sigma_0 u_x + \tau_0(u_z + w_x) + U'\tau \end{aligned}$$

- Dissipation term

$$\varepsilon' = \tfrac{3}{2}\tau_0 U'(e/e_0), \qquad e \equiv \overline{q^2} - e_0$$

Wave Groups

- There are NO $a\cos(kx)$ waves in the sea.
- In sea, waves move in groups which affects:

  (a) How wind flows over the waves;
  (b) How waves break and thus how droplets form.

Instabilites/Interactions

- Weakly non-linear interaction significantly influence average momentum.
- Very small unsteady waves are formed by
- Turbulence; or
- T–S instability in shear airflow over the surface [class A];
- K–H instability over the liquid [class C];
- C–L instability [class B]; and
- NSS instability [class D].
  [Class A–C are Benjamin's three-fold instabilities. Class D needs to be included].

Wave Growth

- Waves $\lambda = 2\pi/k$ grow at the rate kc_i.
- Only when $c_i \neq 0$, wave grows/decays.
- There is a net force on the wave caused by C_L.
- Miles (1957)/Lighthill (1962) calculate growth (γ) for $c_i = 0, \;\; a = \text{const.}, \;\; ka \ll 1$.
- They conclude: there is a net inviscid force on monochromatic non-growing waves – VERY WRONG.

NS-Sheltering Mechanism

- When $c_r > U_*$ then C_L is outside surface S_L.
- This acts to reduce ns-sheltering mechanism.
- When $c_r < U_*$ then C_L is within S_L.
- This increases ns-sheltering mechanism.
- Thus, decrease in γ as c_r/U_* increases is compensated by increase in γ as waves form into a group at higher wind speed U_λ.

Analytical Model

- We consider surface group waves

$$\begin{aligned}\zeta &= \mathrm{Re}\left\{ae^{ik(x-ct)} + \varepsilon(t)\left[e^{ik_2(x-c_2t-\theta_2)} + e^{ik_3(x-c_3t-\theta_3)}\right]\right\} \\ &\equiv \eta + \varepsilon(t)(\eta_2 + \eta_3)\end{aligned}$$

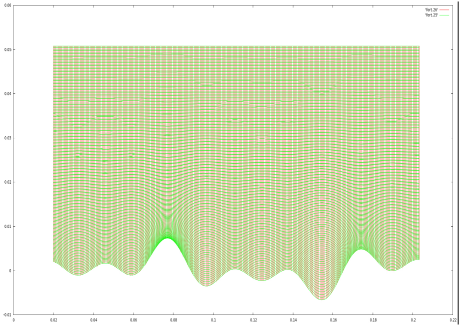
where

$$\varepsilon(t) = \exp\left\{\tfrac{1}{2}\delta(2k^2a^2 - \delta^2)^{1/2}\omega t\right\}$$

$$k_{2,3} = k(K \pm 1) \qquad \omega_{2,3} = \omega(\delta \pm 1)$$

Wave Profile

- Typical group profile comprising of three waves.



- Used as computational domain.

Surface Pressure

- We assume mean velocity profile above the surface is logarithmic.
- We pose the surface pressure in the form

$$p_a = \rho_a U_1^2 k \{(\alpha + i\beta)\eta + \varepsilon(t)[(\alpha_2 + i\beta_2)\eta_2 + (\alpha_2 + i\beta_2)\eta_2]\}$$

where

$$c_{2,3} = c_{2w,3w}\left[1 + \frac{1}{2}\frac{k^2}{k_{2,3}^2}(\alpha_{2,3} + i\beta_{2,3})\left(\frac{U_1}{c_{2w,3w}}\right)^2 s\right]$$

$$s \equiv \rho_a/\rho_w, \qquad U_1 = U_*/\kappa$$

Group Velocity

- Need total contribution of each wave speed to group.
- Use resonant interactions of 2 gravity waves [L-H] to 3 waves, in conjunction with phase velocity effects in tertiary wave interactions [L-H & Phillips].
- Two secondary waves interacting gives

$$c_g = \frac{g}{2}\left[2\sqrt{gk(1+K)} - \sqrt{gk(1-K)}\right]^{-1}$$

- This integrating with the primary waves yields

$$C_g = \frac{g}{2}\left[2\sqrt{gk(1+k^2a^2)} - \sqrt{gk_3}\right]^{-1}$$

Airflow Perturbation

- Perturbations to airflow is modelled by eddy viscosity
- Vertical velocity perturbation satisfies IH-Rayleigh equ

$$\frac{\partial^2 \hat{\mathscr{W}}}{\partial z^2} - \left(k^2 + \frac{U''}{U - ic_i}\right)\hat{\mathscr{W}} = \frac{i}{U - ic_i}\frac{\partial^2}{\partial z^2}\left(\nu_e \frac{\partial^2 \hat{\mathscr{W}}}{\partial z^2}\right)$$

- In the middle layer advection term $\ll$ curvature term. Thus

$$\frac{\partial^2 \hat{\mathscr{W}}}{\partial z^2} - \frac{U''}{U - ic_i}\hat{\mathscr{W}} \sim 0$$

and the solution is regular since $U > 0$. If $c_i = 0$, then singularity is resolved by inertial effects.

CL Energy Transfer I

- To calculate the energy-transfer parameter we use

$$[\nu_e(\mathscr{V}\mathscr{M}'' + 2U'\mathscr{M}' + U''\mathscr{M})]'' = ik[(\mathscr{V}^2\mathscr{M}')' - k^2\mathscr{V}^2\mathscr{M}]$$

- In the quasi-laminar limit the complex amplitude of the surface pressure is given by the variational integral

$$a\mathscr{P}_0 = -\int_0^\infty \mathscr{V}^2(\mathscr{M}'^2 + k^2\mathscr{M}^2)\, dz$$

- We use the simplest admissible trial function

$$\mathscr{M} = ae^{-kz/\varsigma}$$

where ς is a free parameter.

CL Energy Transfer II

- The approximation $\mathscr{V} \approx U_1 \ln(z/z_c) - ic_i$ yields

$$\hat{\mathscr{P}}_0 \equiv \mathscr{P}/kaU_1^2 = -k(\varsigma^{-2}+1)\int_0^\infty e^{-2kz/\varsigma}\mathscr{F}(z)\,dz$$

- Where

$$\mathscr{F}(z) = \ln^2(z/z_c) - 2i\hat{c}_i \ln(z/z_c) - \hat{c}_i^2$$

and $\hat{c}_i = c_i/U_1,\ U_1 = U_*/\kappa$.

CL Energy Transfer III

- Evaluating the integral gives

$$\hat{\mathscr{P}}_0 = -\frac{\varsigma + \varsigma^{-1}}{2}\left\{\frac{\pi^2}{6} + \ln^2\left(\frac{2\gamma\xi_c}{\varsigma}\right) - 2i\hat{c}_i \ln\left(\frac{2\gamma\xi_c}{\varsigma}\right) + \hat{c}_i^2\right\}$$

- The variational condition $\partial\hat{\mathscr{P}}_0/\partial\varsigma = 0$ yields ($\xi_c = kz_c$)

$$\varsigma^2 = \frac{L_\varsigma^2 - 2(1 + i\hat{c}_i)L_\varsigma + (\hat{c}_i^2 + 2i\hat{c}_i + \pi^2/6)}{L_\varsigma^2 + 2(1 - i\hat{c}_i)L_\varsigma + (\hat{c}_i^2 - 2i\hat{c}_i + \pi^2/6)}$$

where $L_\varsigma = -(L_0 + \ln\varsigma)$ and $L_0 = \gamma - \ln(2\xi_c) = \Lambda^{-1}$.

CL Energy Transfer IV

- The corresponding CL approximation to energy-transfer parameter β is calculated from

$$\mathscr{W}_c = \mathscr{P}_c / U'_c \approx \mathscr{P}_0 / U'_c.$$

- Thus we obtain

$$\begin{aligned}
\beta_c &= \pi \xi_c |\mathscr{W}_c / U_1 a|^2 = \pi \xi_c^3 |\hat{\mathscr{P}}_0|^2 \\
&= \tfrac{1}{4}\pi(\varsigma + \varsigma^{-1})^2 \left| \left(L_\varsigma^2 - 2i\hat{c}_i L_\varsigma + \hat{c}_i^2 + \tfrac{1}{6}\pi^2 \right) \right|^2 \\
&= \pi \xi_c^3 L_0^4 \left[1 + \left(4 - \tfrac{1}{3}\pi^2 + 10\hat{c}_i^2 \right) \Lambda^2 + \mathcal{O}(\Lambda^3) \right].
\end{aligned}$$

Energy Transfer (Turbulence) I

- For energy-transfer parameter due to turbulence

$$\int_0^\infty \mathscr{M}\,\mathscr{T}''\,dz = ka[\mathscr{T}_0 - i\mathscr{P}_0] + i(kac)^2 + \int_0^\infty \mathscr{M}''\,\mathscr{T}\,dz$$
$$= i(kac)^2 + ik\int_0^\infty \mathscr{V}^2\left(\mathscr{M}'^2 + k^2\mathscr{M}^2\right)dz.$$

- $\mathscr{T}_0$ is complex amplitude of surface shear stress and

$$\alpha + i\beta \equiv (c^2 - c_w^2)/sU_1^2 = (\mathscr{P}_0 + i\mathscr{T}_0)/kaU_1^2 \equiv (\hat{\mathscr{P}}_0 + i\hat{\mathscr{T}}_0),$$

- c is the complex wave speed, $s = \rho_a/\rho_w \ll 1$ and

$$c_w = \sqrt{g/k} - 2ik\nu_w, \qquad |k\nu_w/c| \ll 1$$

is the speed of water waves in the absence of the airflow.

Energy Transfer (Turbulence) II

- Energy-transfer parameter is then calculated from

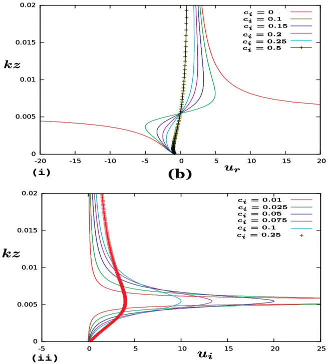
$$\alpha_T + i\beta_T = (kaU_1)^{-2} \int_0^\infty \left\{ i\nu_e \left[\mathscr{V} \mathscr{M}''^2 + 2U' \mathscr{M} \mathscr{M}'' + U'' \mathscr{M} \mathscr{M}'' \right] - k\mathscr{V}^2 \left(\mathscr{M}'^2 + k^2 \mathscr{M}^2 \right) \right\} dz.$$

- Evaluation of integral asymptotically and then taking the imaginary part yields

$$\beta_T = 5\kappa^2 L_0 + \mathcal{O}(\Lambda).$$

Perturbation Velocity

- Leading order solution: $u \propto (U - c)^{-1}$; c is complex



Results I

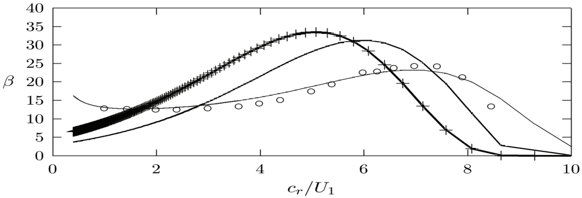


Figure: Total energy transfer parameter, β, due to the combined effect of sheltering and inertial critical layer for growing waves (where $c_i \ll U_*$) as a function of the wave age c_r/U_1. +++++, Miles calculation ($c_i = 0, \nu_e = 0$) from his formula: $\beta = \pi\xi_c \left\{\frac{1}{6}\pi^2 + \log^2(\gamma\xi_c) + 2\sum_{n=1}^{\infty} \frac{(-1)^n \xi_c^n}{n!n^2}\right\}^2$, where $\xi_c = kz_c$ is the critical height $\xi_c = \Omega(U_1/c_r)^2 e^{c_r/U_1}$ and $\Omega = gz_0/U_1^2$ is the Charnock's constant. Thick solid line, Janssen's parameterization of Miles formula, for $c_i = 0, \nu_e = 0$: $\beta = 1.2\kappa^{-2}\xi_c \log^4 \xi_c$, where $\xi_c = \min\left\{1, kz_0 e^{[\kappa/(U_*/c+0.011)]}\right\}$. Thin solid line, present formulation: $(\beta_T + \beta_c)$ for $c_i \neq 0, \nu_e \neq 0$. $\circ$, Numerical simulation using LRR Reynolds-stress closure model for $c_i \neq 0, \nu_e \neq 0$.

Results II

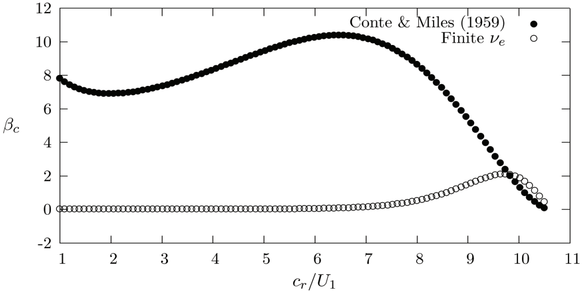


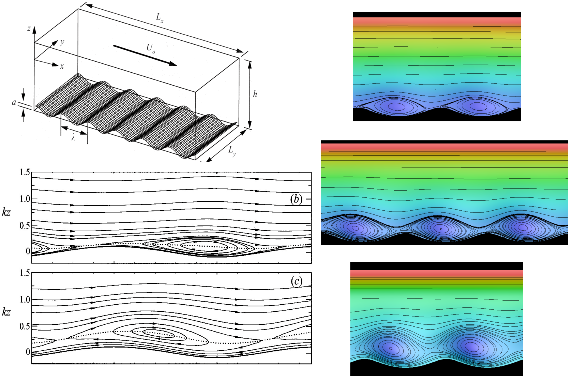
Figure: Component of energy transfer parameter, β_c, due to inertial critical layer for growing waves (where $c_i \ll U_*$) as a function of the wave age c_r/U_1. $\bullet$, numerical solution of inviscid Orr-Sommerfeld equation for $c_i = 0$ and $\nu_e = 0$ using the singular critical layer approach; $\circ$ numerical solution of equation IN-Rayleigh equation for $c_i \neq 0$ and $\nu_e \neq 0$.

Turbulence Model

- We adopt full realizable Reynolds-stress turbulence closure (TCL) [Sajjadi, Craft & Feng].
- Coupled to water motion below (through orbital velocities of deep water – Stokes drift).
- Fully implicit, collocated, general curvilinear coordinates finite volume.
- Also adopted semi-implicit FD solver with LRR turbulence model for comparison.

Comparison with Sullivan

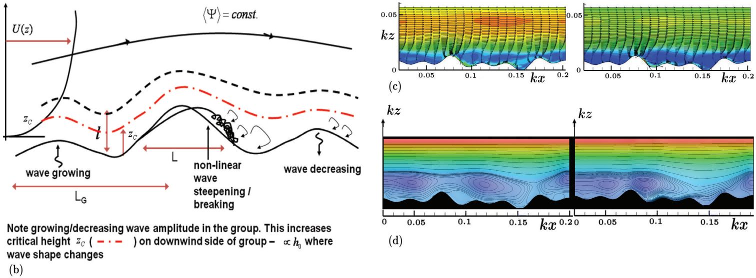
- CL elevates as c_r/U_* increases. No flow separation at the surface.



- Flow over moving waves is attached to the mean flow.

Results for Wave Groups

- Initial computations of turbulent flows over specified groups of 3 dynamic waves.



- Streamwise velocity profiles shows how z_c is higher on downwind than on upwind side of wave groups.

Conclusions

- Is Miles' CL mechanism wrong?
- Not really! – It is always there, but it is NOT the only mechanism.
- Need to consider Unsteady-CL in conjunction with NSS mechanism (both operate together).
- CL plays an important role on sheltering.
- Asymmetrical sheltering leads to reduction in wind speed.
- Growing/decaying wave amplitude in the groups increases CL on the downside of group where wave shape changes.