IMAGE REGISTRATION IN THE PRESENCE OF DISCONTINUITIES

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Image registration is one of the challenging problems in image processing and specifically medical imaging. Given two images taken, for example, at different times, and from different devices or perspectives, the goal is to determine a reasonable geometric transformation that aligns two images in a common frame of reference.

In general, image registration can be modeled as an inverse problem and addressed using optimization schemes. Given a reference image $\mathcal{R}$ and a template image $\mathcal{T}$ defined on $\Omega \subset \mathbb{R}^d \to \mathbb{R}$, the goal of image registration is to find a transformation $y : \mathbb{R}^d \to \mathbb{R}^d$ that minimizes

$$\mathcal{J}[y] := \mathcal{D}[\mathcal{T}[y], \mathcal{R}] + \alpha \mathcal{S}[y],$$

where $\mathcal{D}$ is a function that measures the dissimilarity between the reference image $\mathcal{R}$ and the transformed template image $\mathcal{T}[y]$, $\mathcal{S}[y]$ is a regularization term that guarantees the minimization problem is well-posed and that the resulting transformation is in some way smooth, and α is a parameter that can be used to vary the impact of the dissimilarity measure and the regularization term.

In image registration, researchers rely generally on continuous and smooth transformations (for example, by using Tikhonov regularization) to describe the alignment process relating two images. However, in the clinical setting, there are many situations where the "true" deformation may have discontinuities. For example, one would expect the transformation between the images of a patient before and after tumor resection would contain a singularity (in the forward direction) or a hole (in the backward direction). In situations where organs slide relative to one another or to the chest wall, one would expect the transformation between images to contain a "rip" or "tear". Some researchers tackle the problems of rips/tears by employing TV regularization, but this typically only works well for extremely small discontinuities. Furthermore, these approaches do not at all address the clinical problems where singularities or holes are present in the true deformation.

Given these limitations of current approaches to image registration, an open problem is to define an approach that would enable deformable registration in the presence of various types of large-scale discontinuities. One potential approach would be to investigate whether it is possible to develop a well-posed registration technique that explicitly parameterizes each discontinuity. Of even greater impact would be the development of a registration technique that implicitly allows topological changes in the deformation field, much like the use of level sets in image segmentation allows implicitly for topological changes in regions boundaries.

A combined segmentation and registration may seem to be a feasible approach which segments the reference and the template images and independently registers the corresponding pieces. The challenge in this approach may remain in finding the corresponding pieces in the two images.

Initially one may also consider related simple toy problems to address. For example, how to match a circle to the case where it is split into two half-circles and the two pieces are moved apart from each-other?

What features of the image may be used to identify the cuts and how this could be used in an optimization scheme?

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