Modeling linearized dissipation in the laboratory and in ocean swell

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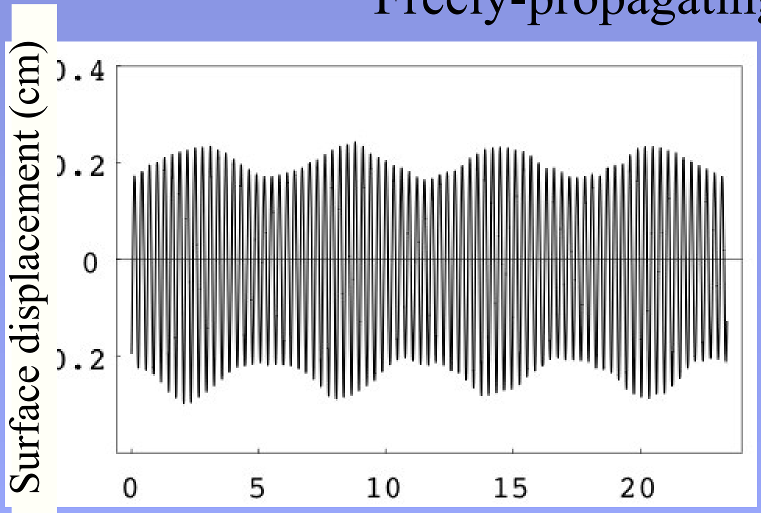
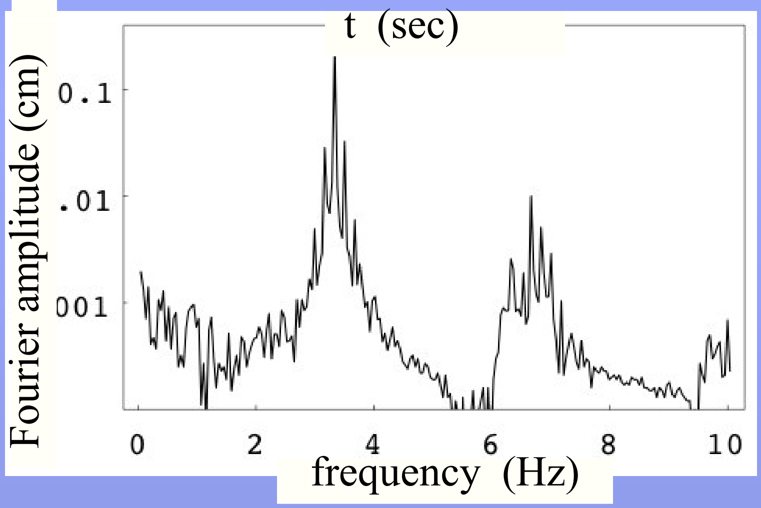
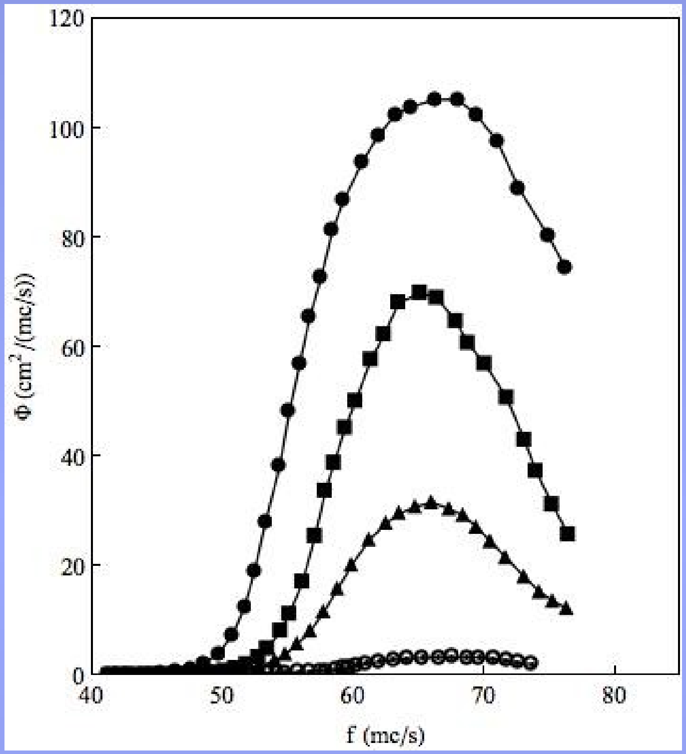
Motivation for looking at dissipation

Freely-propagating narrow-banded spectra.







Snodgrass *et al.* 1966.

What is a model that agrees with predictions for measurements of decay rates in lab and ocean for moderate amplitude waves and that can be used for nonlinear models?

Sources of Dissipation

Lab:

- Dissipation in the bulk.
- Air-water interface.
- The wetted perimeter of the wavetank - bottom and sidewall surfaces.
- Contact line dynamics.

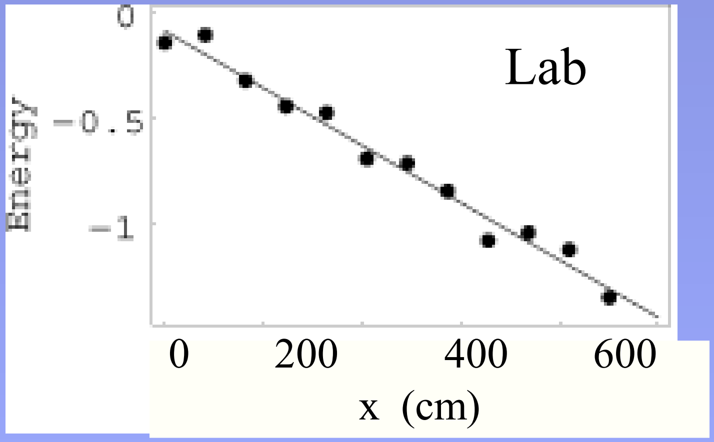
Ocean Swell:

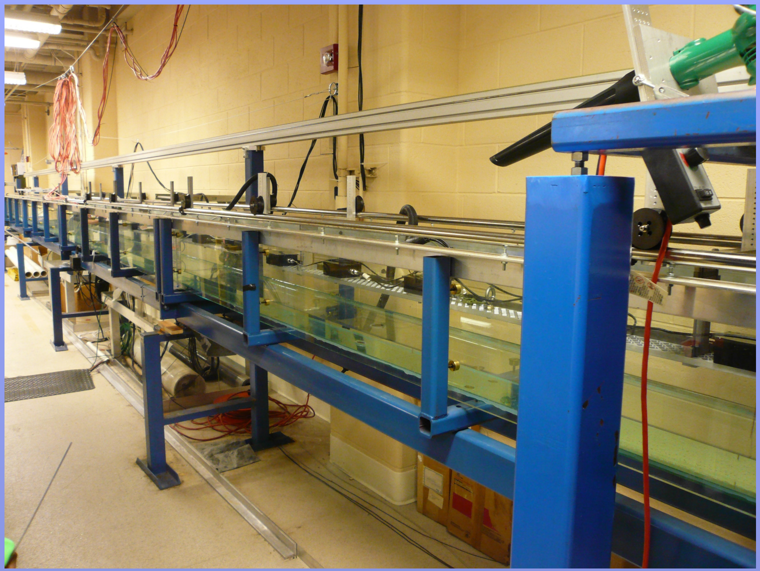
- Dissipation in the bulk.
- Air-water interface.
- Interaction with other wave systems.
- Geometric spreading.
- Breaking (not so common for swell).

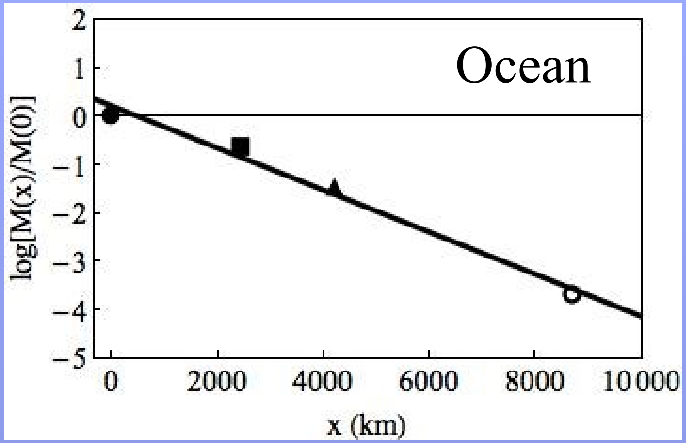
Exponential decay

$$a(x) = a(0)\,Exp[-\Delta x]$$

(In the previous talk, Δ was the decay rate for energy. Here it is the decay rate for amplitude.)

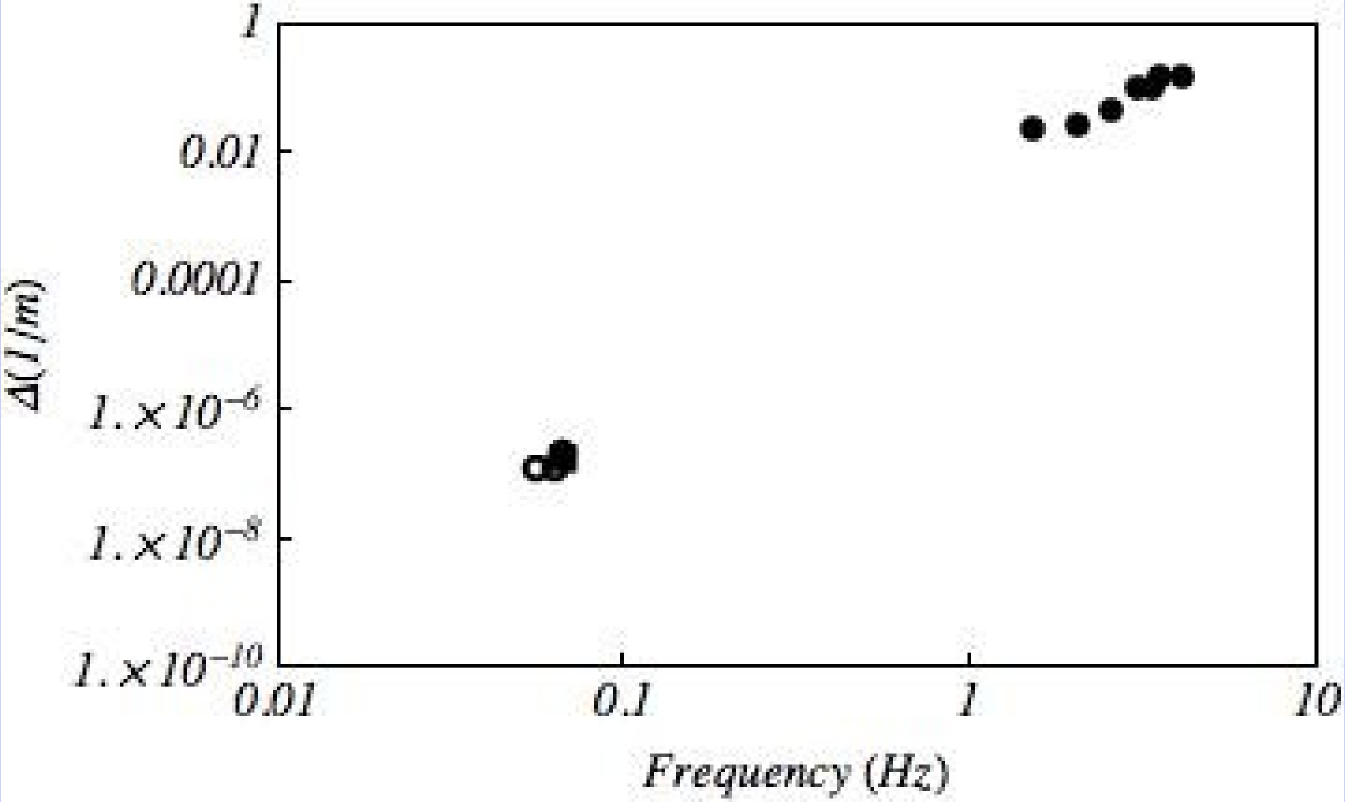






Snodgrass *et al.* 1966.

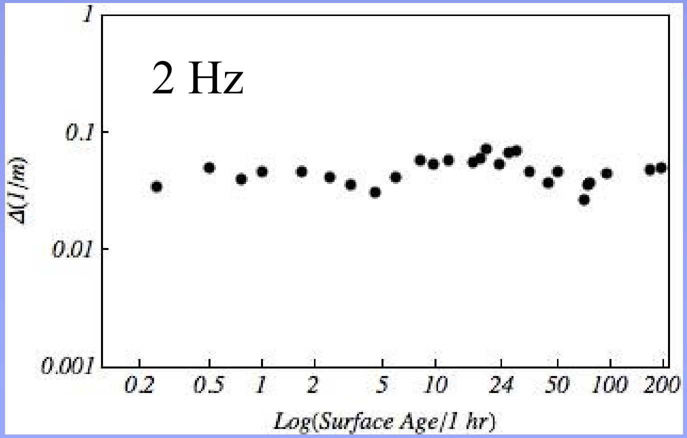
Measured damping rates

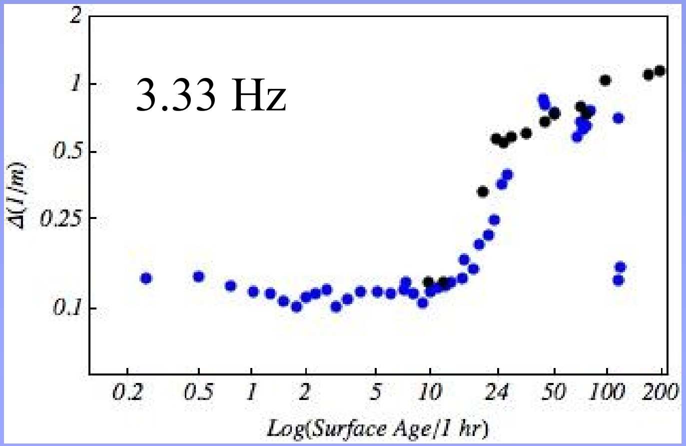


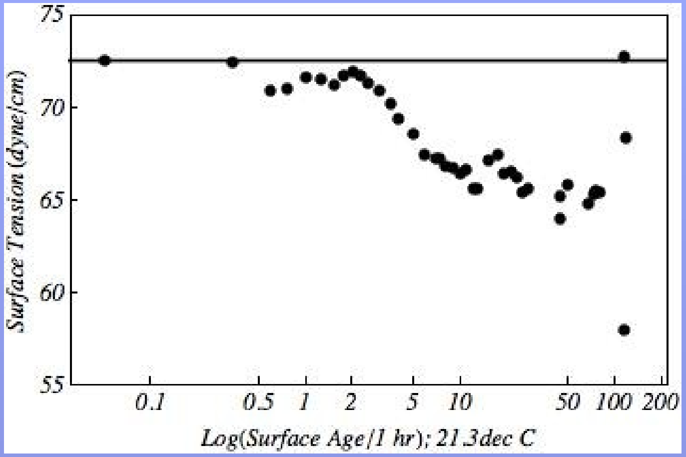
Ocean: hollow circles, Snodgrass *et al*. (1966);
solid circles, Collard *et al.* (2009)

Lab: "clean surface"

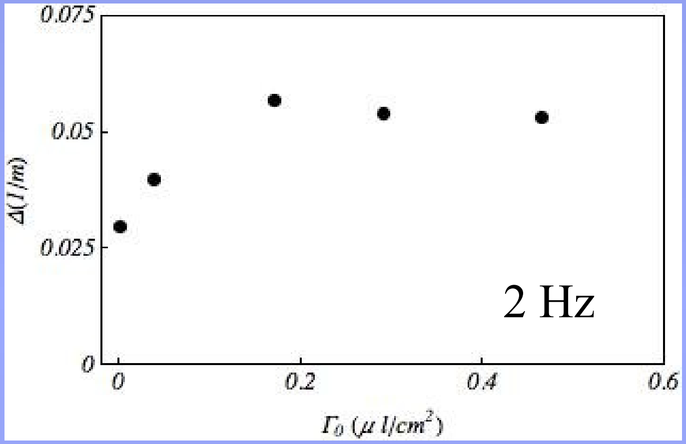
Damping rates in the lab as a function of surface age.

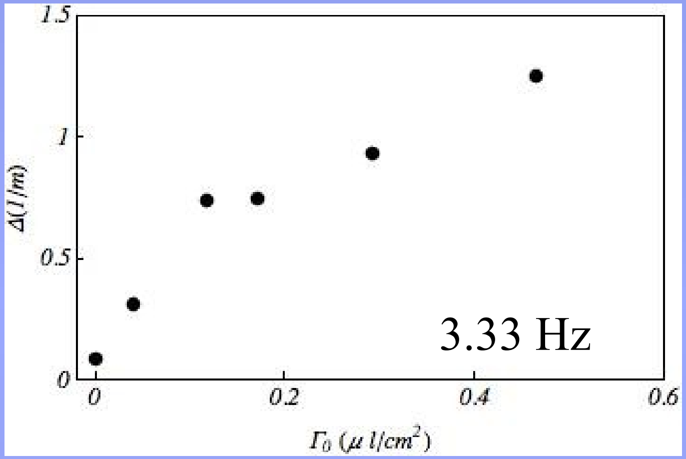




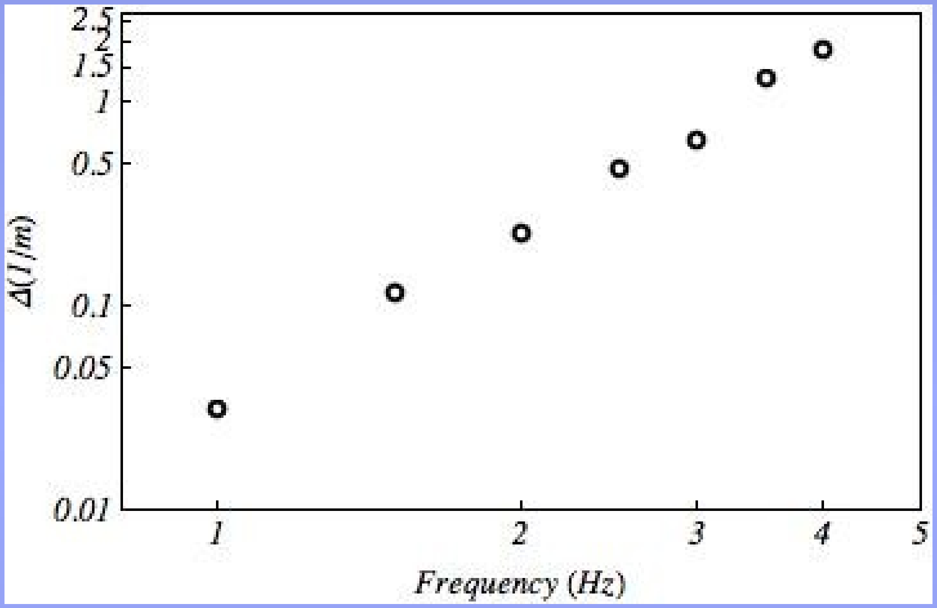


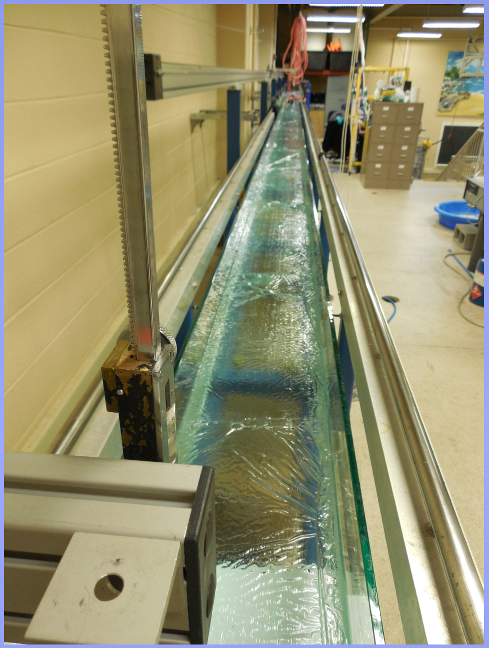
Damping rates in the lab as a function of concentration of oil added to the surface.





Damping rates in the lab as a function of frequency with a saran-wrap surface.





BTW, decay rate ~ $f^{2.9}$

Stole this idea from Guillemette Caulliez, Mediterranean Institute of Oceanography.

Laboratory - tank sidewalls and bottom

Van Dorn (1966)

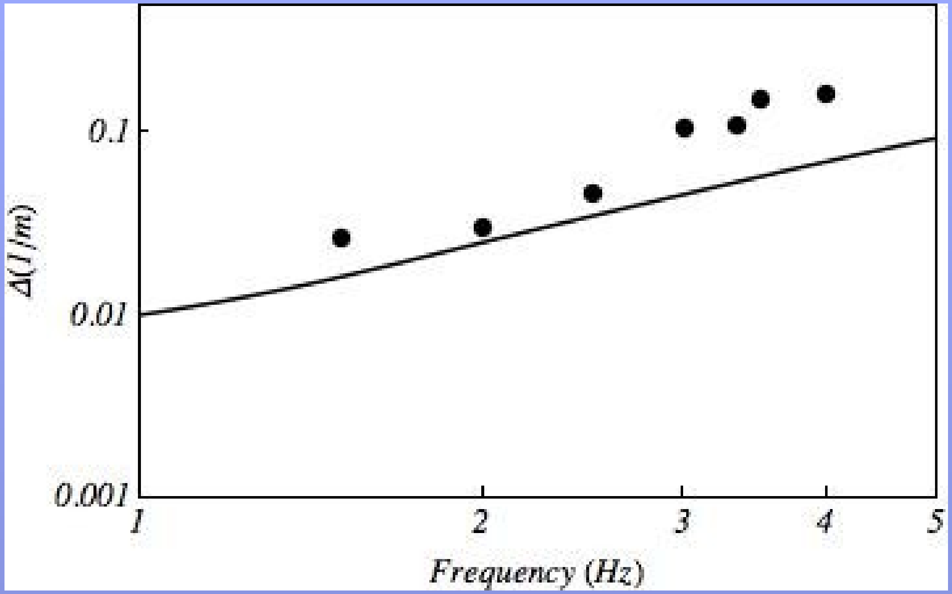
Boundary layers on the sidewalls and bottom.
The rotational motion inside dissipates energy.

$$\Delta_{sb} = \left(\frac{\nu}{2\omega}\right)^{\frac{1}{2}} \left(\frac{2k}{b}\right) \left(\frac{kb + \sinh(2kh)}{2kh + \sinh(2kh)}\right)$$

$b = 25.4\,cm$

$h = 20\,cm$

$\nu = 0.01\,cm^2/s$



Clean Surface (Lamb, 1932)

Free surface beneath a vacuum. Dissipation due to viscosity in the bulk.

$$u(x,z,t) = ik[a e^{ikx+\omega t} - a^* e^{-ikx+\omega^* t}]e^{|k|z} - [m C e^{ikx+\omega t+mz} + m^* C^* e^{-ikx+\omega^* t+m^* z}]$$

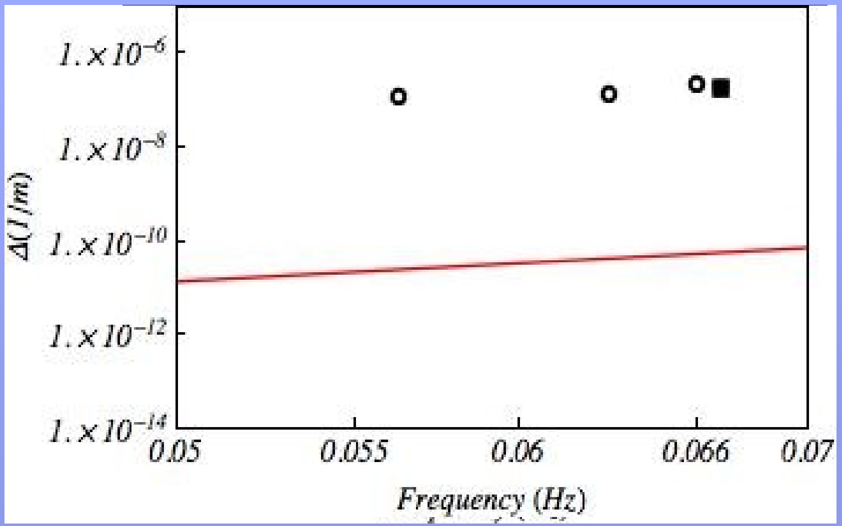
$$w(x,z,t) = |k|[a e^{ikx+\omega t} + a^* e^{-ikx+\omega^* t}]e^{|k|z} + ik[C e^{ikx+\omega t+mz} - C^* e^{-ikx+\omega^* t+m^* z}]$$

Laplace's equation for irrotational part.
Diffusion equation for rotational part: $\omega = \nu(m^2 - k^2)$
Normal stress vanishes at the interface (or balanced with capillary pressure).
Tangential stress vanishes at the interface.

$$\left(\frac{m}{|k|}\right)^4 + 2\left(\frac{m}{|k|}\right)^2 - 4\left(\frac{m}{|k|}\right) + 1 + \frac{g|k| + T|k|^3}{(\nu k^2)^2} = 0$$

$$\Delta_{cs} = \frac{2\nu k^2}{C_g}$$

Dias, Dyachenko, Zakharov (2008) derived a dissipative NLS eqn with this decay rate;
Kharif, Kraenkel, Manna, Thomas (2011) use this in a damped/driven NLS equation.



Ocean: hollow circles, Snodgrass *et al*. (1966); solid circles, Collard *et al.* (2009)

Clean Surface between two fluids (Dore, 1978) Air-water interface.

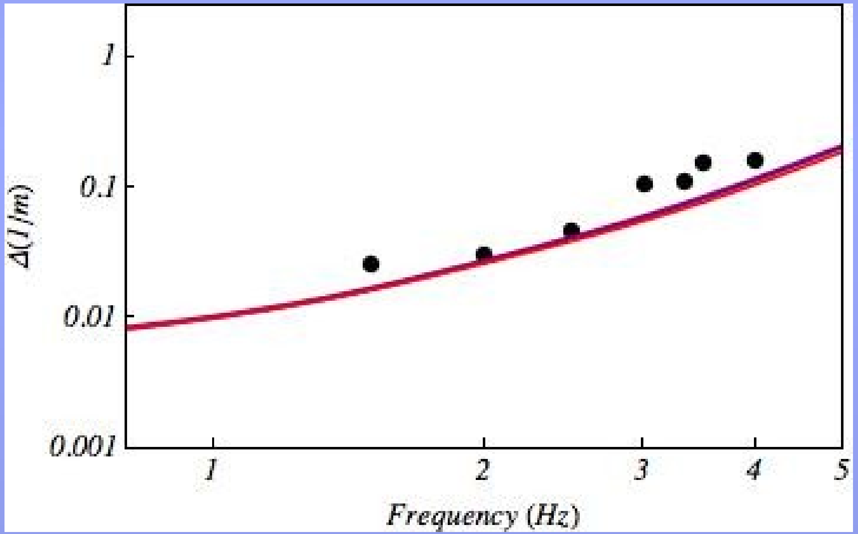
"Virtually no stresses can exist in the air…" Huhnerfuss, *et al.* 1985

Continuity of normal and tangential velocities.
No jump in normal stress at the interface (or balanced with capillary pressure).
No jump in tangential stress at the interface.

$$\Delta_{2f} = \frac{1}{C_g}\left(\sqrt{2\nu_{water} k^2 \omega R^2 V}\right)$$

Purple: Air over water, clean; $\Delta_{2f} + \Delta_{cs} + \Delta_{sb}$

Red: Vacuum over water, clean; $\Delta_{cs} + \Delta_{sb}$



$\nu_{air} = 0.15\, cm^2/s$

$\rho_{air} = 0.0012\, g/cm^3$

$\nu_{water} = 0.010\, cm^2/s$

$\rho_{water} = 1.0\, g/cm^3$

$R = \rho_{air}/\rho_{water} = 0.0012$

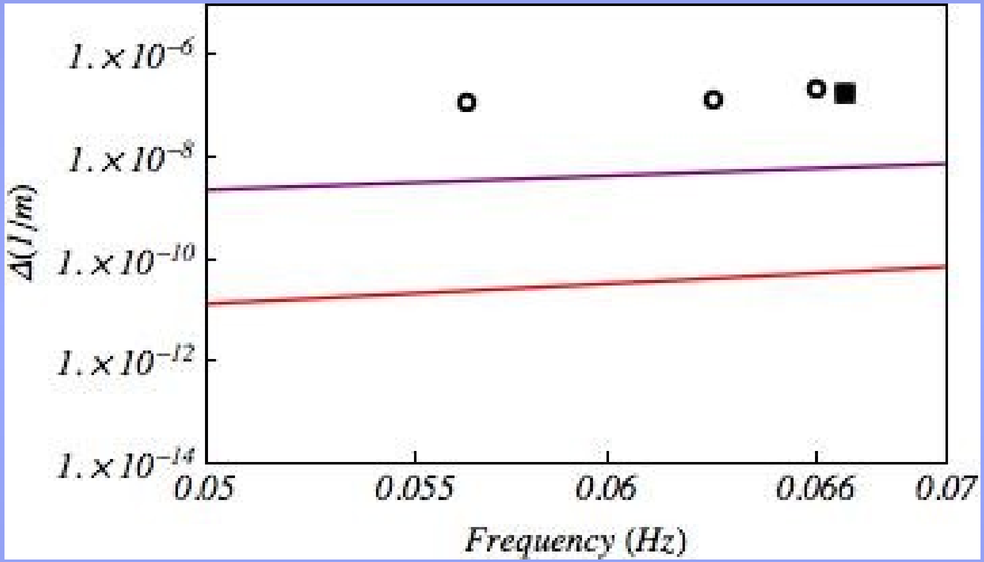
$RV = \mu_{air}/\mu_{water} = 0.018$

$V = \nu_{air}/\nu_{water} = 15$

Ocean Data

Observations from Snodgrass *et al.* (1966) (open circles) and Collard *et al*.(2009) (solid square).

$$\Delta_{2f} = \frac{1}{C_g}\left(\sqrt{2\nu_{water}k^2\omega R^2 V}\right)$$



Purple: Air over water, clean; $\Delta_{2f} + \Delta_{cs}$

Red: Vacuum over water, clean; Δ_{cs}

$\nu_{air} = 0.15\, cm^2/s$

$\rho_{air} = 0.0012\, g/cm^3$

$\nu_{water} = 0.010\, cm^2/s$

$\rho_{water} = 1.0\, g/cm^3$

$R = \rho_{air}/\rho_{water} = 0.0012$

$RV = \mu_{air}/\mu_{water} = 0.018$

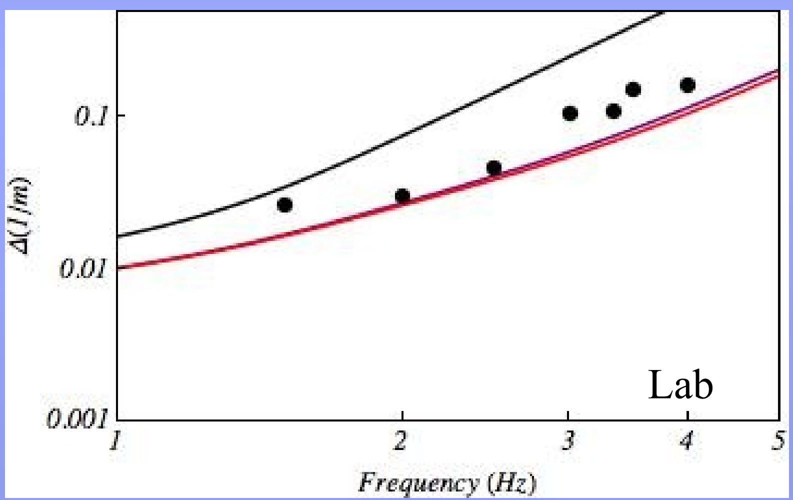
$V = \nu_{air}/\nu_{water} = 15$

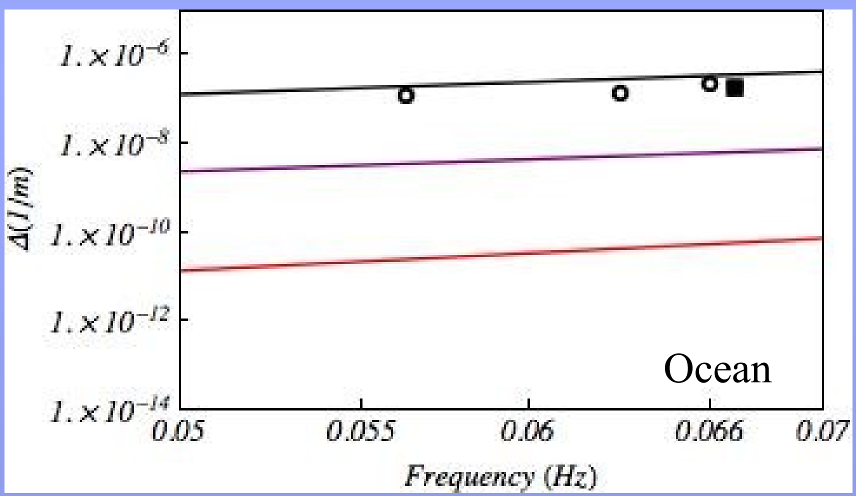
A “dirty” surface model - Inextensible surface (Lamb, 1932)

Vacuum over water with an inextensible interface:
it can oscillate vertically, but cannot stretch horizontally.

Normal stress vanishes (or balanced with capillary pressure).
Tangential stress not constrained.
Tangential velocity is zero.

$$\Delta_{in} = \frac{k}{2C_g}\left(\sqrt{\frac{\nu_{water}\omega}{2}}\coth kh\right)$$





Black: Inextensible surface, Vacuum over water; $\Delta_{in} + \Delta_{cs} + \Delta_{sb}$

Purple: Air over water, clean; $\Delta_{2f} + \Delta_{cs} + \Delta_{sb}$

Red: Vacuum over water, clean; $\Delta_{cs} + \Delta_{sb}$

Van Dorn (66), Miles (67) included depth.

The inextensible model seems to work well for the ocean? Only in the linearized problem.

Kinematic interface condition. $$\frac{\partial \eta}{\partial t} + u \frac{\partial \eta}{\partial x} = w$$

Rewritten in terms of velocity normal to the interface.
Zakharov (1968)

$$\frac{\partial \eta}{\partial t} = q_n \sqrt{1 + (\partial \eta / \partial x)^2}$$

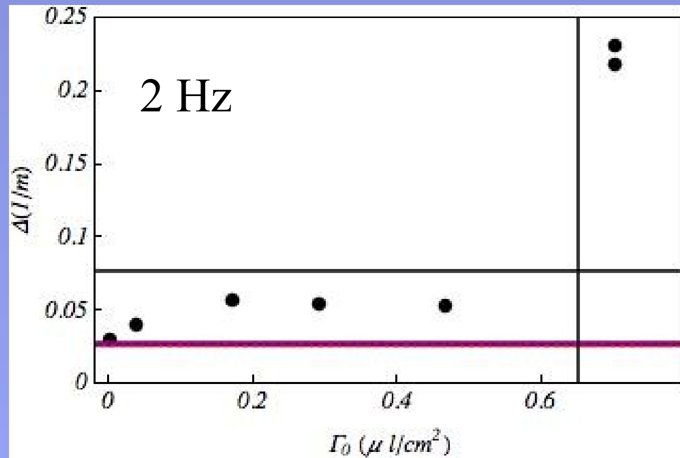
q_n: normal velocity at the interface.

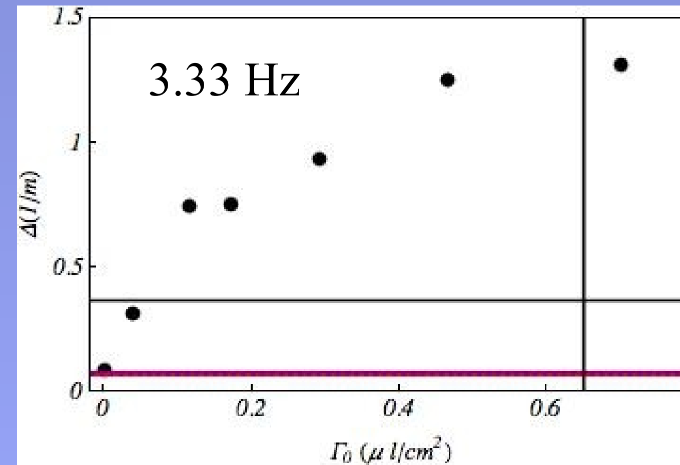
- The interface speed is usually greater than the normal velocity.
- The extra has to come from the tangential velocity component.
- So, requiring the tangential component to vanish at the interface will only work in the linear approximation.

You can't use the inextensible film model to pursue nonlinear effects.

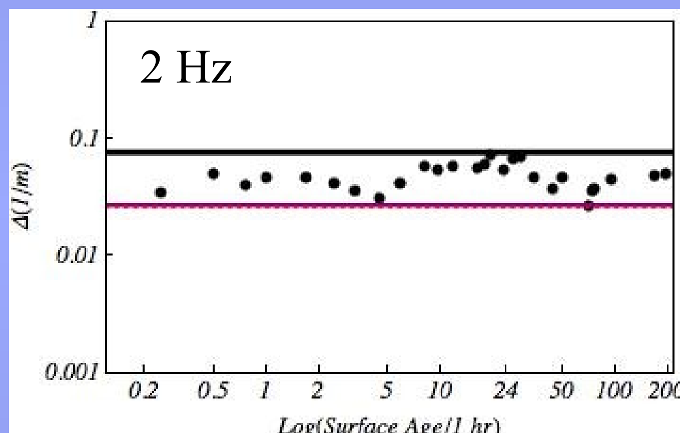
One can imagine many nonlinear generalizations that give this model at linear order. Not unique.

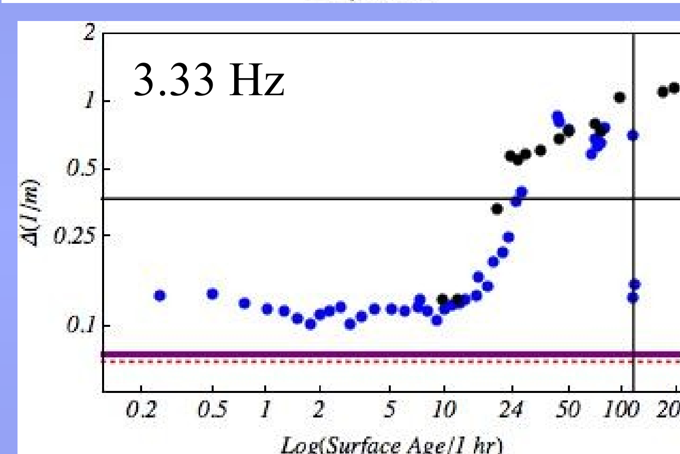
Added oil

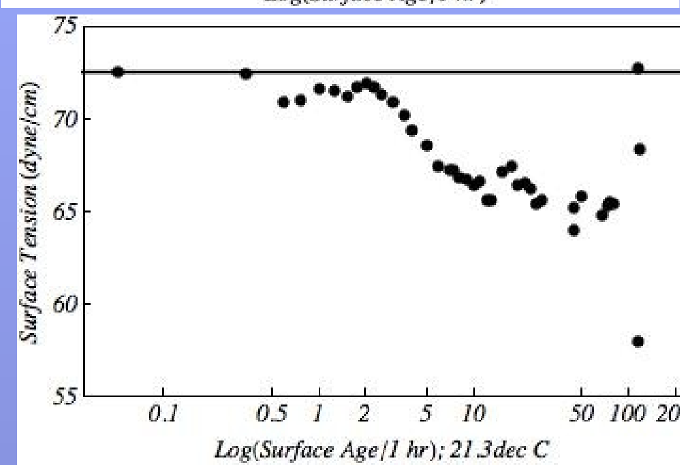




Surface age







Black: Inextensible surface, Vacuum over water; $\Delta_{in} + \Delta_{cs} + \Delta_{sb}$

Purple: Air over water, clean; $\Delta_{2f} + \Delta_{cs} + \Delta_{sb}$

Red: Vacuum over water, clean; $\Delta_{cs} + \Delta_{sb}$

A visco-elastic surface model

(Levich, 1941; Miles, 1967; Lucassen-Reynders & Lucassen, 1969…1982; Hunherfuss *et al.*, 1976…1985)

Vacuum over water with a visco-elastic surface film; contaminants cause surface tension gradient, which causes a surface flow, which takes energy from the wave.

Jump in normal stress balanced by curvature due to surface tension.
Jump in tangential stress balanced by surface tension gradient.
Need a constitutive law for the film.

$$\vec{\tau} = \nabla T + \sigma_1 \nabla\nabla\cdot\vec{u} + \sigma_2 \nabla^2 \vec{u}$$

∇T Elasticity σ_1 Dilational viscosity σ_2 Shear viscosity γ Solubility

D Diffusion into the bulk

$$\nabla T = \left(\frac{dT}{d\Gamma}\right)_0 \nabla(\Gamma - \Gamma_0)$$

$$\frac{\partial\Gamma}{\partial t} + \Gamma_0 \nabla\cdot u = D\left(\frac{\partial\gamma}{\partial z}\right)_0$$

$$\xi = -k^2 \Gamma_0 \left(\frac{dT}{d\Gamma}\right)_0 \sqrt{\frac{2}{\nu\omega^3}}$$

$$\Delta_{ve} = \frac{1}{2Cg} k \sqrt{\frac{\nu\omega}{2}} \left(\frac{\xi(\xi+\sigma) + \zeta(\zeta+2)}{(\xi-1)^2 + (1+\sigma)^2 + \zeta(\zeta+2)}\right) \coth(kh)$$

$$\zeta = -\left(\frac{2}{\nu\omega}\right)^{1/2} k^2 (\sigma_1 + \sigma_2)$$

$$\sigma = \left(\frac{2D}{\omega}\right)^{1/2} \left(\frac{d\Gamma}{d\gamma}\right)_0^{-1}$$

Lucassen, 1982, J. Colloid Interface Sci., vol 85, p52**.**

Cini & Lombardini, 1982, J. Colloid Interface Sci., vol 81, p125**.**

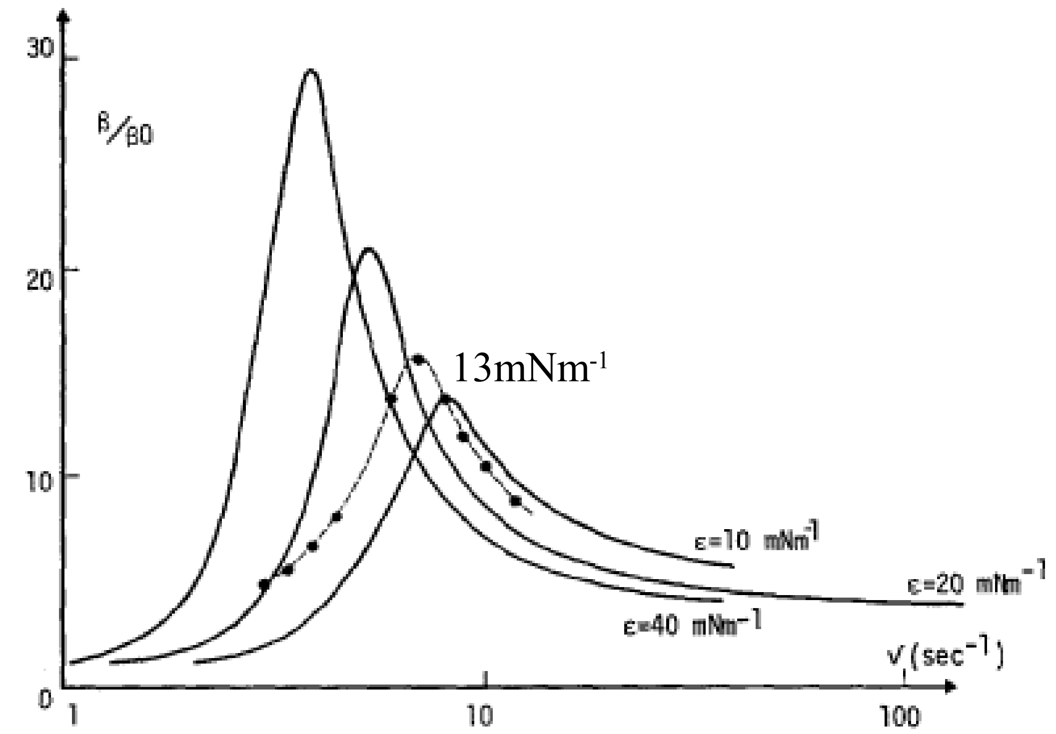


FIG. 4. Relative increase of damping coefficient compared to water—as a function of wave frequency for different values of the surface dilational modulus. Points and dashed line: Ref. (17).

Maximum dissipation rates for an insoluble surfactant. $\sigma=0$

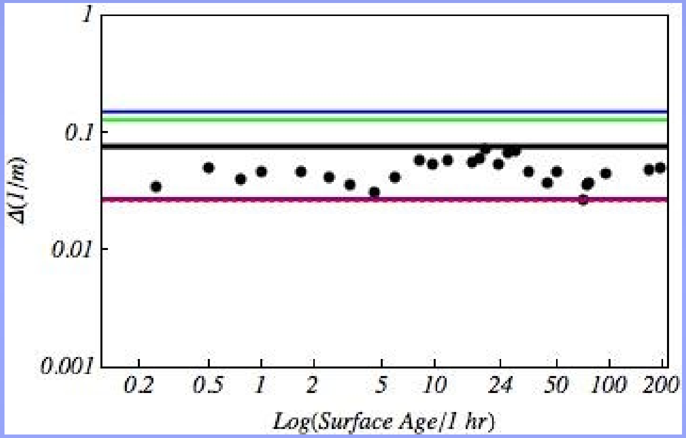
Limit as elasticity and viscous shear become infinite: $\Delta_{ve,\sigma=0} \to \Delta_{in}$

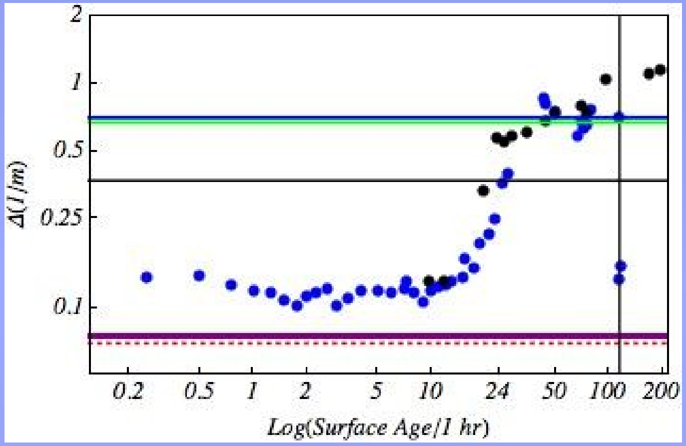
Max damping rate: Miles, 1967: $\Delta_{ve,\sigma=0} \to 2\Delta_{in}$ Lucassen, 1982 $\Delta_{luc} \to \sqrt{2}(\nu^3 k^7/g)^{1/4}$

2 Hz

3.33 Hz

Surface Age





Blue $2\Delta_{in}+\Delta_{sb}+\Delta_{cs}$

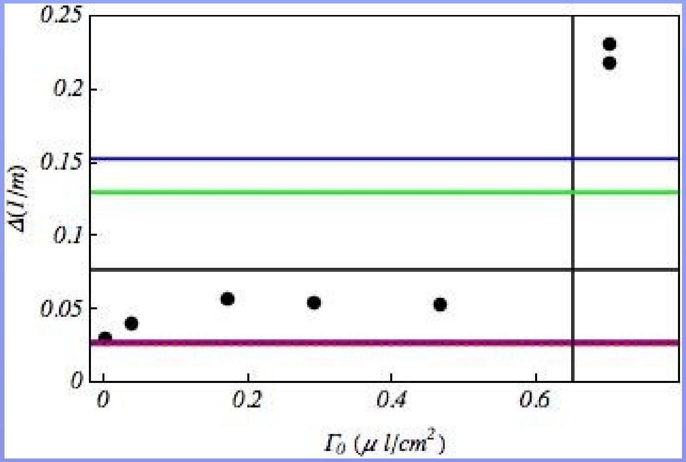
Green $\Delta_{luc}+\Delta_{sb}+\Delta_{cs}$

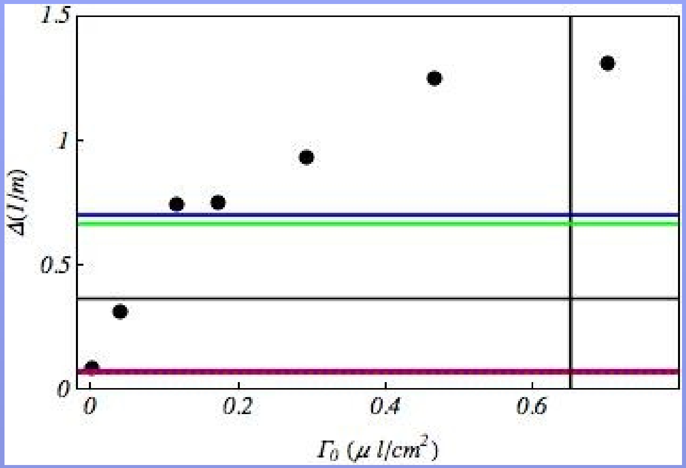
Black $\Delta_{in}+\Delta_{sb}+\Delta_{cs}$

Purple $\Delta_{2f}+\Delta_{sb}+\Delta_{cs}$

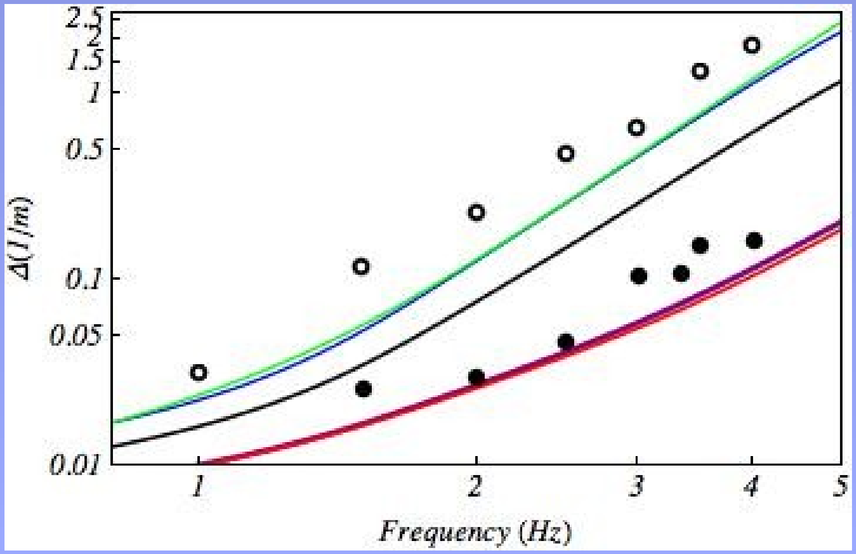
Red (dotted) $\Delta_{cs}+\Delta_{sb}$

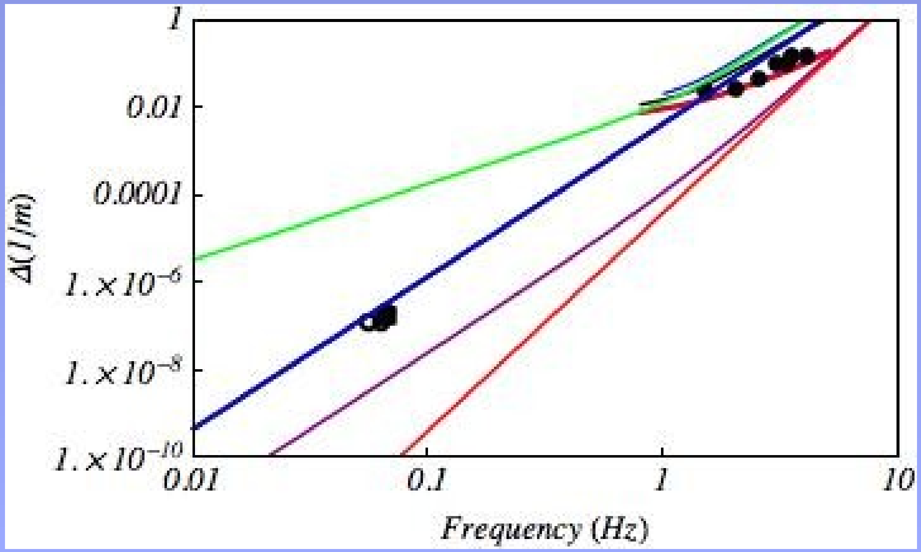
Oil





Lab and Ocean





Lab: hollow circles, saran wrap;
solid circles, clean surface.

Blue $2\Delta_{in} + \Delta_{sb} + \Delta_{cs}$

Green $\Delta_{luc} + \Delta_{sb} + \Delta_{cs}$

Black $\Delta_{in} + \Delta_{sb} + \Delta_{cs}$

Purple $\Delta_{2f} + \Delta_{sb} + \Delta_{cs}$

Red $\Delta_{cs} + \Delta_{sb}$

Ocean: hollow circles, Snodgrass *et al*.;
solid circles, Collard *et al.* (2009)

Blue $2\Delta_{in} + \Delta_{cs}$

Green $\Delta_{luc} + \Delta_{cs}$

Black $\Delta_{in} + \Delta_{cs}$

Purple $\Delta_{2f} + \Delta_{cs}$

Red Δ_{cs}

Summary and Conclusions

Ocean:

The one-fluid clean-surface model does not predict measurements of damping rates in the ocean.
The two-fluid clean-surface model (Dore, 78): air matters!

The inextensible film model predicts linear dissipation rates pretty well.
Cannot be used for nonlinear models.

The visco-elastic model- in the limit of infinite elasticity/shear (Miles, 67) is the inextensible film model, but does not have the offending boundary condition of zero tangential velocity. It has no free parameters.

Lab:

Inextensible film model is ok for 2hz.
For $f >$ hz? Next up: air-water interface with viscoelastic film.