ANALYTICAL, GEOMETRICAL AND NUMERICAL ASPECTS OF CURVATURE DRIVEN FLOWS

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Abstract. I will present several approaches for curvature motions of the form

$$u_t + |Du| F\left(\operatorname{div}\left(\frac{Du}{|Du|}\right)\right) = 0, \qquad \text{in } \Omega \times (0,\infty) \tag{1}$$

$$u(\cdot,0) = u_0, \qquad \text{in } \Omega \tag{2}$$

These equations are particularly interesting since they can be axiomatically obtained from the *image multiscale theories*, as the partial differential equations satisfying the most desired invariance properties in computer vision: causality, locality, isometry and contrast invariance. This characterization was given in 93 by Alvarez, Guichard, Lions and Morel. In particular, these are geometric evolution equations. In the past twenty five years, several methods have been used for the study of these equations, and very interesting results have been obtained regarding existence of solutions, regularity, and many geometric properties.

In the first part of the lecture, I will present three of these approaches: parametric methods from differential geometry, level set formulations from viscosity solutions theory and variational approaches from the BV setting. In addition, I will discuss old and new connections between these methods and the consistency of the geometric and variational approach with the level-set-viscosity formulation.

In the second part of the talk, I will present a review, analysis and comparison of numerical methods implementing the curvature motions for 2D images, shapes, and curves. These can be viewed as *curvature scale spaces*, which allow in principle to compute an accurate multiscale curvature in digital images. We introduce a new numerical chain, which is fast and preserves the invariance mentioned above, that we term Level Lines Shortening. The Level Lines Shortening numerical chain stands for the simultaneous and independent curvature evolution of *all* the level lines for a given function. This chain will provide not only an exact numerical implementation, but also an analytical framework, and it runs on-line on any image at http://www.ipol.im/.

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