Numerical simulations of lateral dispersion of a passive tracer in a field of oceanic internal waves

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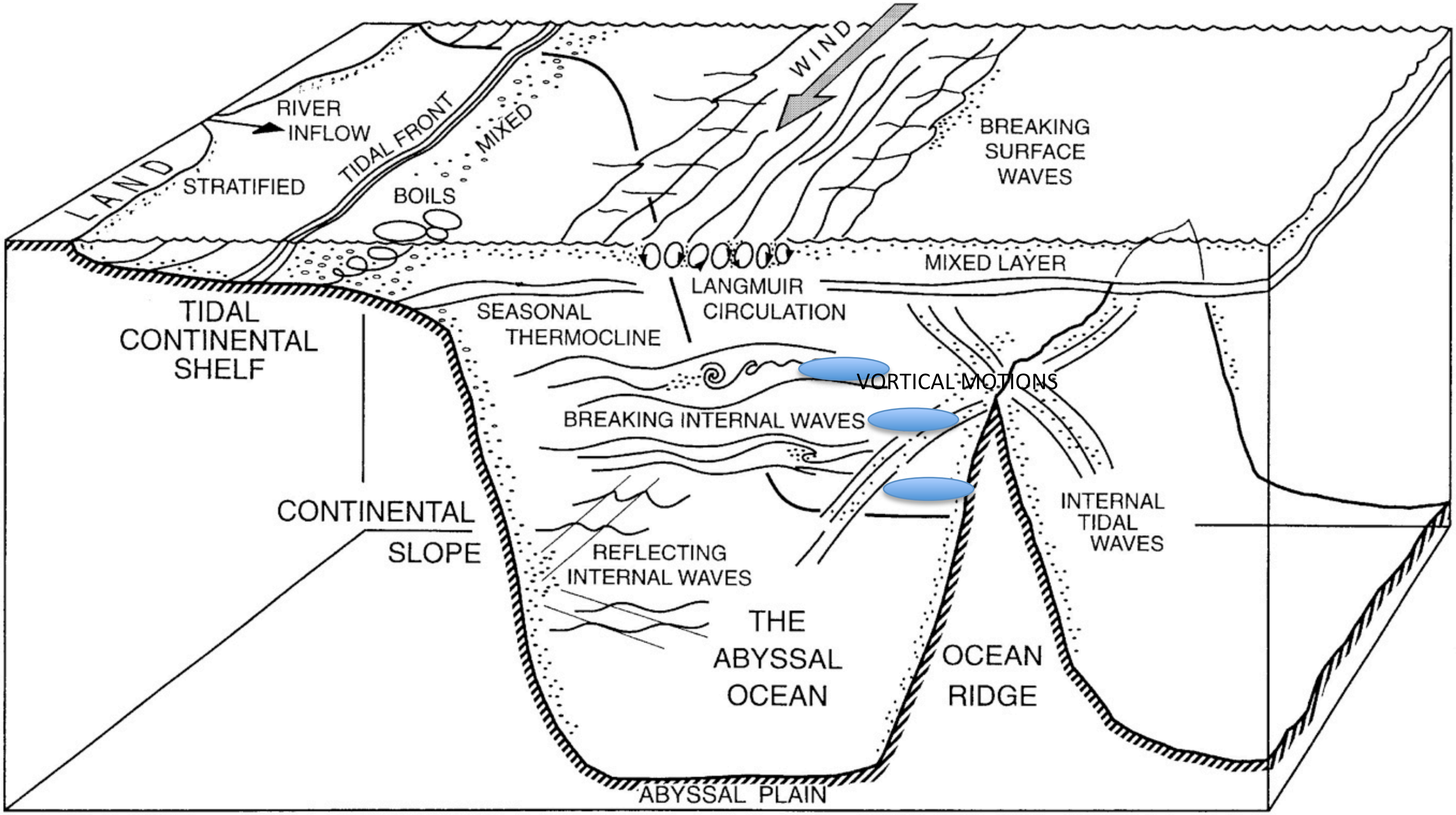
Outline

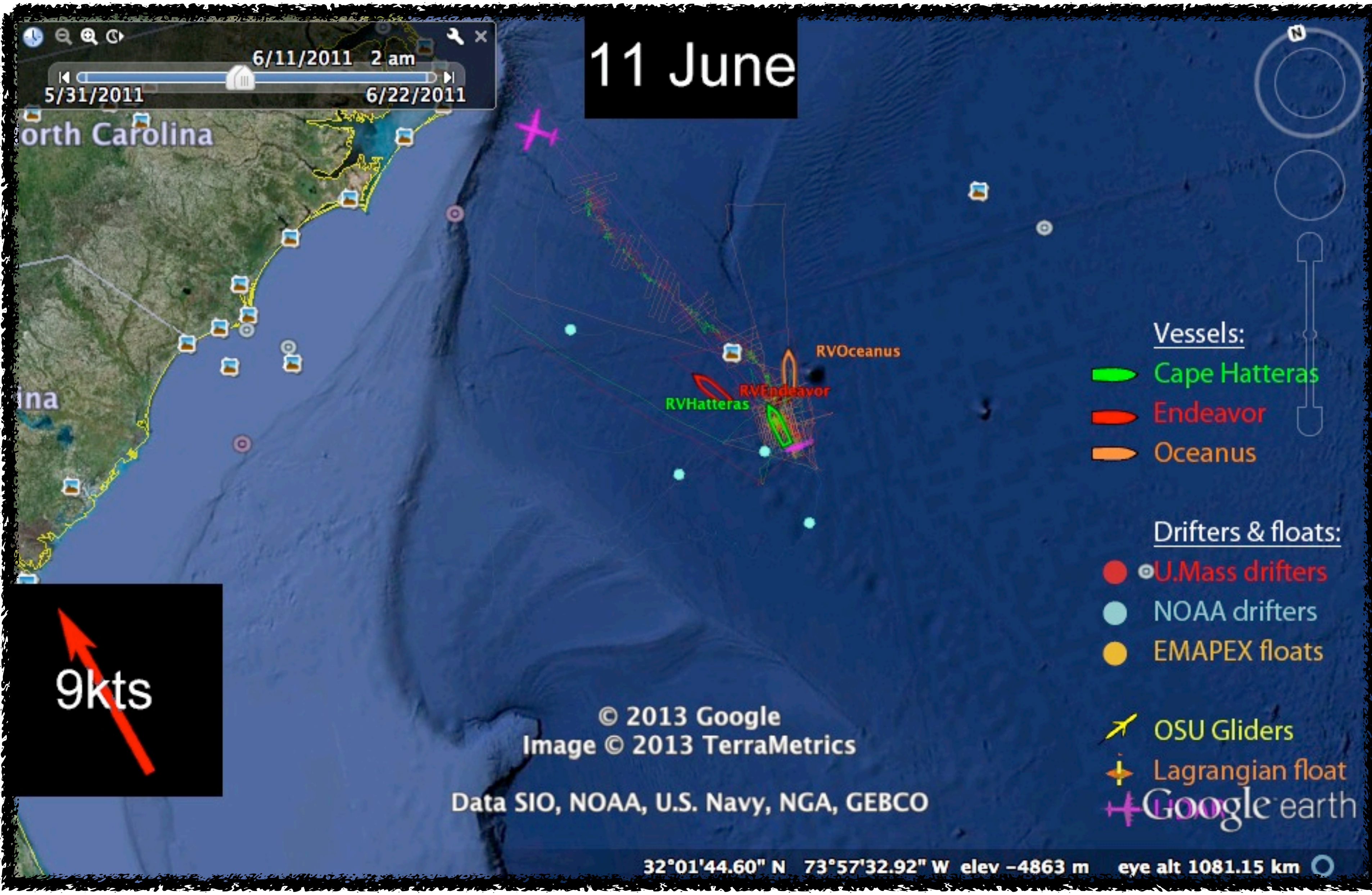
- Overview of LatMix 2011 observations at site I
- hypothesis to be tested
- role of potential vorticity/local Richardson number
- Numerical strategy
- Winters Boussinesq model
- Garrett-Munk initial condition
- Results
- comparison of 2 sets of simulations (unresolved/ resolved wave breaking)
- dependence on GM spectral level
- dependence on latitude
- conclusions
- Future simulations / analysis

The oceanic submesoscale (100m-10 km horizontal scales)

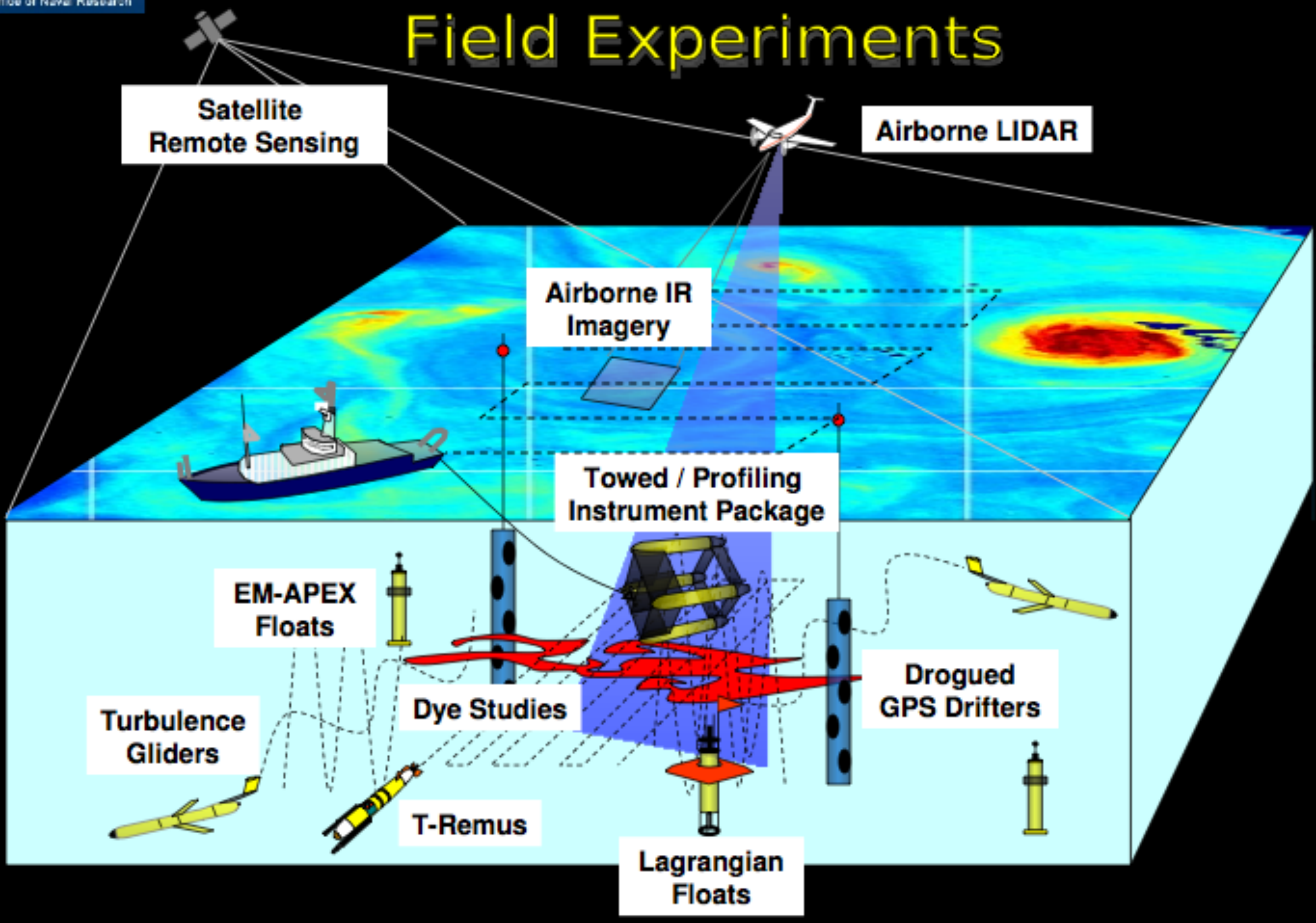


adapted from Thorpe (2005)

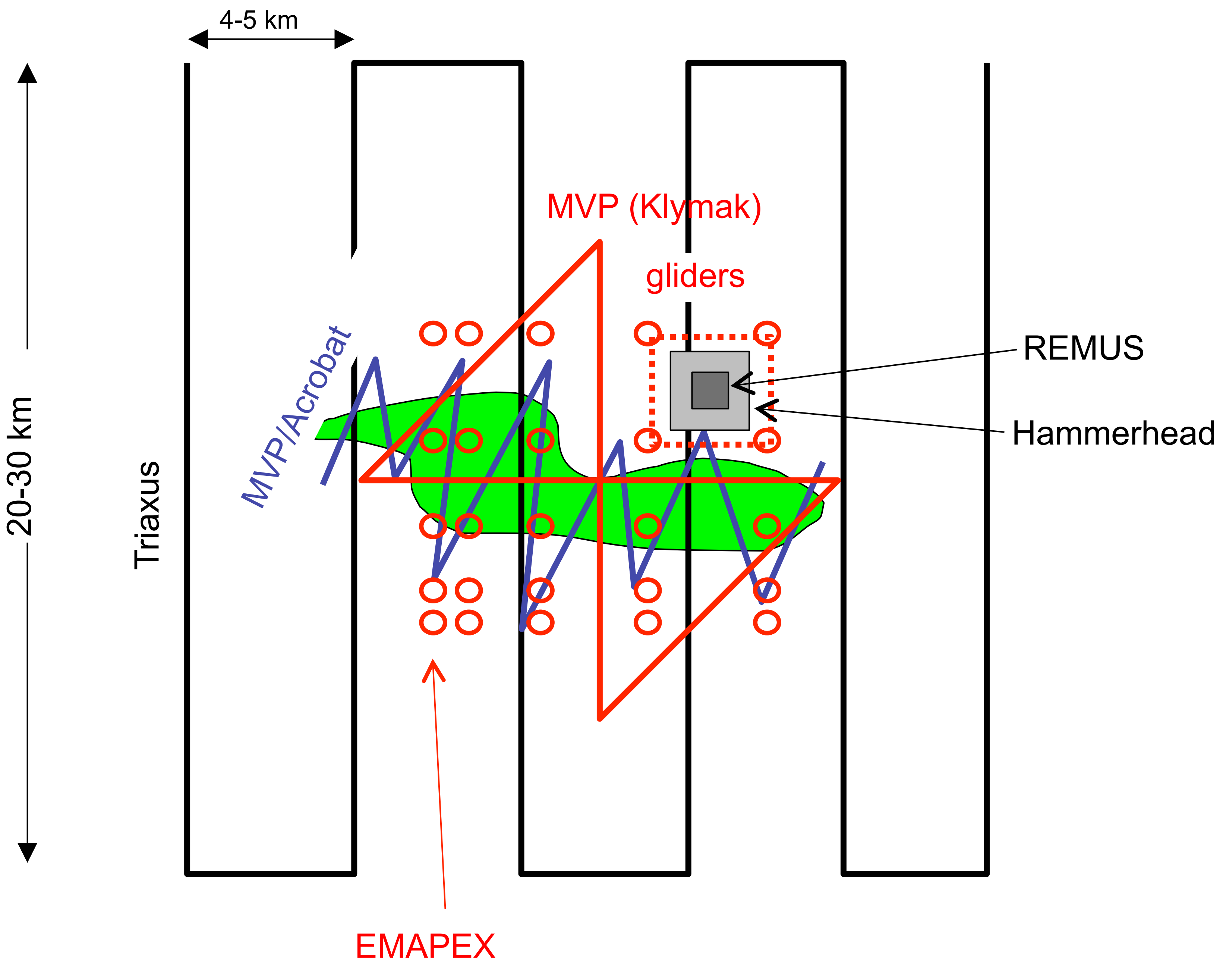
LatMix 2011, site I (aka The Big Nothing)



ONR LatMix
Field Experiments



Dartmouth Acrobat glider


4-5 km
20-30 km
Triaxus
MVP/Acrobat
MVP (Klymak)
gliders
REMUS
Hammerhead
EMAPEX

What are the dynamical mechanisms responsible for observed submesoscale lateral dispersion?

- Hypothesis I:

Inhomogeneous internal wave mixing creates potential vorticity anomalies that are responsible for significant submesoscale isopycnal mixing.

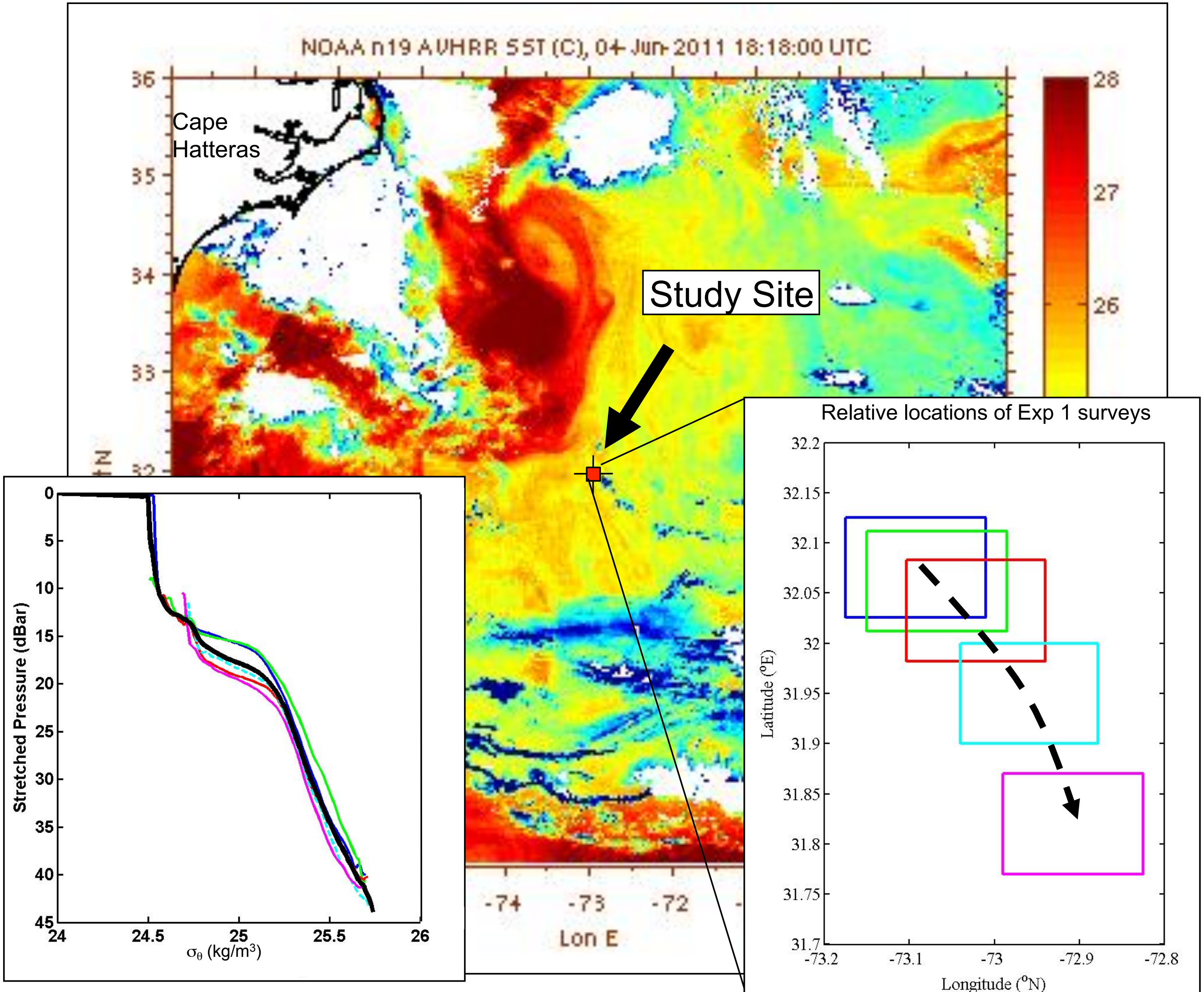
- Hypothesis II:

Mesoscale straining leads to a cascade of both tracer and PV variance to submesoscales that is responsible for significant submesoscale isopycnal mixing.

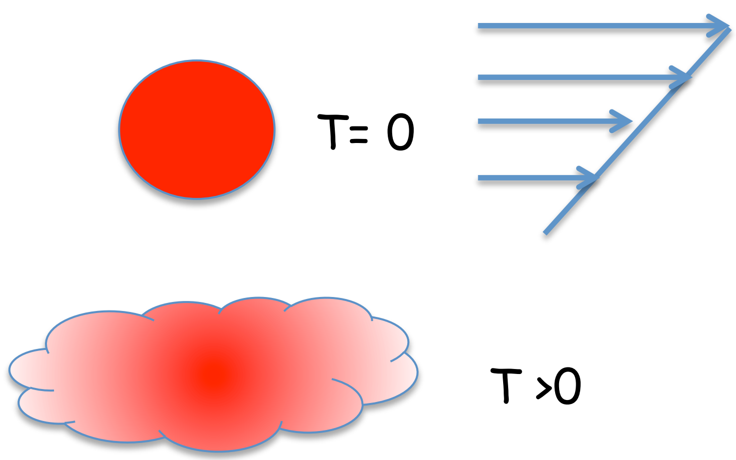
- Hypothesis III

Non-QG submesoscale instabilities feed a forward cascade of energy, scalar and tracer variance which enhances both isopycnal and diapycnal mixing.

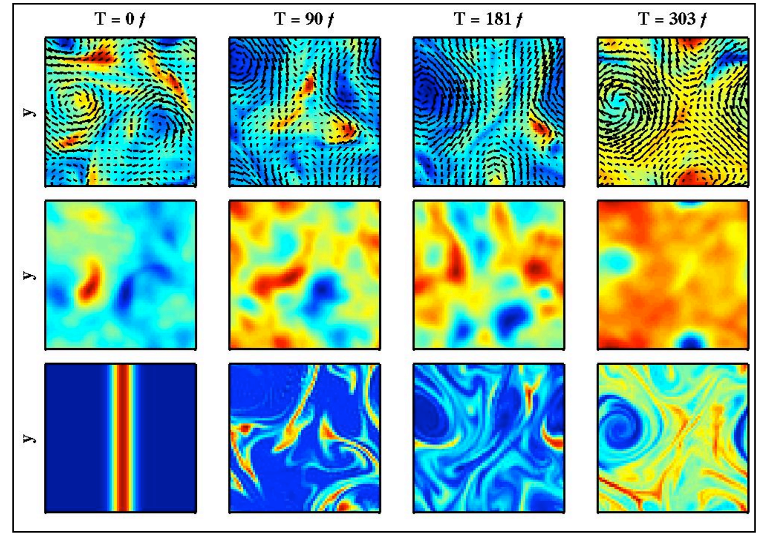
LatMix I: Low to Intermediate Strain
(June 1 – June 21, 2011)



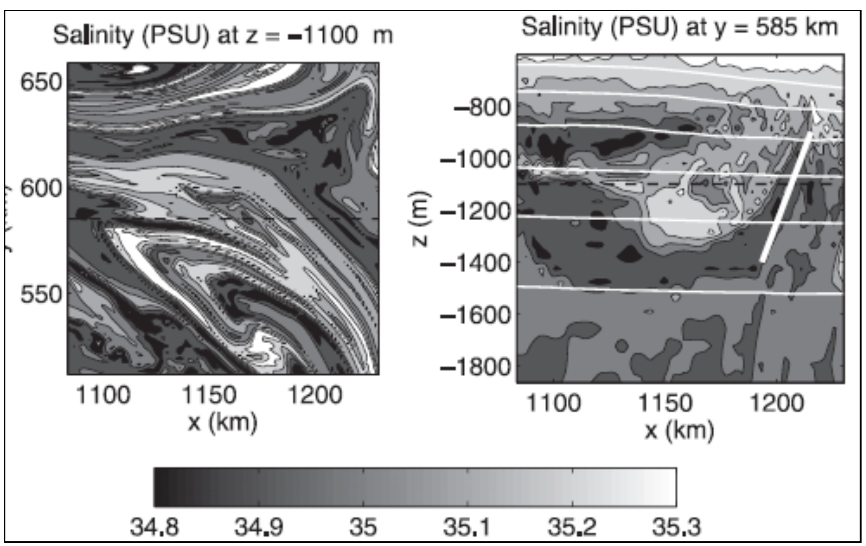
Submesoscale mixing processes in stratified ocean interior



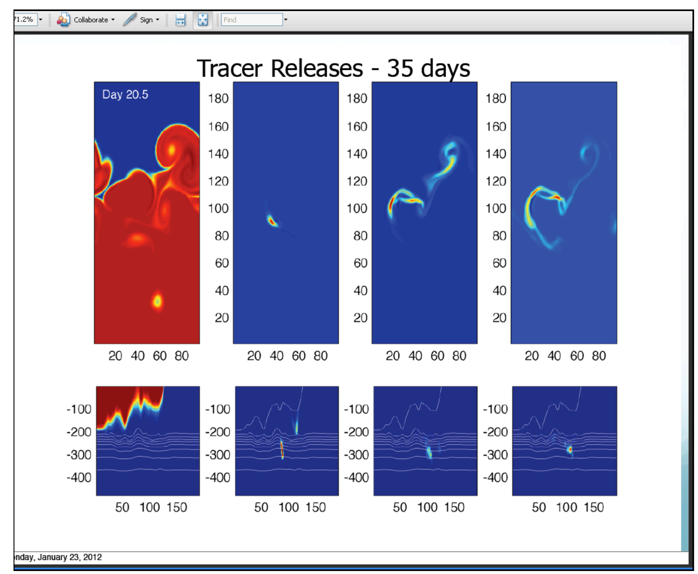
Shear dispersion, Taylor, 1923,Young et al. 1982



Sundermeyer and Lelong, 2005



Smith and Ferrari, 2009



Badin et al., 2011

Analytic study of wave-induced dispersion in a rotating Boussinesq flow (Holmes-Cerfon et al., 2011)

- Holmes-Cerfon et al. (JFM, 2011)

one-particle diffusivity (Taylor, 1921) to infer effective diffusivity in a field of random internal waves.

- For a Garrett-Munk spectrum, effective diffusivity D scales as,

$$D \approx 0.08 \frac{E_0^2 m_*^2}{N^2 f} \{2\, ln\, (\frac{m_c}{m_*}) - \frac{\pi^2}{4}\}$$

Hypothesis I of the Lateral Mixing DRI:

Inhomogeneous internal wave mixing creates potential vorticity anomalies that are responsible for significant submesoscale isopycnal mixing.

What we know from observations...

Instabilities lead to wave breaking and create intermittent mixed turbulent patches in space and time that exhibit high dissipation.

What we speculate from observations of submesoscale dispersion..

Mixed patches adjust geostrophically to form small-scale vortices which can stir tracers efficiently along isopycnal surfaces.

What we don't know...

How prevalent is this mechanism in the ocean?
How efficient is it at mixing loang isopycnals?
How does it compare with other stirring mechanisms?

Direct knowledge of small-scale vortical modes is inferred primarily from theory and numerical studies!

from theory...

- The vortical mode exists: it’s the linear, balanced PV eigenmode of the equations of motion.

It facilitates energy transfer between waves through a catalytic effect (Lelong and Riley, 1991; Bartello, 1995; Waite and Bartello, 2005, 2006)

Ensembles of vortex modes can be modeled as random walks and are efficient at stirring fluid laterally (Sundermeyer, 1998; Sundermeyer and Lelong, 2005)

334 M. L. Waite and P. Bartello

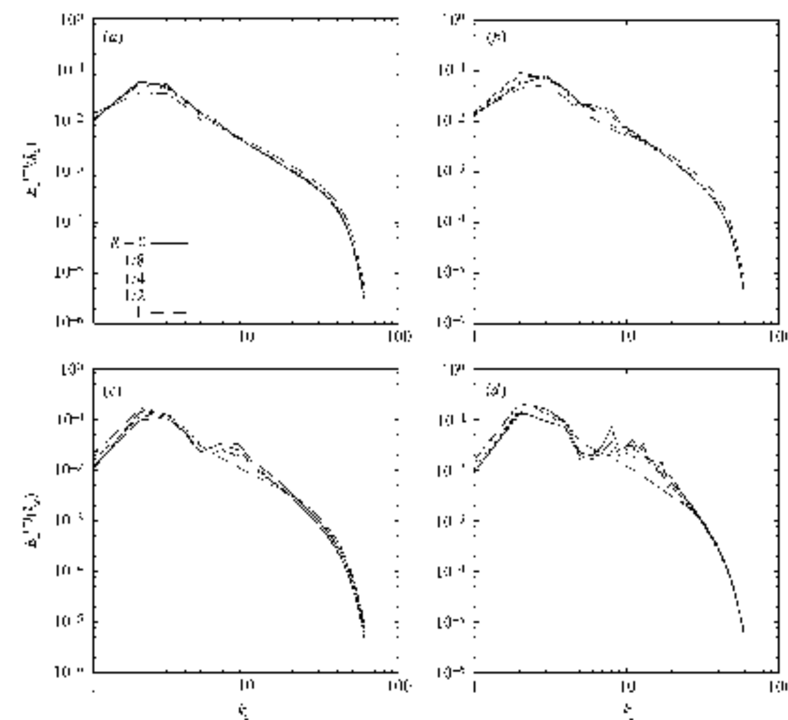


FIGURE 15. Vertical wavenumber spectra of wave energy for $R = 0$, 1/8, 1/4, 1/2 and 1 when $M = 160$ and (a) $N = 4$, (b) $N = 8$, (c) $N = 16$ and (d) $N = 32$.

Waite and Bartello, 2005

Dye dispersion in fields of internal waves in low-strain, low-shear environment characteristic of LatMix11 Site I.

- Winters 3D nonlinear pseudo-spectral Boussinesq model (Winters et al, 2003)
- Initial condition: Broadband internal wave field (GM)
- Domain dimensions: 15 km x 15 km x 300 m
10 km x 10 km x 100 m
- Ambient conditions specified from LatMix 2011, site I (no background shear/strain).
- Simulations run for 48-72 hours (unforced)
- Gaussian dye streak injected after initial flow adjustment (about 10 hours) into thermocline.
- Some simulations resolve wave breaking, others do not.

The Garrett-Munk spectrum representation of the internal wave field

GM energy spectrum $$b^2 N_0 N(\omega^2 + f^2) B(\omega) H(m) E_0/\omega^2$$

$$b = 1500\, m, N_0 = 5.2 \times 10^{-3} s^{-1} \quad B(\omega) = (f/\omega)/\sqrt{\omega^2 - f^2}$$

$$H(m) = (\pi/b)(N/N_0) j_* \qquad m_* = (\pi/b)(N/N_0) j_*$$

Assumption of horizontal isotropy

Spectral energy amplitude is adjusted by varying E0

(adapted from Flatté et al. 1979; Winters and D' Asaro1997)

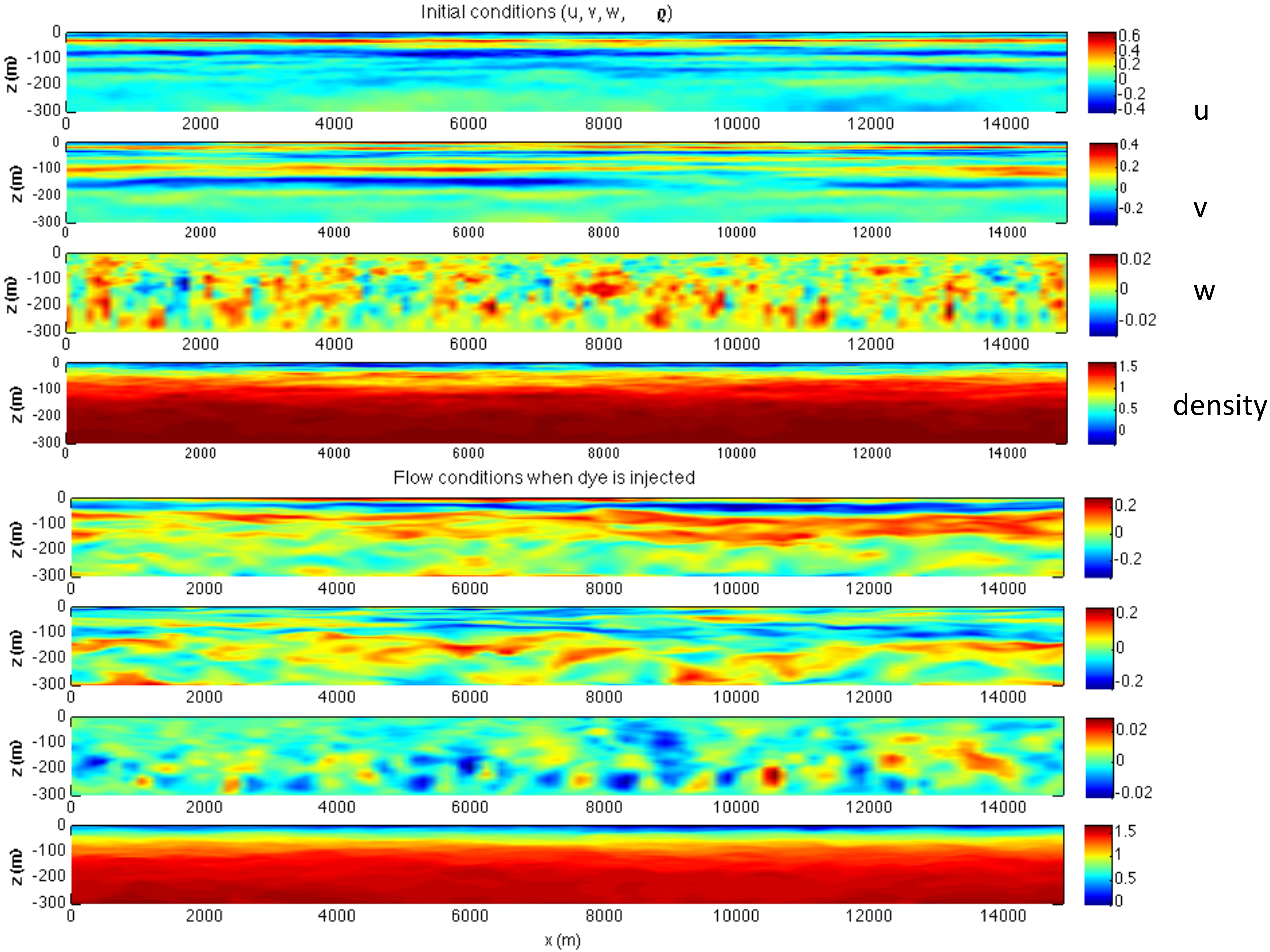
Solving an eigenvalue problem (adapted from Winters and D'Asaro, 1997)

$$\hat{\zeta}''_{jkl} + k\frac{N^2(z)-\omega^2_{jkl}}{\omega^2_{jkl}-f^2}\hat{\zeta}_{jkl} = 0$$

$\hat{\zeta}_{jkl}$ is Fourier transform of vertical displacement ζ of mode (j,k,l)

$$\hat{\zeta}_{jkl}(z=0) = \hat{\zeta}_{jkl}(z=L_z) = 0$$

All other variables are obtained through linear polarization relations



Initial conditions (u, v, w, ϱ)
u
v
w
density
Flow conditions when dye is injected
z (m)
x (m)

The role of breaking waves in PV production

Potential vorticity Π

$$\Pi \equiv (\omega + f\hat{z}) \cdot (N^2\hat{z} + \nabla b) = fN^2 + (N^2\omega_z + f\partial_z b) + \omega \cdot \nabla b$$

Π provides a measure of vorticity perpendicular to isopycnals.

Evolution equation for Π:

Waite, (2013)

$$\frac{D\Pi}{Dt} = (N^2\widehat{z} + \nabla b) \cdot (\nu\nabla^2\vec{\omega} + \nabla \times \vec{F}) + \kappa(f\widehat{z} + \vec{\omega}) \cdot \nabla(\nabla^2 b)$$

Π is conserved following the flow in the absence of viscous and diffusive effects and forcing.

Dissipation and diffusion provide both sources and sinks of PV.

At turbulent scales, dissipation produces PV.

When waves break, they create turbulence, dissipate energy and provide a source of PV at small scales.

Diagnosing internal waves and vortical motions...

Linear eigenmode decomposition, Müller et al. (Reviews of Geophysics,1988), Bartello (JAS, 1995), Waite and Bartello, (2005, 2006)

$$\phi_k^{(0)} = \frac{1}{\sqrt{2}\sigma_k k} \begin{pmatrix} N k_h \\ 0 \\ -i f k_z \end{pmatrix} \qquad \phi_k^{(\pm)} = \frac{1}{\sqrt{2}\sigma_k k} \begin{pmatrix} i f k_z \\ \sigma_k k \\ \mp N k_h \end{pmatrix}$$

where, $\sigma_k = \|\omega_k\| = \sqrt{\frac{N^2 k_h^2 + f^2 k_z^2}{k_h^2 + k_z^2}}$ internal wave frequency

$\phi_k^{(0)}$: vortical mode

$\phi_k^{(\pm)}$: internal wave modes

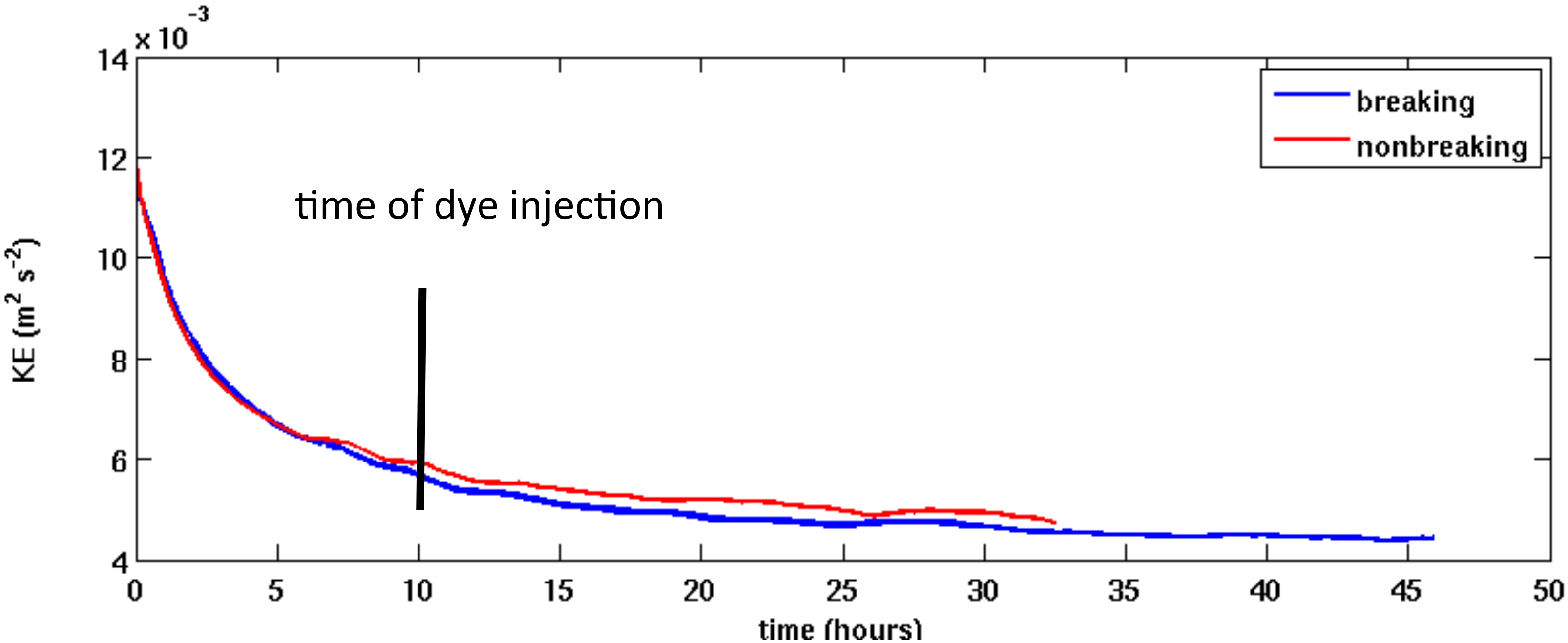
N: buoyancy frequency

f: Coriolis frequency

k_h and k_z are horizontal and vertical wave numbers

KE: contrast of non-breaking vs breaking wave fields

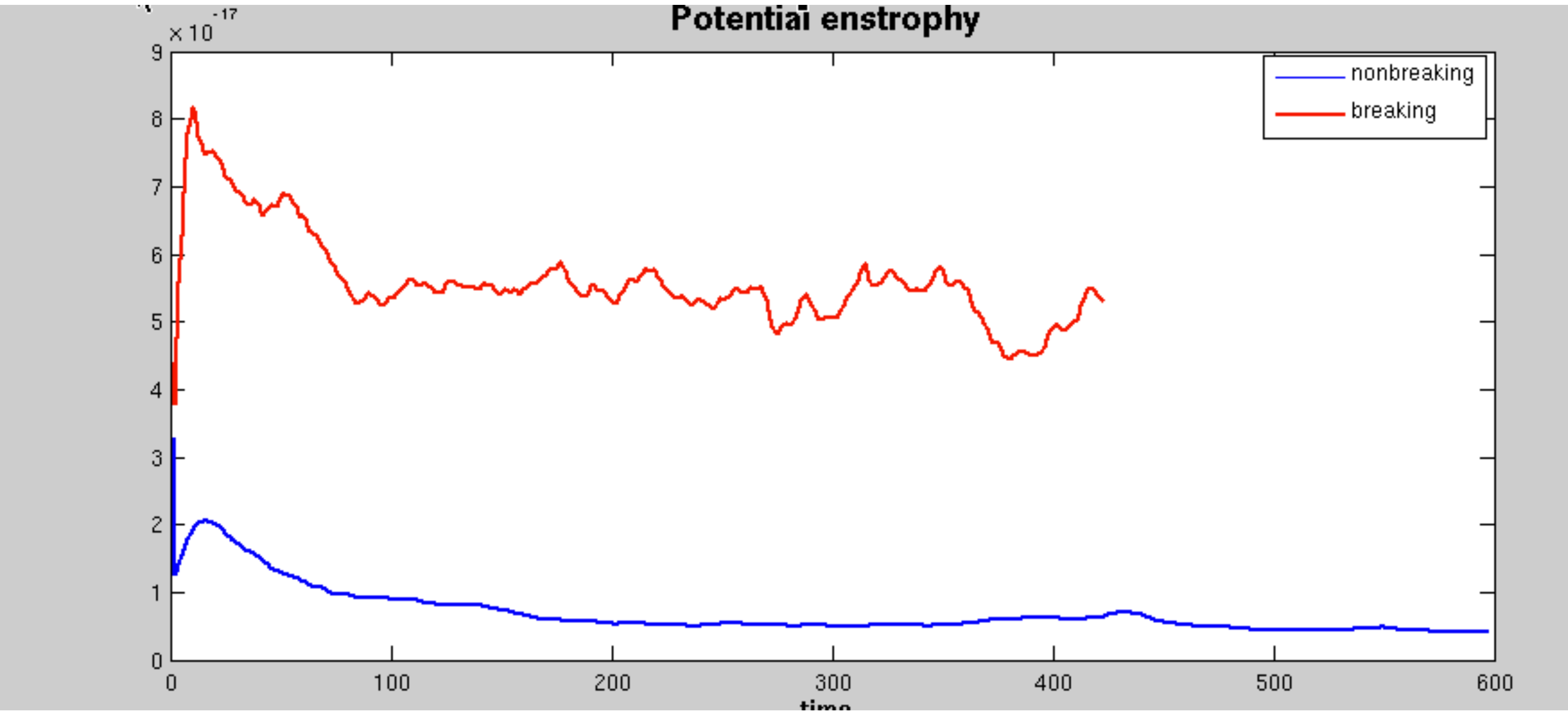
- non-breaking simulation: low vertical resolution
- breaking simulation: high vertical resolution



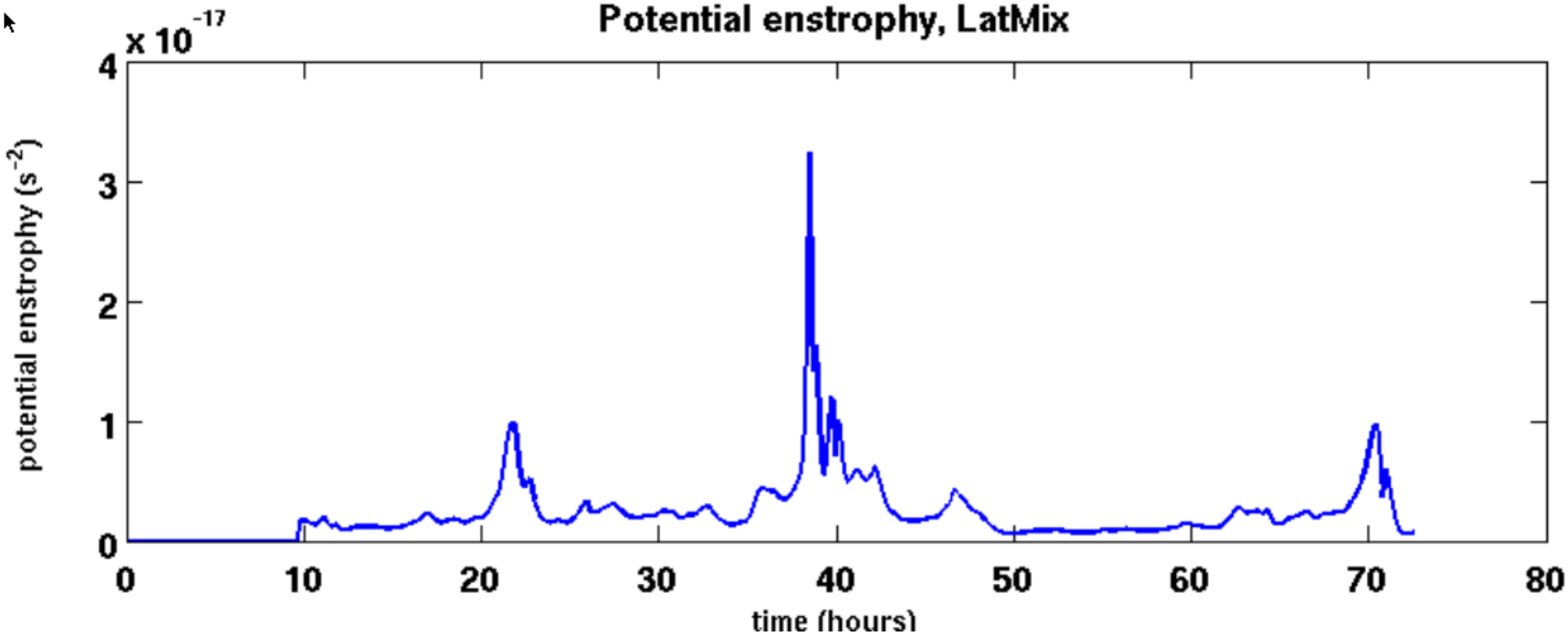
Potential enstrophy production (non-breaking vs breaking waves)

Simulations use identical wave field but with different vertical resolutions

- Coarse resolution (wave breaking not resolved)
- Fine resolution (wave breaking resolved)

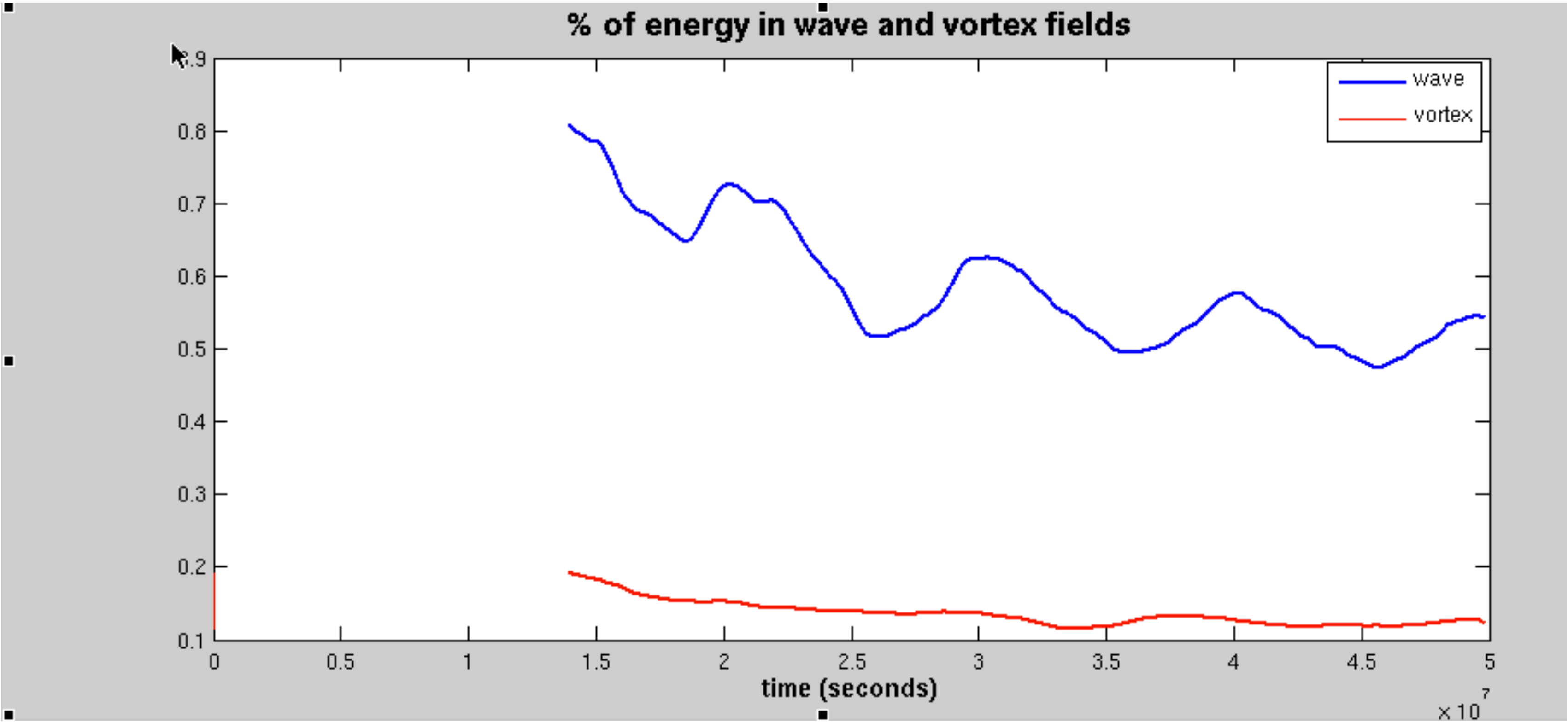


Production of potential enstrophy as a function of time



conversion of wave energy into vortex energy

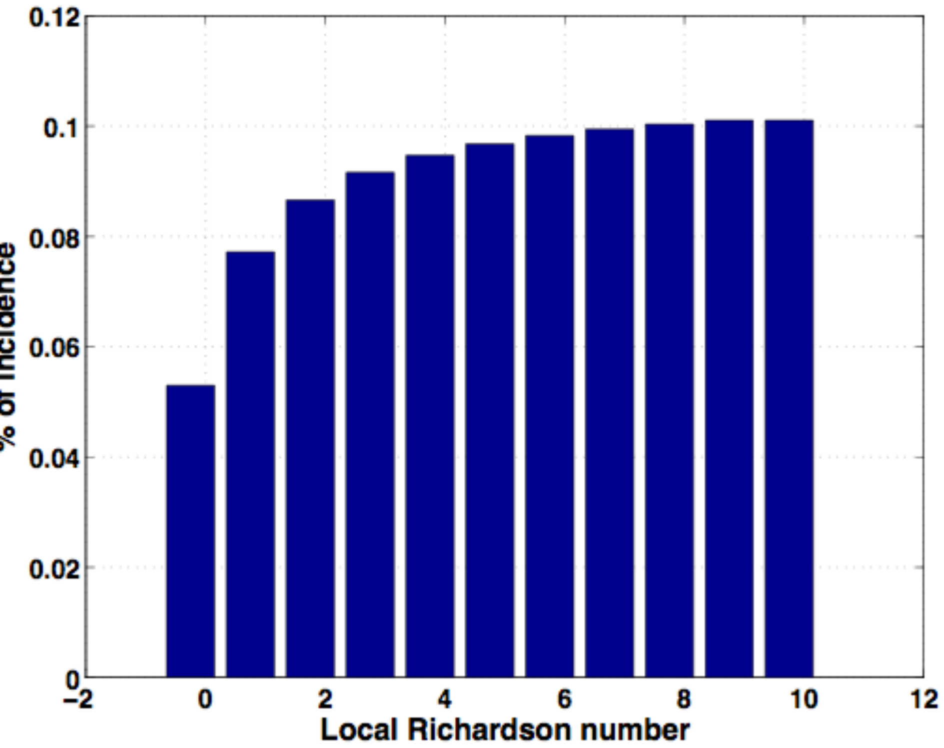
Energies normalized with total energy at time of dye injection



About 12% of initial wave energy is converted to vortex motions.

Diagnosing wave breaking

$$Ri_l = \frac{-\frac{g}{\rho_0}\left(\frac{d\bar{\rho}}{dz} + \frac{\partial \rho'}{\partial z}\right)}{\left|\frac{\partial u}{\partial z}\right|^2 + \left|\frac{\partial v}{\partial z}\right|^2} < 0.25$$



On average for GM-levels:

$Ri_l \leq 1$, **13% of the time**

$Ri_l \leq 0.25$, **5% of the time**

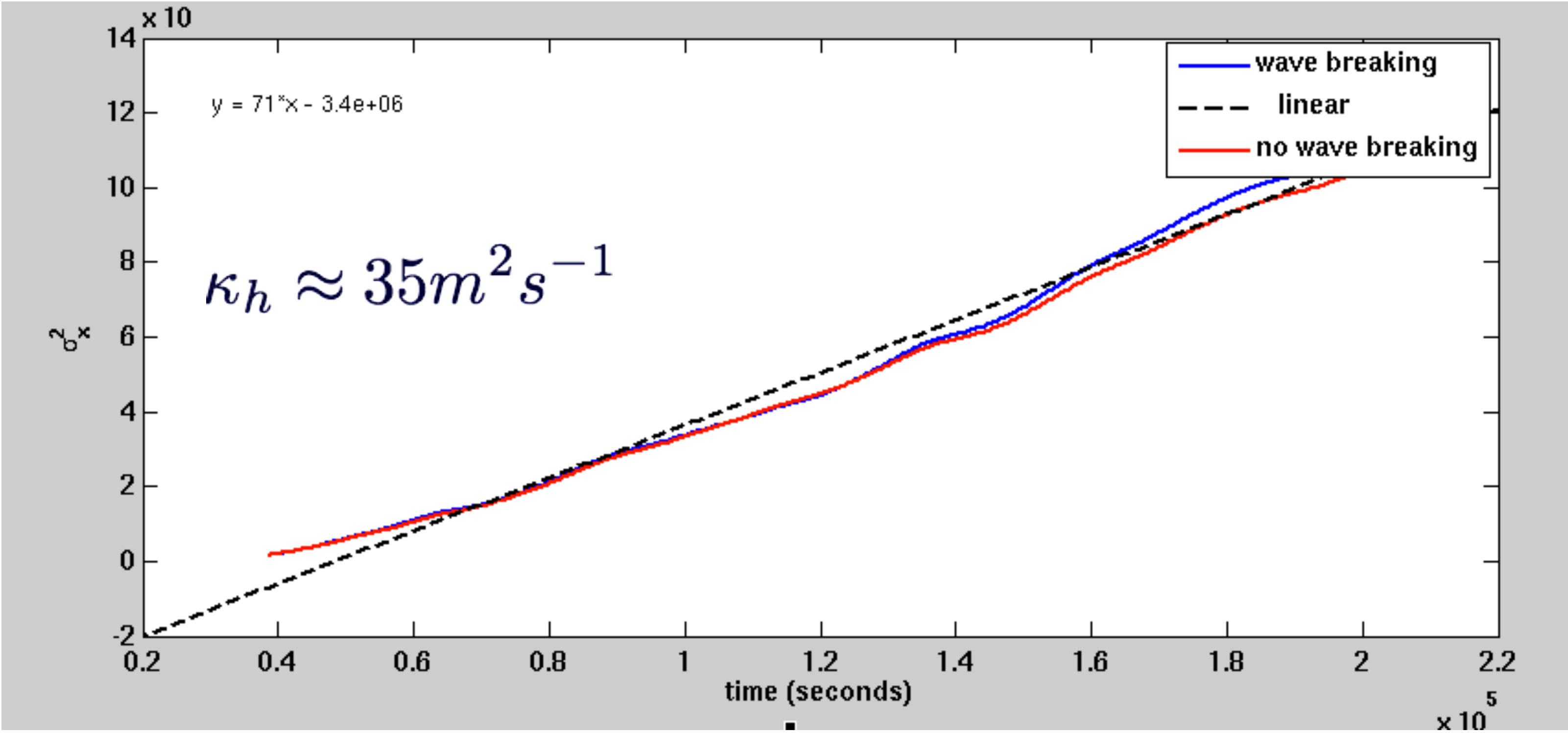
Inferring effective horizontal diffusivity from behavior of second moment of tracer concentration C

$$\kappa_H = \frac{1}{2}\frac{\partial \sigma_x^2}{\partial t},$$

where

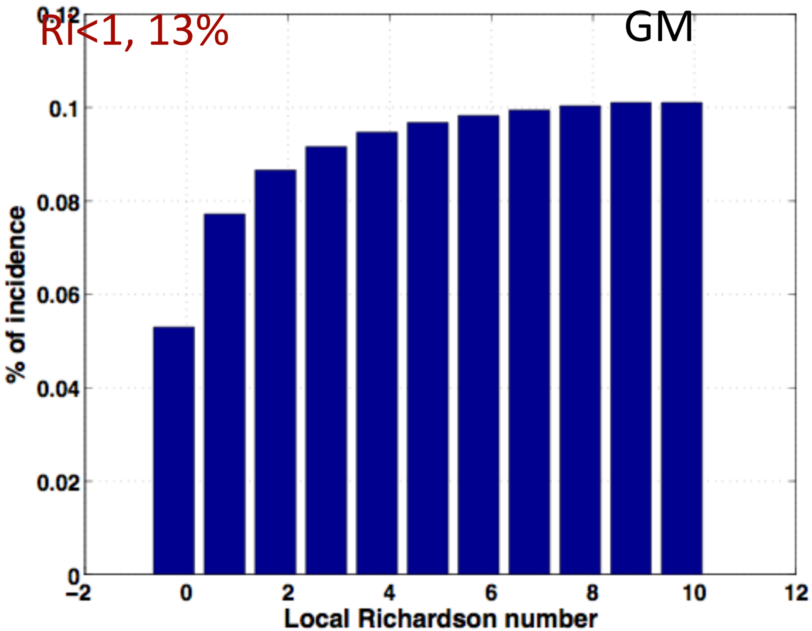
$$\sigma_x^2 = \frac{\int_0^{L_x} x^2 C\,dx - \left(\int_0^{L_x} xC\,dx\right)^2}{\int_0^{L_x} C\,dx},$$

Impact of wave breaking on lateral dispersion

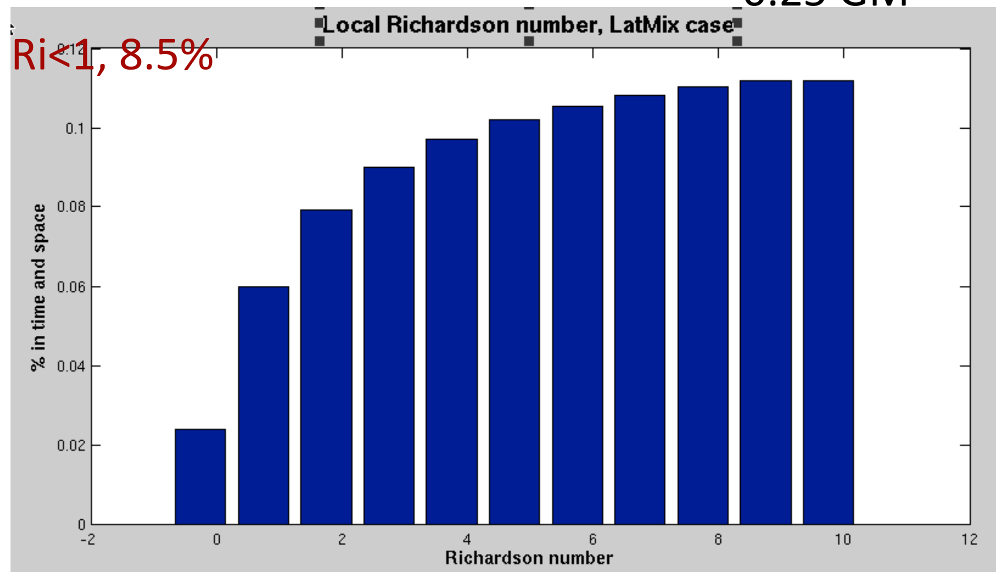


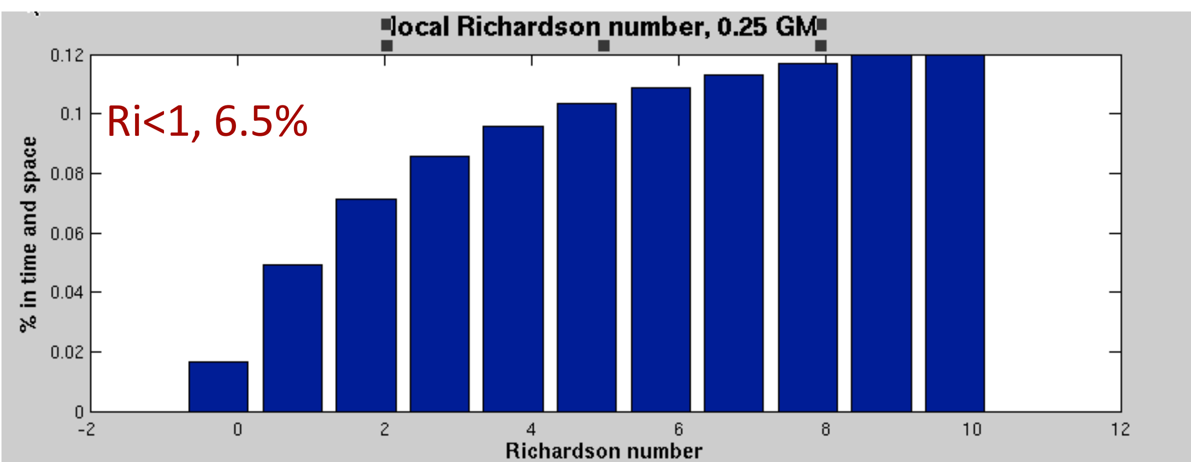
Not much difference in lateral dispersion with breaking/ non-breaking waves. This suggests the role of vortical mode may not be important.

Diagnosing wave breaking with a local Richardson criterion



0.25 GM

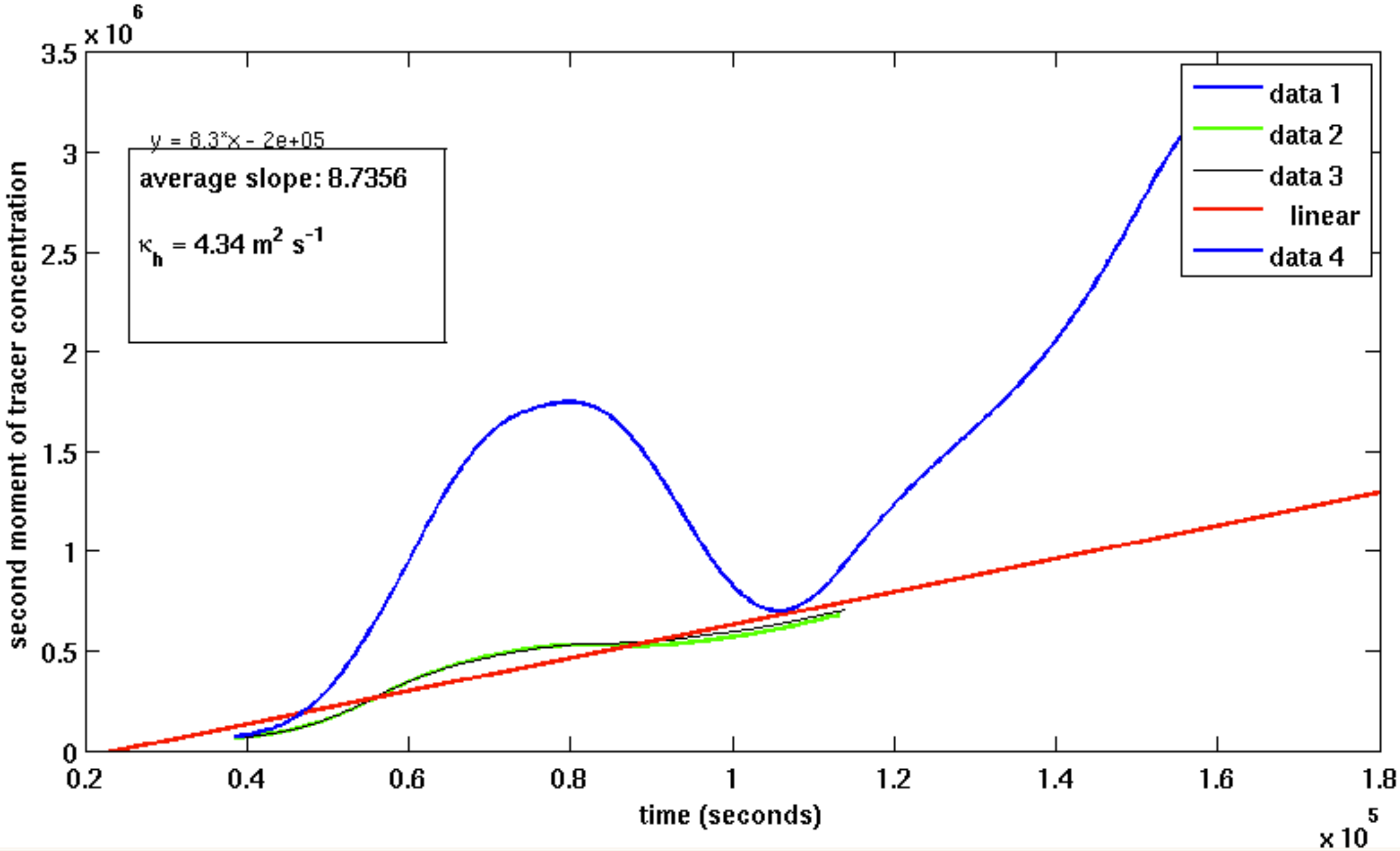




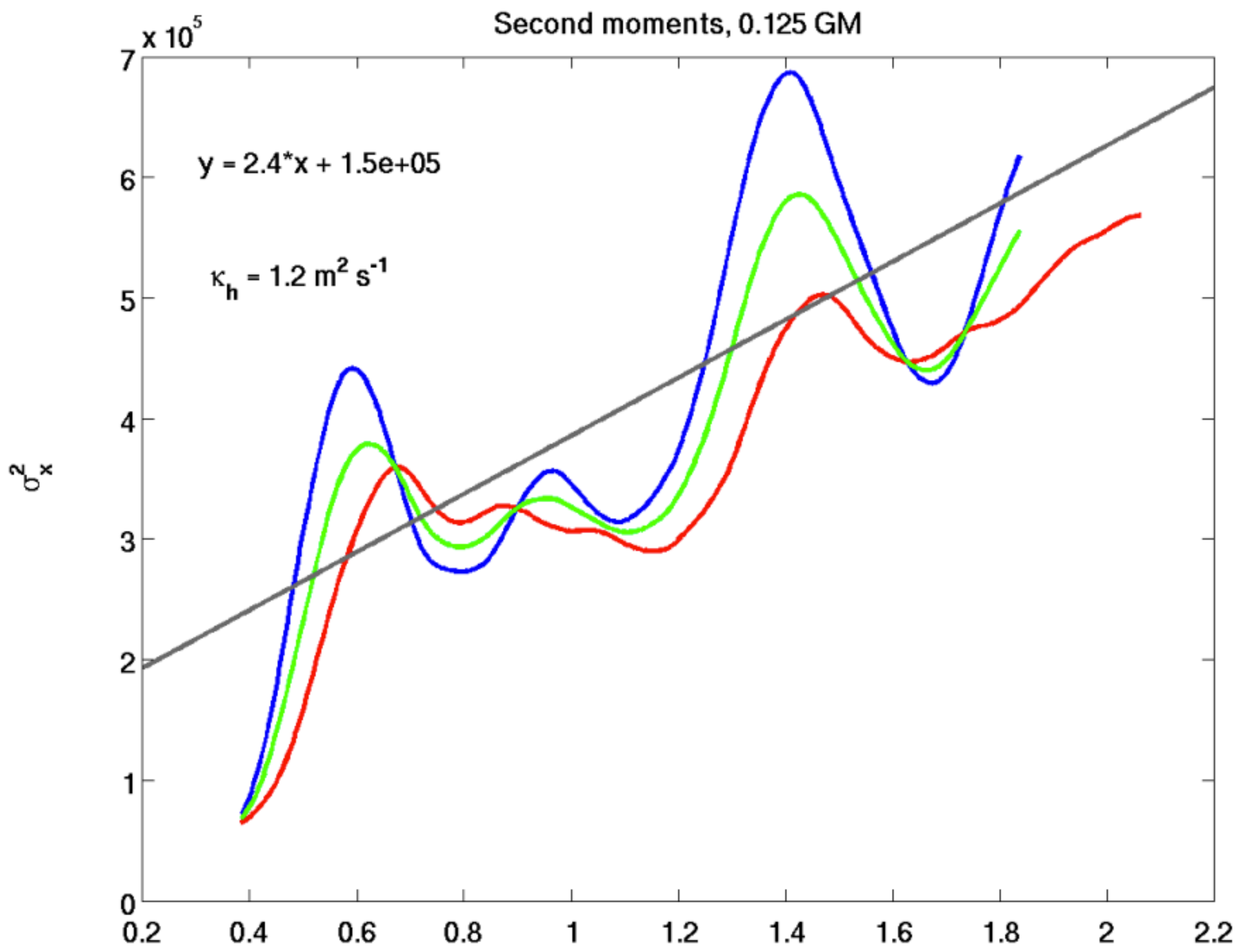
0.125 GM

observations suggest that turbulence patches occupy, on average, 10% of the ocean in space and time.

Evolution of 2nd moment of tracer concentration for LatMix case (0.25 GM)



Evolution of second moment of tracer, weak wave field: 0.125 GM

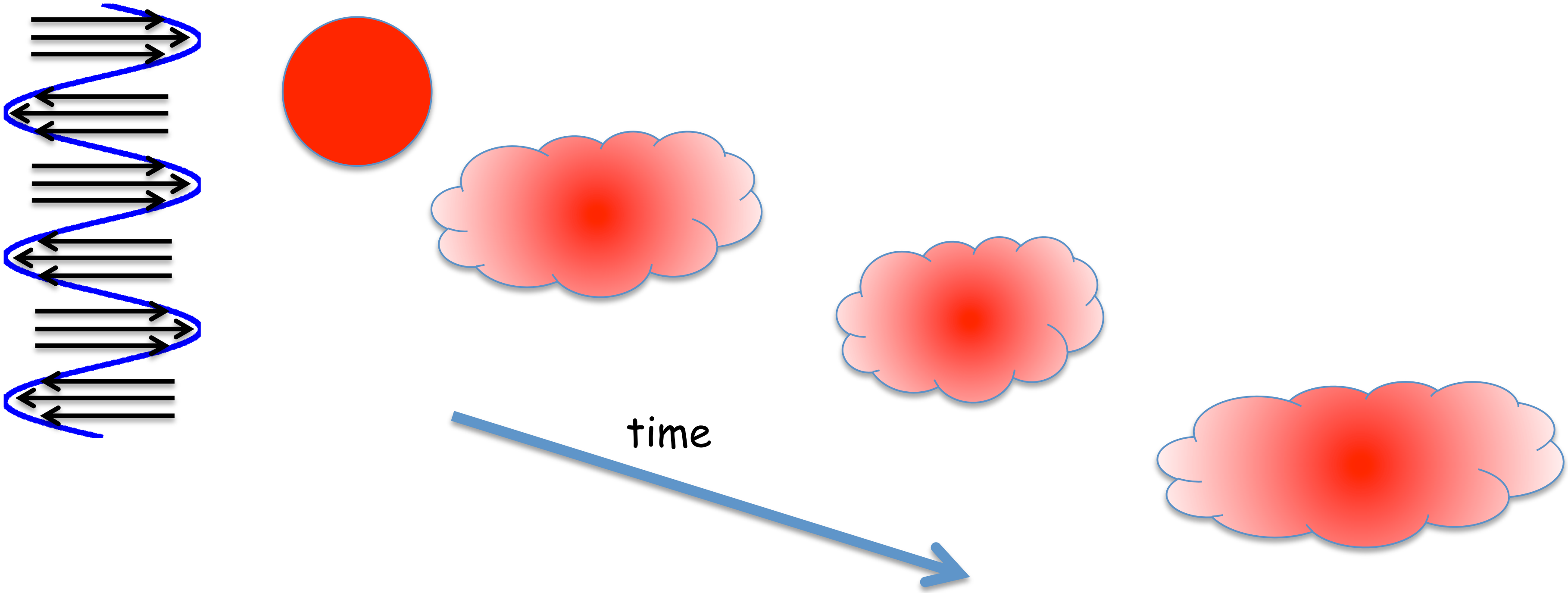


GM spectrum parameter dependence, comparison with Holmes-Cerfon et al

$$D \approx 0.08 \frac{E_0^2 m_*^2}{N^2 f} \{2\, ln\, (\frac{m_c}{m_*}) - \frac{\pi^2}{4}\}$$

- Model effective diffusivities scale as 1/f and $E_0{}^2$
- Consistent with Holmes-Cerfon et al. 2011
- Need to verify scaling with stratification.

Behavior of a blob of tracer as a function of time in an oscillatory shear flow

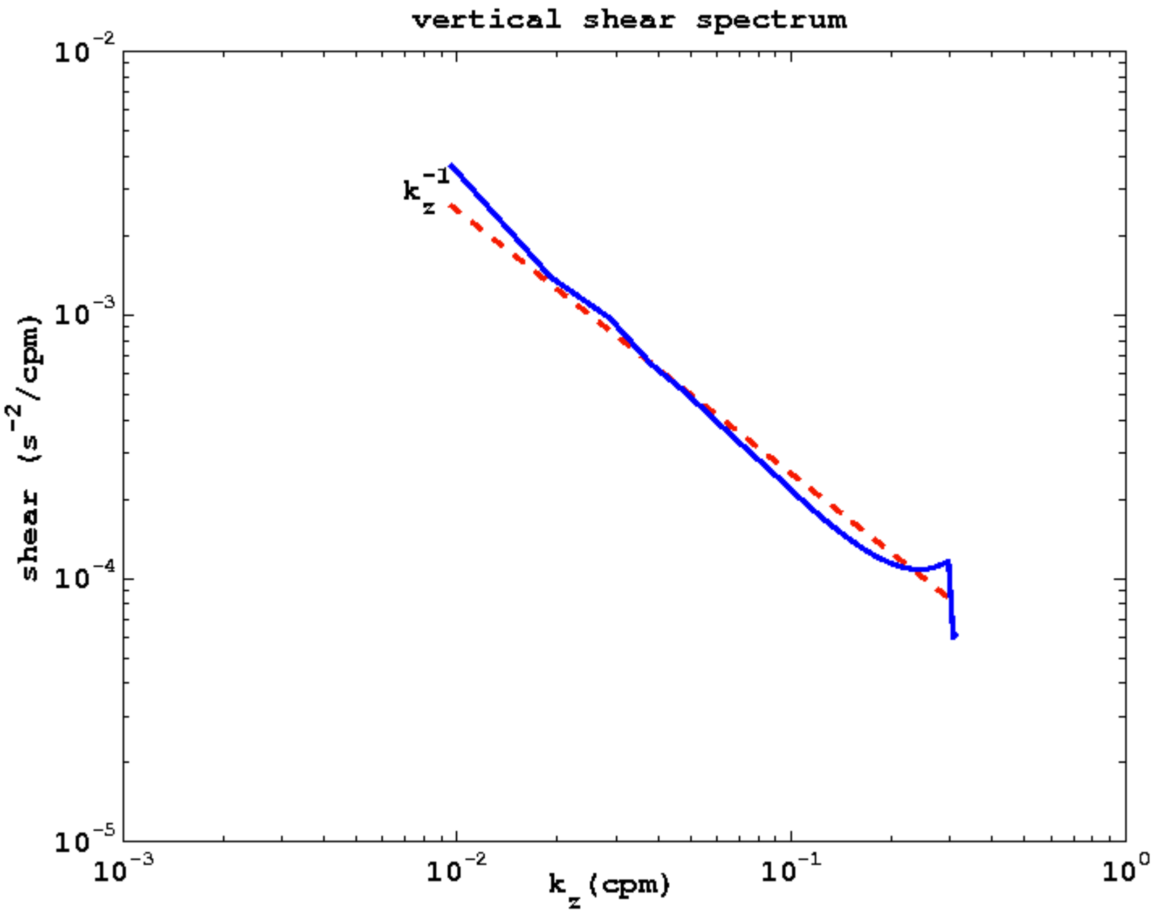


In a wave field, most efficient shear dispersion is due to lowest frequencies, i.e. near-inertial waves.

Estimating shear dispersion from model vertical shear spectrum

$$\eta_e = \eta + \kappa \int_f^N d\omega \int_0^\infty dm(\omega^2 + m^4\kappa^2)^{-1} \times (f\pi^{-1})(\omega^2 - f^2)^{-1/2}\omega^{-1} S(m)$$

(Young et al. 1982)



η_e : effective lateral diffusivity

η : model horizontal diffusivity

κ : model vertical diffusivity

ω : frequency

m : vertical wavenumber

$S(m)$: vertical shear spectrum

$$\kappa = \gamma\epsilon/N^2$$

$$\eta_e = 0.02 m^2 s^{-1}$$

$$\eta_{model} = O(5) m^2 s^{-1}$$

What is causing observed lateral dispersion at LatMix site I?

- Vortical modes stirring? Unlikely since production from wave breaking is too weak to have much impact.
- Shear dispersion? Still a possibility since near-inertial motions are not well maintained in model (lack of forcing in present simulations).
- Wave-induced dispersion? Estimates from present simulations indicate that this is a strong possibility. Effective model-computed diffusivities are in the range of diffusivities inferred from observations.

Summary and conclusions from observations (courtesy M. Sundermeyer)

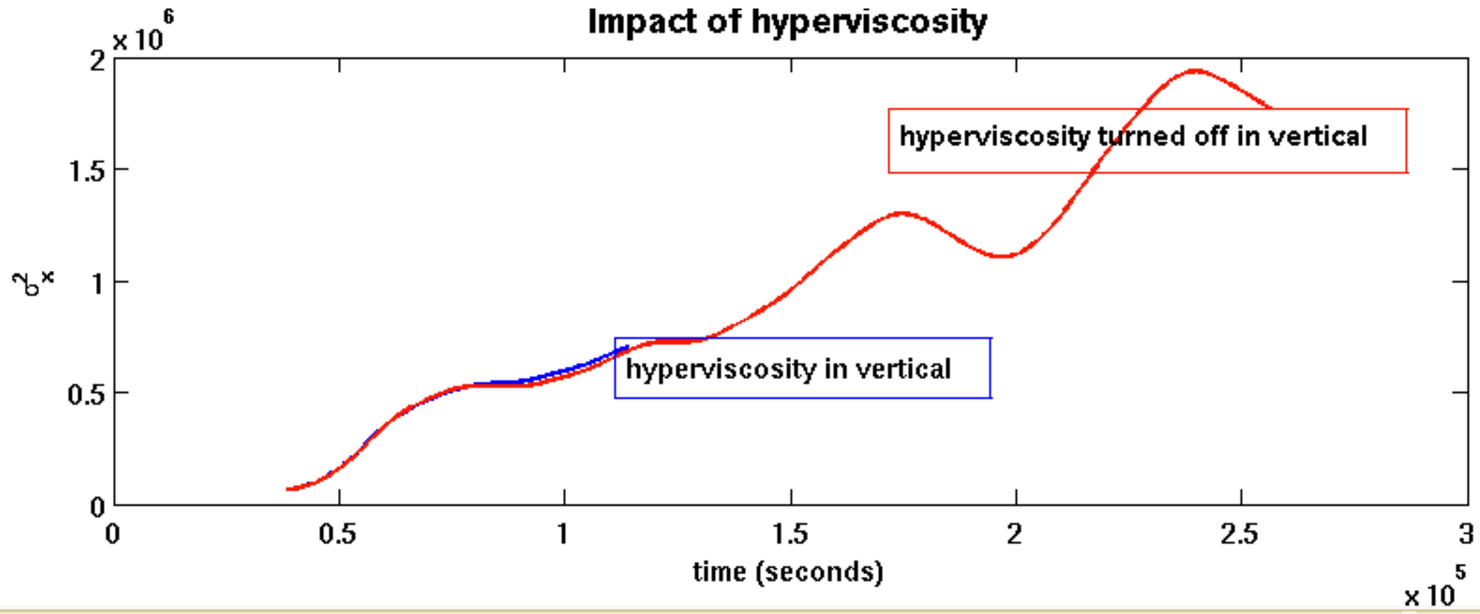
- Diffusivity and strain estimates from dye and drifters
- Experiment 1: $\kappa_z \sim 5 \times 10^{-6}\ m^2\ s^{-1}$, $\kappa_H \sim 0.5 - 4\ m^2\ s^{-1}$, $\gamma \sim 0.1\text{-}2 \times 10^{-5}\ s^{-1}$
- Experiment 2: $\kappa_z \sim 2 \times 10^{-6} - 5 \times 10^{-6}\ m^2\ s^{-1}$ $\kappa_H \sim 0.5 - 4\ m^2\ s^{-1}$, $\gamma \sim 1\text{-}3 \times 10^{-5}\ s^{-1}$
- Vertical shear dispersion by low frequency shears and internal waves may explain some of this κ_H, but more work needed
- Complex T & S structure, as well as dye details reveals more complicated story – collaboration with other LatMix field and modeling work needed to understand possible scale dependence and underlying dynamics
- Lidar dye surveys reveal even more rich structure of dye patches … lots still to be done with these data …

Conclusions and future work

- even weak nonbreaking wave fields are capable of reproducing lateral dispersion characteristics observed during LatMix 2011.
- vortical mode contribution does not appear to be significant.
- particle studies to infer scale dependence of dispersion (comparison with Okubo diagram)
- comparison with other dispersing mechanisms. Mixed layer instabilities? Tandon et al. reproduce the same ballpark values without waves.

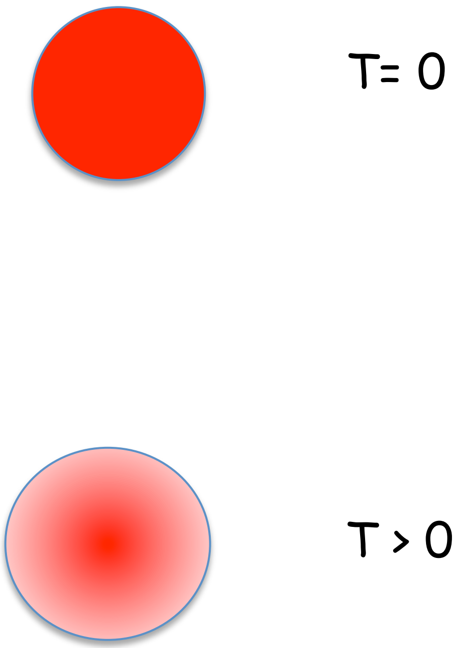

Thank you for your attention…

Impact of hyperviscosity



hyperviscosity has a negligible impact on tracer dispersion

Diffusion of a passive tracer (no mean flow)



Diffusion in presence of vertical shear