Is swimming a limit cycle?

Henry O. Jacobs

Geometry, Mechanics, and Dynamics:
The Legacy of Jerry Marsden
at the Fields Institute, July 23, 2012
THIS IS A BOARD TALK (GET CLOSE)

names

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- Yu Zheng *
- Dennis Evangelista *
- Sam Burden **
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Outline

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Does this make sense?

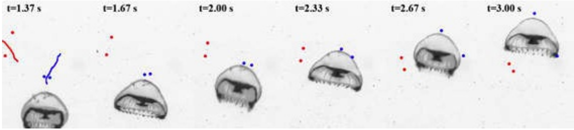


Figure : figure courtesy Kakani Katija

Courtesy Nikita Nester

Figure : Jellyfish in Palau (video by Naoki Inoue posted on YouTube Feb 2007)

The Averaging Theorem

Theorem

Let

$$\dot{x} = f(x)$$

be a dynamical system with an asymptotically stable fixed point at x_0*. Then for any* T*-periodic vector field,* $g(x, t)$*, the dynamical systems*

$$\dot{x} = f(x) + \epsilon g(x, t)$$

admits a limit cycle near x_0 *with period* T *for sufficiently small* ϵ*.*

[1]

[1] Guckenheimer & Holmes, *Nonlinear Oscillations and Chaos*, 2nd Ed, Springer (1983).

The passive system has a stable point

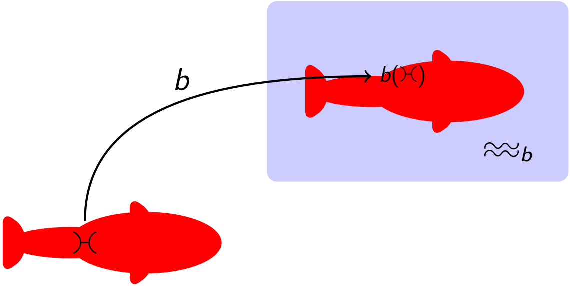
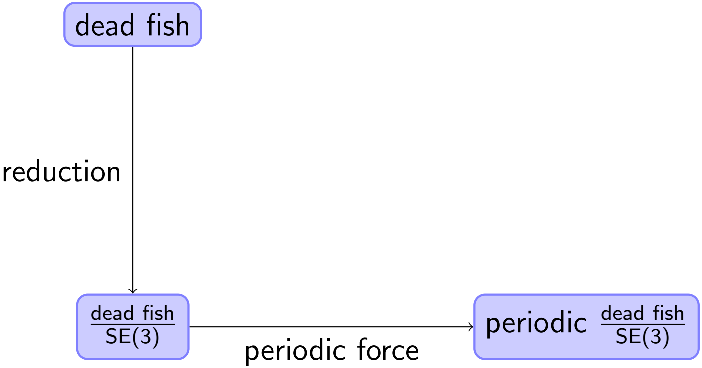


Figure : embedding of a dead fish in $\mathbb{R}^3$

a motionless corpse in stagnant water is a stable state.

The Big Picture



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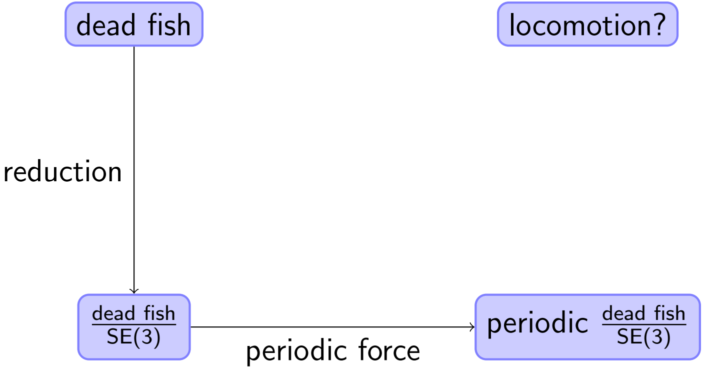

An analogous system

Consider the system on $\mathbb{R}^3$

$$\begin{aligned}\dot{x} &= y \\ \dot{y} &= -x - \nu y + \epsilon \sin(t) \\ \dot{z} &= \dot{x} + x\dot{y}\end{aligned}$$

The first two equations are that of a forced/damped oscillator. Note that this ODE has z symmetry so we can "ignore" z. (draw diagram on board)

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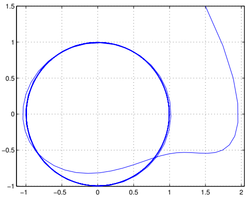

Figure : reduced trajectory

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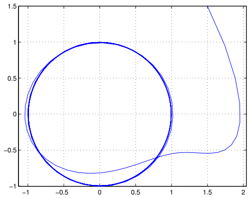

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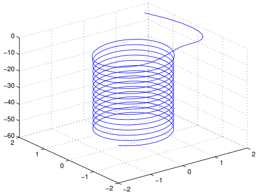

Figure : unreduced trajectory

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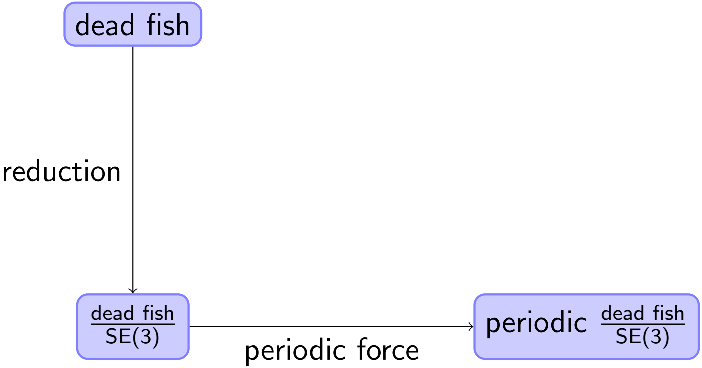

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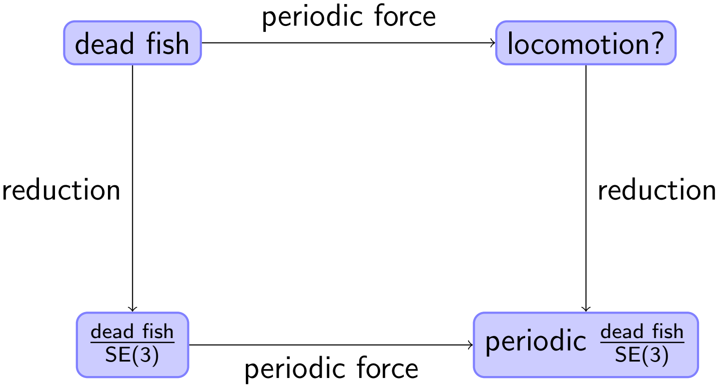

Previous Work

1. Liao et. al. *Fish exploiting vortices decrease muscle activity*, Science **302** (2003).
2. S. Alben, M. J. Shelley, *Coherent locomotion as an attracting state for a free flapping body*, PNAS **102** (2005).
3. A. Shapere, F. Wilczek, *Geometry of self-propulsion at low Reynolds number*, JFM **198** (1989).
4. S. D. Kelly, *The mechanics and control of robotic locomotion with applications to aquatic vehicles*, PhD thesis, Caltech, (1998).
5. Kanso et. al., *Locomotion of articulated bodies in a perfect fluid*, J. Nonlinear Sci **15** (2005).
6. H. Cendra, J. Marsden, T. Ratiu, *Lagrangian Reduction by Stages*, Memoirs of the AMS, (2001).
7. A. Weinstein, *Lagrangian Mechanics on Groupoids*, Mechanics Day, Fields Inst, (1995).

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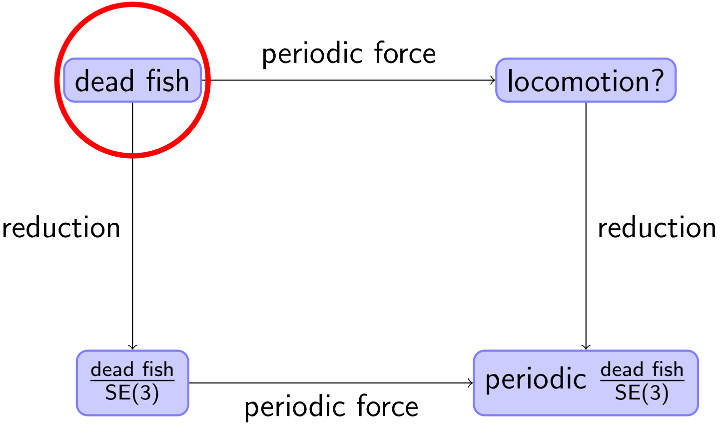

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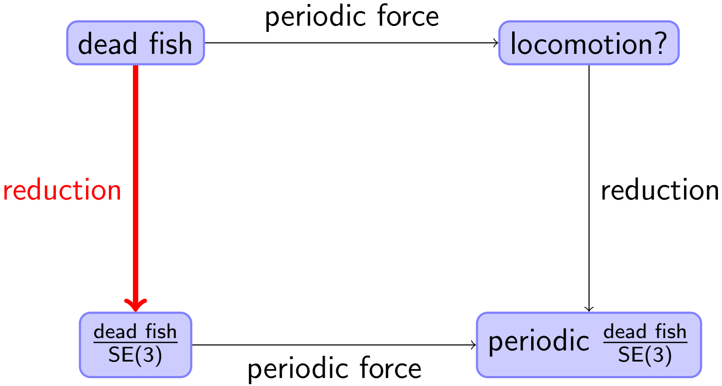

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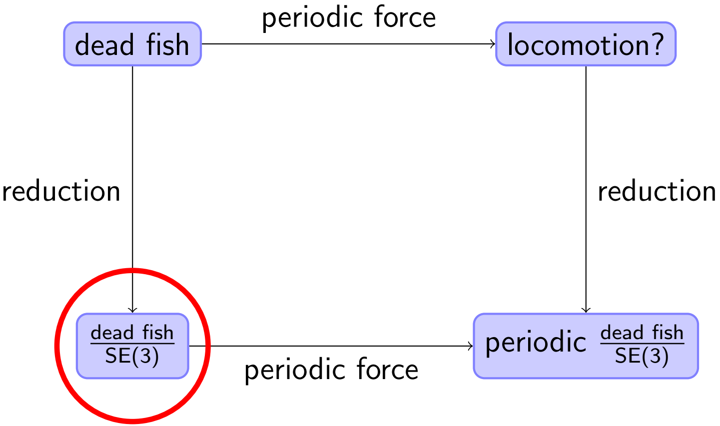

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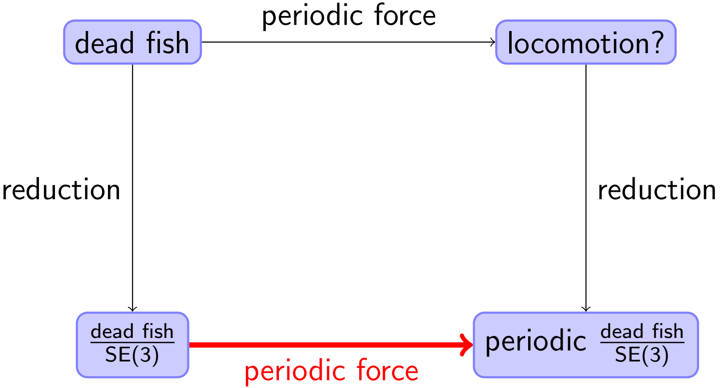

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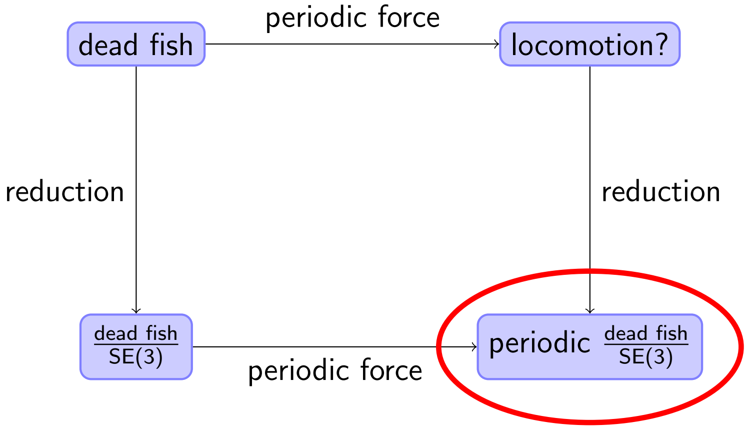

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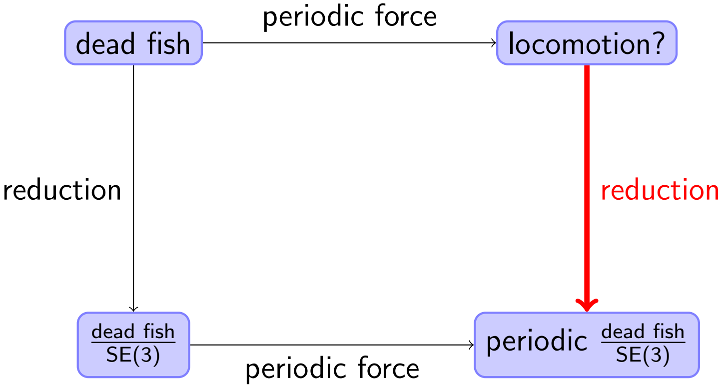

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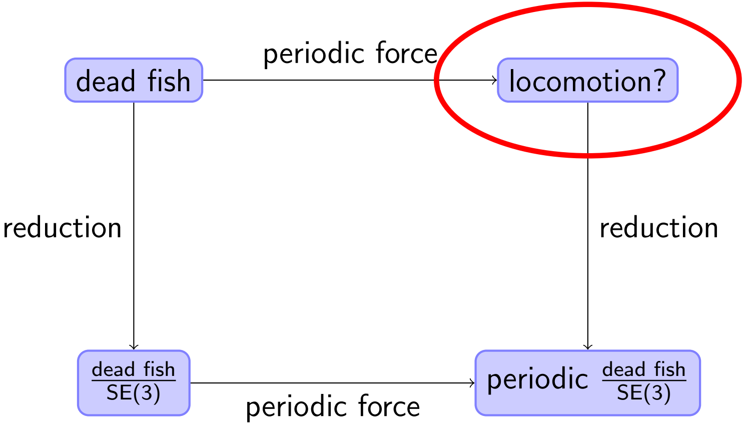

What just happened?

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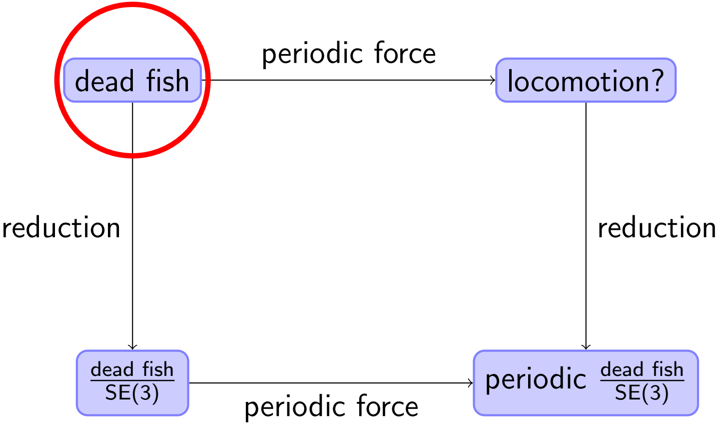

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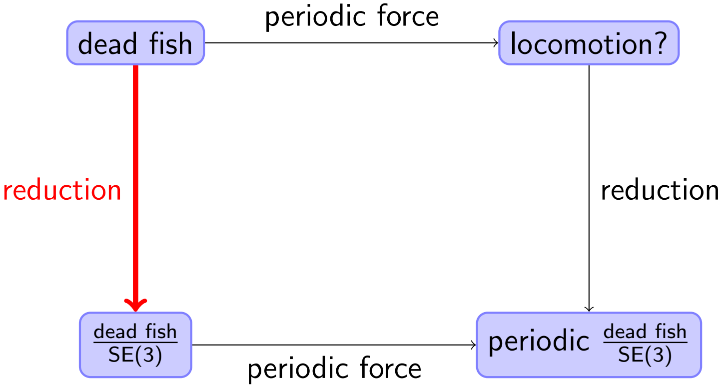

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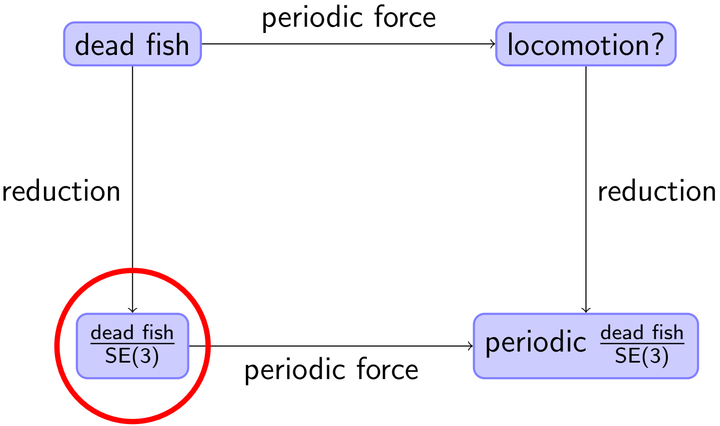

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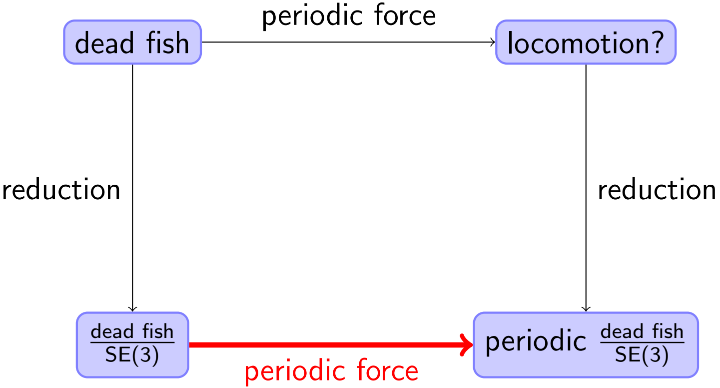

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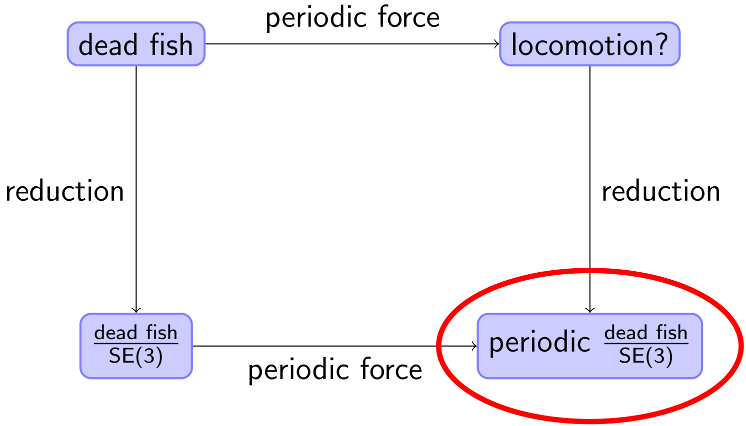

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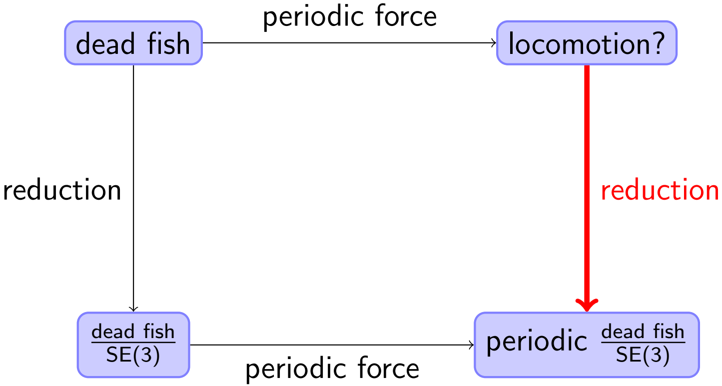

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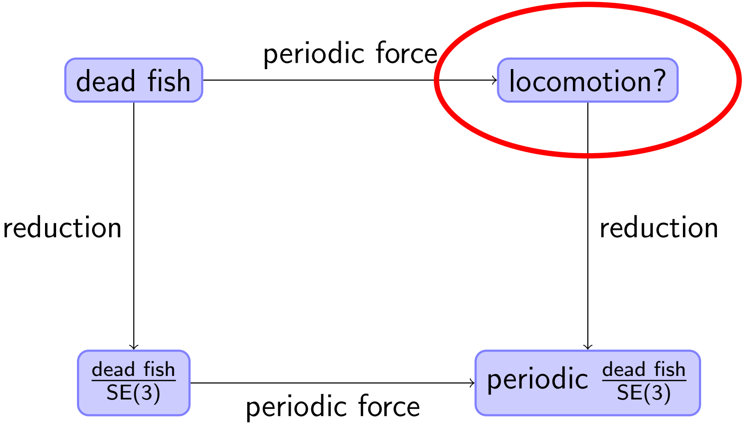

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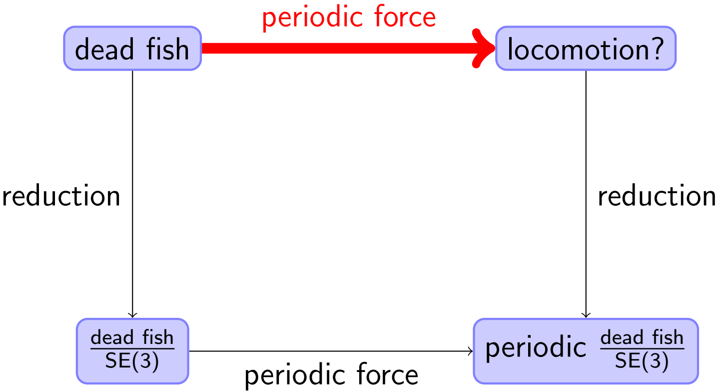

Some sad news

The Averaging theorem requires that we be in a Banach space. Here are some musings

1. We can use the completion of Q? This involves non-differentiable mappings.
2. We can search for a set of feasible perturbations?
3. We may construct a sequence of finite dimensional models.

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Motivation

Math

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8. ... almost.

Primary References

video by Naoki Inoue

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1. Lagrangian Reduction by Stages [Cendra, Ratiu & Marsden, 1999].
2. A. Weinstein, *Lagrangian mechanics and groupoids*, Mechanics Day, Fields Inst. Proc., vol. 7, AMS, 1995.
3. H. J. , *Geometric Descriptions of Couplings in Fluids and Circuits* , Caltech PhD thesis, 2012.