Nonlinear long waves over a muddy beach

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Abstract

There are many sea coasts in the world where the seabed is covered with fluid-mud which is a mixture of water, fine and cohesive clay particles, and sand. Dissipation in the mixture is considerably higher than in the pure sea water and causes very effective damping of waves arriving from the open sea. In turn the waves can induce changes of the muddy seabed and alter the coastal morphology in the long run.

For steady flows in muddy rivers numerous experiments have shown that the fluid-mud can be approximated as a Bingham-plastic material. However, for fluid-mud in oscillatory flows rheological experiments are relatively scarce. Limited laboratory tests of natural mud by oscillatory rheometers have however indicated that viscoelastic properties are prevalent. In existing theoretical models different idealizations have been proposed mostly for wave/mud interaction over a horizontal seabed. Some authors have adopted the mathematically convenient Newtonian viscous model with vastly contrasting viscosities. Others have proposed Bingham plastic models based only on rheometer tests of unidirectional flows. It is known that under waves fluid-mud behaves more as a visco-elastic material. So far simple Kelvin-Voigt models with only two or three constant coefficients have been used.

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Existing laboratory tests of natural fluid-mud by using dynamic rheometers performing simple harmonic motions have however shown that the constitutive coefficients in the Kelvin-Voigt model depend strongly on frequency. With increasing frequency, the shear modulus increases only mildly, but the viscosity μ decreases significantly and monotonically. Hence the Kelvin-Voigt coefficients depend on the motion itself and are not just material properties. A remedy necessary for treating transient but non-sinusoidal problems is to use the generalized viscoelastic model relating the stress τ and strain $\mathcal{E}$, which reads, in one dimensional motion,

$$\tau + \sum_{n=1}^{N} \alpha_n \frac{\partial^n \tau}{\partial t^n} = \beta_0 \mathcal{E} + \sum_{n=1}^{N} \beta_n \frac{\partial^n \mathcal{E}}{\partial t^n} \tag{1}$$

The coefficients α_n and β_n can be chosen to match the data of simple-harmonic tests for the entire range of measured frequencies. In this way the constitutive coefficients depend only on material properties such as the mineral composition, particle size, concentration, etc and not on the frequency. For sinusoidal motions, the formally Kelvin-Voigt model with frequency-dependent coefficients is still meaningful. Using the measured data by Huang & Huhe (1992), Mei, Chan & Liu (2010) examined theoretically the effects of contrasting rheologies of fluid-mud samples from two sites on the eastern coast of China on the evolution of narrow-banded waves in water of intermediate but constant depth. Asymptotic equations by Stokes-like expansions are derived for narrow-banded waves in water of finite depth. Damping rate and mud motion are examined at the first order of wave steepness, and long waves induced by radiation stresses at the second order. However, as is known for long waves over a rigid seabed, the Stokes expansion fails in very shallow water.

A prominent feature of a muddy beach is its ability to damp out essentially all incoming waves before they reach the shoreline. Wells (1978) first reported systematic field observations over a mud bank near the mouth of Surinum River, Brazil, that incoming waves were all diminished to naught with no reflection or breaking. This was confirmed by extensive measurements by Matthiew et al (1995) on the Southern Coast of India.

The objective of this work is to develop a nonlinear theory suitable for long waves in a shallow seabed covered by fluid-mud. Since generation of higher harmonics is expected in such a setting, dependence of constitutive coefficients in frequencies is crucial for a realistic model of natural mud and is accounted for. Specifically we aim to examine the physical differences between two natural fluid-muds taken from two field sites on the eastern coast of China. An asymptotic theory extending the Korteweg-de Vries approximation is derived for examining the effects of contrasting rheological mud properties on wave evolution under the influence of nonlinearity, dispersion, dissipation and shoaling. While acoustic streaming is known to occur in an oscillating boundary layer in a Newtonian viscous fluid, as a results of Reynolds stresses, we shall show that an analogous phenomenon exists in a mud layer under periodic waves. Specifically, waves can force a steady displacement in mud, leading to changes of mud depth, which can affect the morphology of the muddy coast.