Image registration in the presence of discontinuities

Nathan Cahill, Mehran Ebrahimi, Khaled Issa, Evgeniy Lebed, Alvin Ihsani, Dustin Steinhauer, Ali M. Montaz, Rolf Clackdoyle, Karteek Popuri, Yunho Kim

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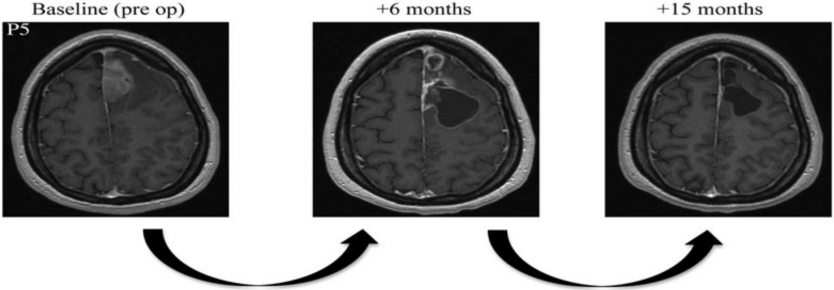
Problem Statement

We would like to perform nonrigid image registration in the presence of local growth, local shrinkage, or missing objects.

- Can the variational approach be modified to handle situations where the true physical deformation has a hole due to an object that is present in the reference image but absent in the floating image?
- What about situations where an object is present in the floating image but absent in the reference image?

Problem Statement

For instance,



Discussions

Problem

Given two 2D images, $A, B : \Omega \to \mathbb{R}$, and a manually identified simply connected region $\Theta \subset \Omega$ that corresponds to a location in A,

- Register A to B by identifying a transformation $\Phi : \Omega \to \Omega$ such that $\exists x \in \Omega$ with $\Phi(\Theta) = x$ that minimizes a prescribed cost function,
- Register B to A by identifying a transformation $\Psi : \Omega \to \Omega \backslash \Theta$ that minimizes a prescribed cost function.

There are questions to ask to make the above problems more clear such as regularity and similarity measures. We have been trying to lay down a mathematical foundation.

Discussions

Below are the ideas to tackle the proposed problem.

1. Cloaking
2. FEM Model
3. Level set Method
4. Block Matching Method
5. Shape Matching via Currents

Cloaking based approach (1)

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- "Cloaking" is an analogy with the eponymous field of study in PDE that attempts to shield objects or charges from electromagnetic detection

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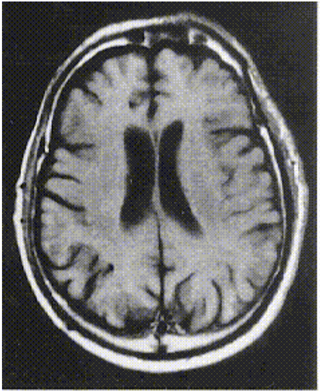
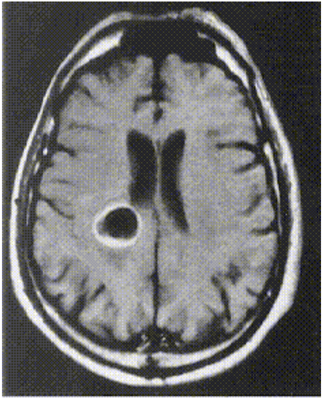
- "Cloaking" is an analogy with the eponymous field of study in PDE that attempts to shield objects or charges from electromagnetic detection
- Since we want to create a diffeomorphism between a region with a hole and one without, we use change of variables to shrink the hole to a single point

Cloaking based approach (2)



No tumor present



A tumor has developed

Figure: Images of the same brain taken at different times

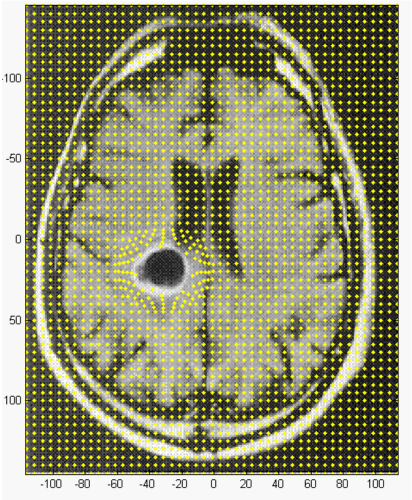


Figure: The sampling grid on the target image

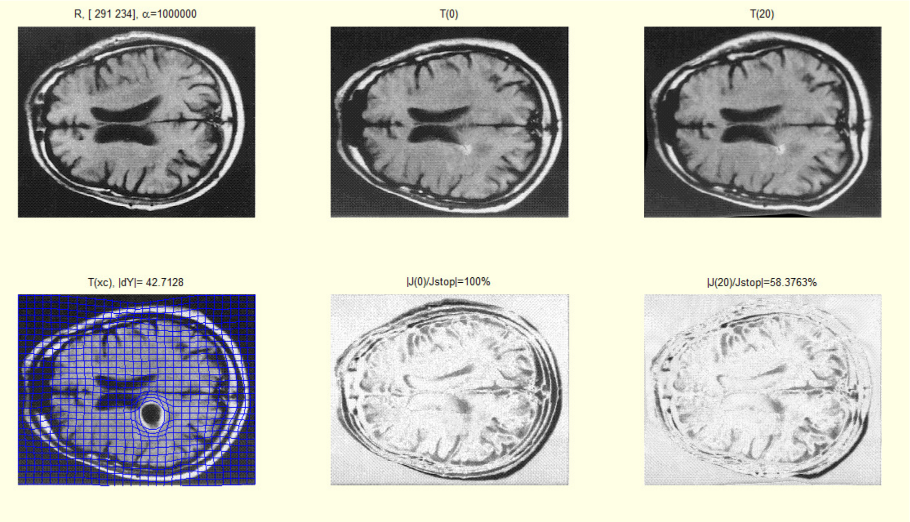
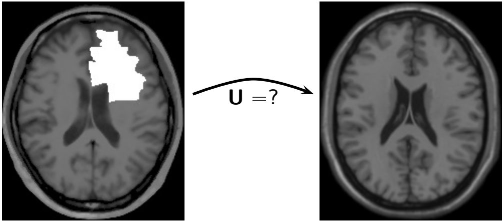
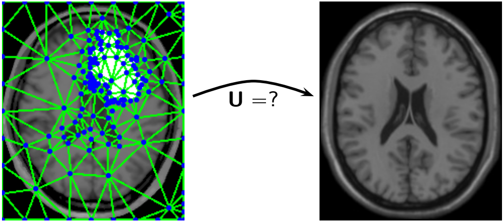


Figure: The deformation from the source to the registered image

FEM-based approach (1)



$I_A : \Omega_A \to \mathbb{R}$ $\qquad$ $I_B : \Omega_B \to \mathbb{R}$



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- Approximate the deformation field as: $\mathbf{U}(\mathbf{x}) = \sum_{n \in \mathcal{N}} \mathbf{U}_n \phi_n(\mathbf{x}) \qquad \forall \mathbf{x} \in \Omega_A$

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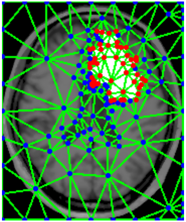

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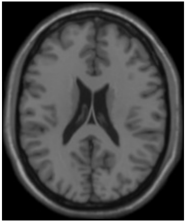
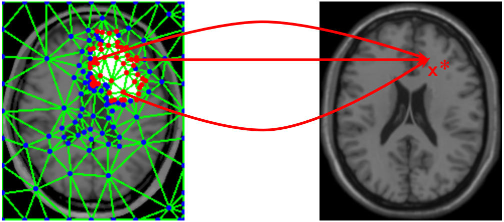


$I_B : \Omega_B \to \mathbb{R}$

- Approximate the deformation field as: $\mathbf{U}(\mathbf{x}) = \sum_{n \in \mathcal{N}} \mathbf{U}_n \phi_n(\mathbf{x}) \qquad \forall \mathbf{x} \in \Omega_A$
- Partition the nodes into two disjoint sets as: $\mathcal{N} = \mathcal{N}_1$ (normal) $\cup$ $\mathcal{N}_2$ (tumor)



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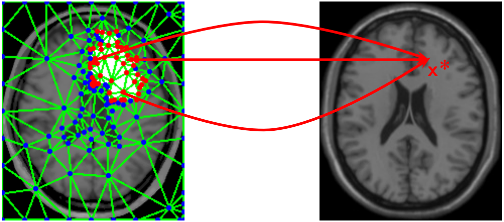

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- Re-write the deformation field as:

$$\mathbf{U}(\mathbf{x}) = \sum_{n \in \mathcal{N}_1} \mathbf{U}_n \phi_n(\mathbf{x}) + \sum_{n \in \mathcal{N}_2} (\mathbf{x}^* - \mathbf{p}_n) \phi_n(\mathbf{x}) \quad \forall \mathbf{x} \in \Omega_A$$

- Need to estimate the unknowns $\{\mathbf{U}_n\}_{n \in \mathcal{N}_1}$ and $\mathbf{x}^*$

FEM-based approach (2)

- Finite-dimensional multivariate energy minimization to estimate the set of unknowns $\mathbf{\Theta} = \{\mathbf{U}_n\}_{n \in \mathcal{N}_1} \cup \{\mathbf{x}^*\}$:

$$\mathbf{\Theta}^* = \operatorname*{argmin}_{\mathbf{\Theta} \in \mathbb{R}^{(|\mathcal{N}_1|+1)d}} E_D(\mathbf{\Theta}; I_A, I_B) + \gamma E_R(\mathbf{\Theta})$$

- The sum of squared differences (SSD) data term and diffusion-based regularizer:

$$E_D \equiv E_D^{\mathrm{SSD}}(\mathbf{\Theta}; I_A, I_B) = \frac{1}{2} \int_{\Omega_A} M(\mathbf{x}) \left(I_B(\mathbf{x} + \sum_{n \in \mathcal{N}_1} \mathbf{U}_n \phi_n + \sum_{n \in \mathcal{N}_2} (\mathbf{x}^* - \mathbf{p}_n) \phi_n) - I_A(\mathbf{x}) \right)^2 d\mathbf{x}$$

$$E_R \equiv E_R^{\mathrm{diff}}(\mathbf{\Theta}) = \frac{1}{2} \sum_{i=1}^{d} \Bigg(\sum_{n \in \mathcal{N}_1} U_{ni} \sum_{m \in \mathcal{N}_1} U_{mi} \Gamma_{nm} + 2 \sum_{n \in \mathcal{N}_1} U_{ni} \sum_{m \in \mathcal{N}_2} (\mathbf{x}^* - \mathbf{p}_m) \Gamma_{nm} \ldots$$

$$\ldots + \sum_{n \in \mathcal{N}_2} (\mathbf{x}^* - \mathbf{p}_n) \sum_{m \in \mathcal{N}_2} (\mathbf{x}^* - \mathbf{p}_m) \Gamma_{nm} \Bigg)$$

$M(\mathbf{x})$: normal region indicator function

$\Gamma_{nm} = \int_{\Omega_A} (D \nabla \phi_n)^{\mathrm{T}} \nabla \phi_m \, d\mathbf{x}$ and $\mathbf{U}_n = [U_{ni}]_{i=1}^d$

- Incremental semi-implicit fixed point iteration scheme to solve the *non-linear* equations:

$$\mathbf{\Theta}_i^{k+1} = \mathbf{\Theta}_i^k + \delta \mathbf{\Theta}_i^k \qquad \mathbf{\Theta} = [\mathbf{\Theta}_i]_{i=1}^d$$

$$(\mathrm{Id}_N + \tau \mathbf{K}) \delta \mathbf{\Theta}_i^k = (-\tau \mathbf{K} \mathbf{\Theta}_i^k + \tau \mathbf{L}_i(\mathbf{\Theta}^k)) \qquad \mathbf{K} = [[\Gamma_{nm}]_{n=1}^{|\mathcal{N}_1|+1}]_{m=1}^{|\mathcal{N}_1|+1}$$

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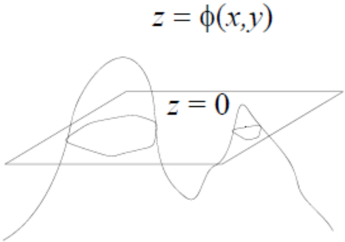

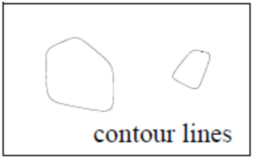

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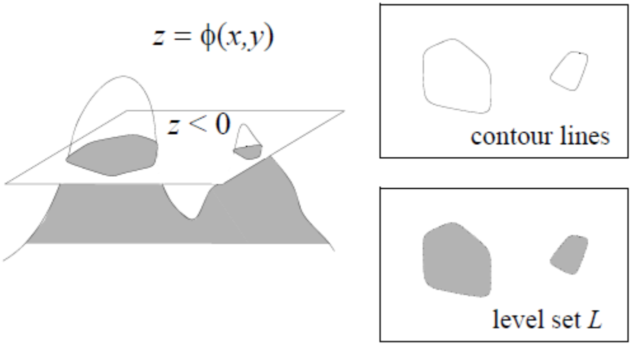

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Level sets and image registration

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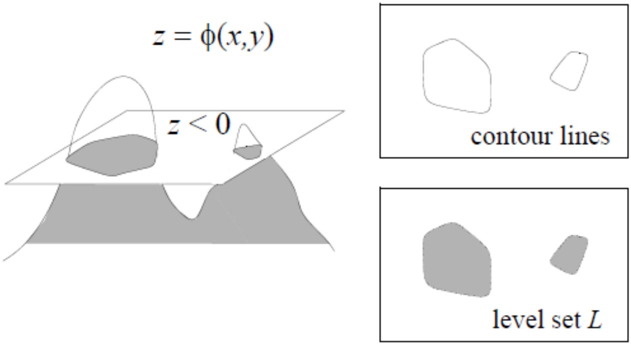

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Two responses. . .
(1) level sets to represent deformations
(2) deforming the level set functions

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For deformations . . . what can we use for Ω such that L is a deformation?
("in what sense is a deformation the same as a subset of something?")

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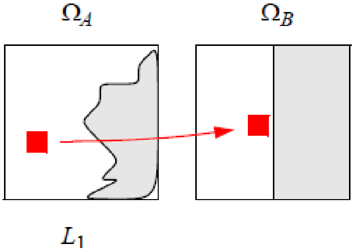

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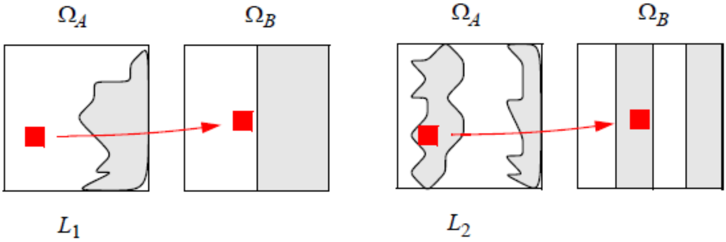

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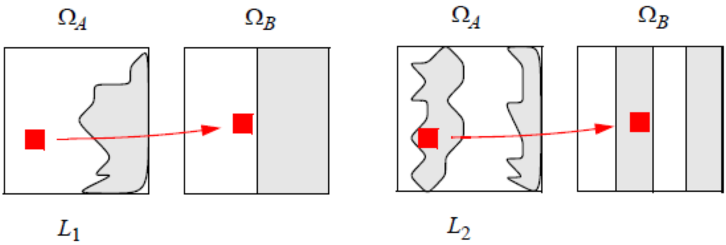


Is this level-set representation of deformations useful for registration?

"Is there an analogy to level-sets? (i.e., can we create a function φ that depends on the transformation for which $\varphi(u) = 0$ when u optimally aligns the images?)"

(2) Deforming the level set functions

Alvin Ihsani's insight (yesterday 11 AM):

- leave the level sets to represent images
- apply deformations to the level set function

Problem Setting

Assume we are given two images:

- a template image $\mathcal{T}$
- a reference image $\mathcal{R}$

such that $\mathcal{T}, \mathcal{R} : \Omega \rightarrow c_i$ where $i = 1, \ldots, N$ and $\Omega \subset \mathbb{R}^2$.

Problem

Find a transformation that is able to map corresponding features in $\mathcal{T}$ to $\mathcal{R}$, but is not necessarily topology preserving.

- Features that exist in $\mathcal{T}$ but not in $\mathcal{R}$ should disappear and be occupied by nearby features in $\mathcal{R}$ in a meaningful way.
- Features that do not exist in $\mathcal{T}$ but exist in $\mathcal{R}$ should appear in $\mathcal{T}$ in a meaningful way.
- Features that exist both in $\mathcal{T}$ and $\mathcal{R}$ should map in a meaningful way, even when they change topology (i.e. bagel and bagel cut in half).

Level-Set Formulation I

Let $\phi_i : \mathbb{R}^2 \to \mathbb{R}$ and

$$\mathcal{L}_i = \{x \in \Omega | \phi_i(x) \leqslant 0\}$$

such that

$$\begin{cases} \mathcal{L}_i \cap \mathcal{L}_j = \varnothing \quad \forall i \neq j \text{ (a.e.)} \\ \bigcup_{i=1}^{N} \mathcal{L}_i = \Omega \end{cases}$$

then indicator functions $K_i : \Omega \times \mathbb{R} \to \{0, 1\}$ can be defined such that

$$K_i(z, x) = \begin{cases} 1, & z < \phi_i(x) \\ 0, & z \geqslant \phi_i(x) \end{cases}$$

where $z \in \mathbb{R}$.

Level-Set Formulation II

Furthermore, let $V : \Omega \times \mathbb{R} \to \mathbb{R}$

$$V(x,z) = \sum_{i=1}^{N} c_i K_i(x,z)$$

such that

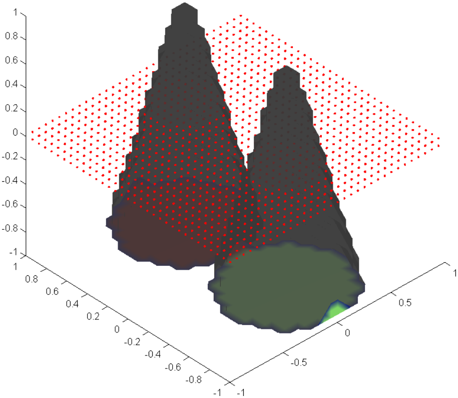
$$\mathcal{T}(x) = V(x,z)|_{z=0}.$$

Let a transformation $u : \Omega \times \mathbb{R} \to \Omega \times \mathbb{R}$ then a new image $\mathcal{T}'$ can be obtained as

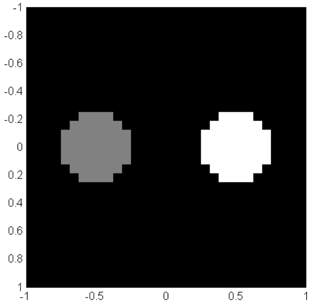
$$\mathcal{T}'(x) = V(u(x,z))|_{z=0}.$$

Model Example

$\mathcal{T}$ is embedded in the zero level-set of V.



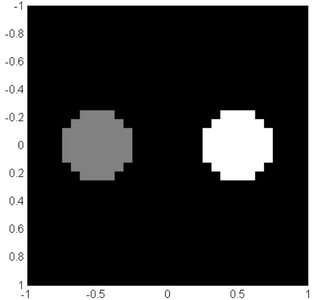
V



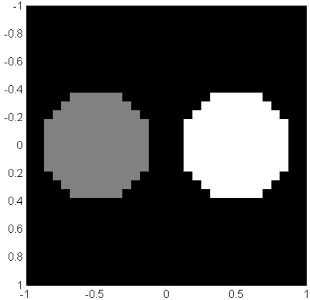
$\mathcal{T}$

Examples of Rigid Transformations (Case 1)

These images can be registered using a rigid transformation.



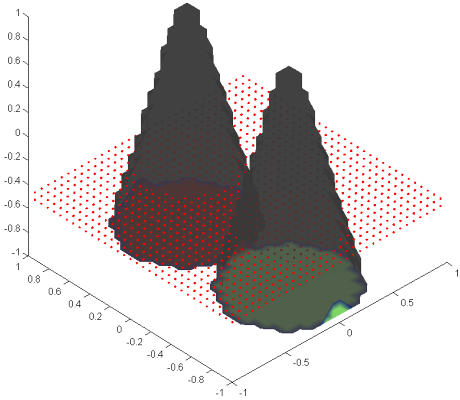
$\mathcal{T}$



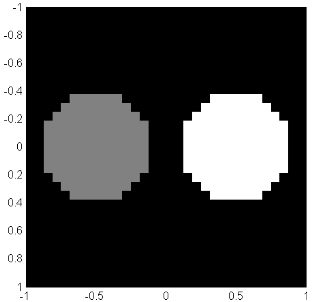
$\mathcal{R}$

Examples of Rigid Transformations (Case 1)

These images can be registered using a rigid transformation by shifting.



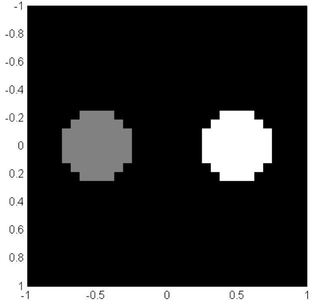
V



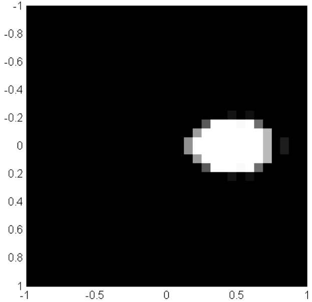
$\mathcal{T}'$

Examples of Rigid Transformations (Case 2)

These images can be registered using a rigid transformation also.



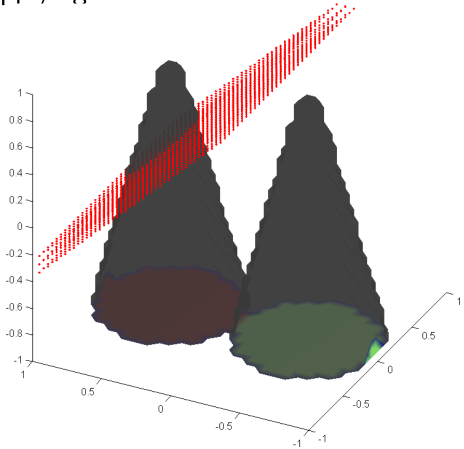
$\mathcal{T}$



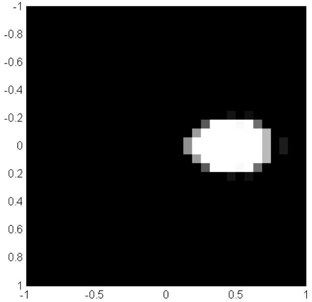
$\mathcal{R}$

Examples of Rigid Transformations (Case 2)

These images can be registered using a rigid transformation also by applying a rotation.



V



$\mathcal{T}'$

Registration Using Level-Sets

A general formulation of the problem where the template image $\mathcal{T}$ is represented by level-sets can be

$$\min_u \int_\Omega (V(u(x,z))|_{z=0} - R(x))^2 dx + \alpha S(u)$$

where

- the first term measures the sums of squared differences between the two images
- the second term regularizes the transformation.

Conclusions

- The advantage of this formulation is that even though u may be diffeomorphic, it will allow for topological changes in the subspace Ω.
- While this formulation may provide some additional flexibility "its abilities" are restricted by the choice of functions ϕ_i, which are constant, and by the regularization of the transformation u.

Local block-matching approach (1)

- Divide Ref and Source images into blocks
- Define the features of each block by calculating all possible correlations between pixels
- Apply block matching to find the total cost of moving from the ref image to the target image

Local block-matching approach (2)

Ref Image

Registered Image

Source Image

Registering curves by currents

[Marc Vaillant and Joan Glaunes - Surface matching via currents]

- Find an "optimal" deformation between two arbitrary surfaces
- Build a norm on the space of surfaces via representation by currents of geometric measure theory
- Why represent surfaces as currents? - They inherit natural transformation properties from differential forms
- Impose a Hilbert space structure on currents, whose norm gives a convenient way to define a matching functional
- Optimal solution to the matching problem is guaranteed to be one-to-one regular map of the entire ambient space
- Found by minimizing a functional consisting of a *regularizing term* + *data attachment term*

Examples