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4D Image-Based CFD Simulation of a Compliant Blood Vessel

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Joint work with M. Piccinelli, T. Passerini (Emory), E. Haber (UBC),
L. Mirabella (GA Tech – Yoganathan's group)

Starting point

More data and images(progressively more accurate) are available for more quantitative medicine

More advanced mathematical models and numerical methods enhance predictions

Merging of data and models for a ***better predictive quantitative medicine***

Different approaches: Nonlinear Kalman filtering, Variational Data Assimilation

Variational Data Assimilation

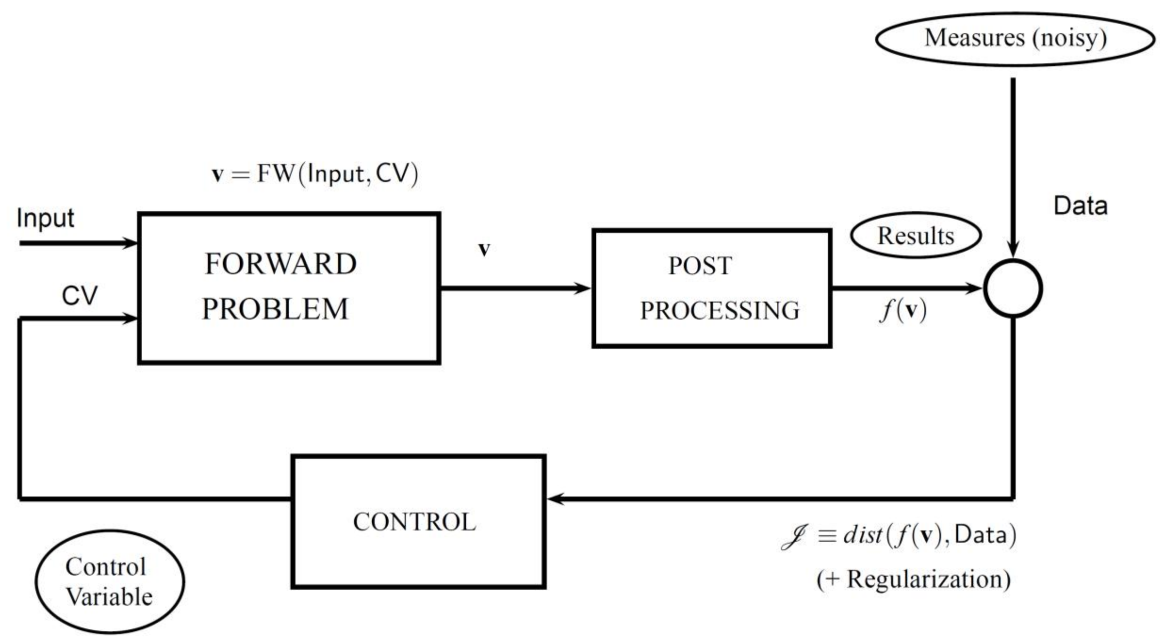
Data Assimilation = methods for merging numerical simulations and available data/images

Variational Approach = constrained minimization, control theory

Find the minimum of

$$\mathscr{J} \equiv dist(f(\mathbf{v}, \text{Data})) \quad (+\ \text{Regularization})$$

under the constraint of FW



Fluid-Structure Interaction (FSI) problems

FSI problems require:

a good mathematical model for the structure (and surrounding tissues)

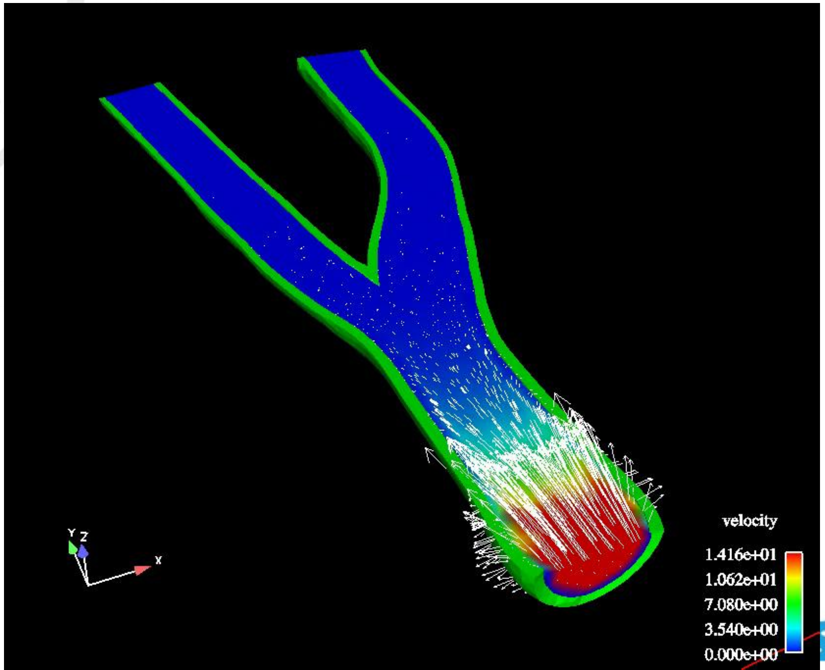
- parameter identification
- superimposition of different effects (e.g. heart movement)

time consuming solvers

- *Monolithic solvers* face ill conditioned (heterogeneous) problems
- *Iterative solvers* need preconditioners, etc

See Gerbeau/Fernandez – Quarteroni/Quaini/Badia – Nobile/Vergara,…

Still a challenging problem (in particular for the heart: Hunter, Pullan, etc.)!



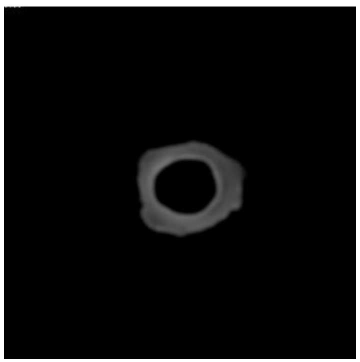
A "Data Assimilation" Procedure

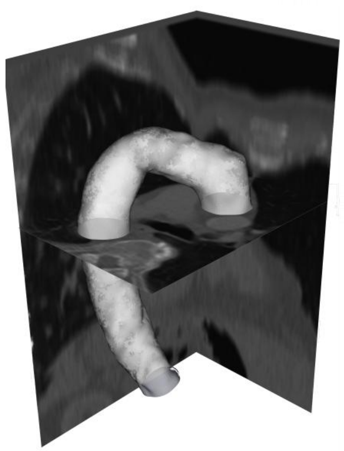
M. Piccinelli, L. Mirabella, T. Passerini, E. Haber, A. Veneziani, in revision

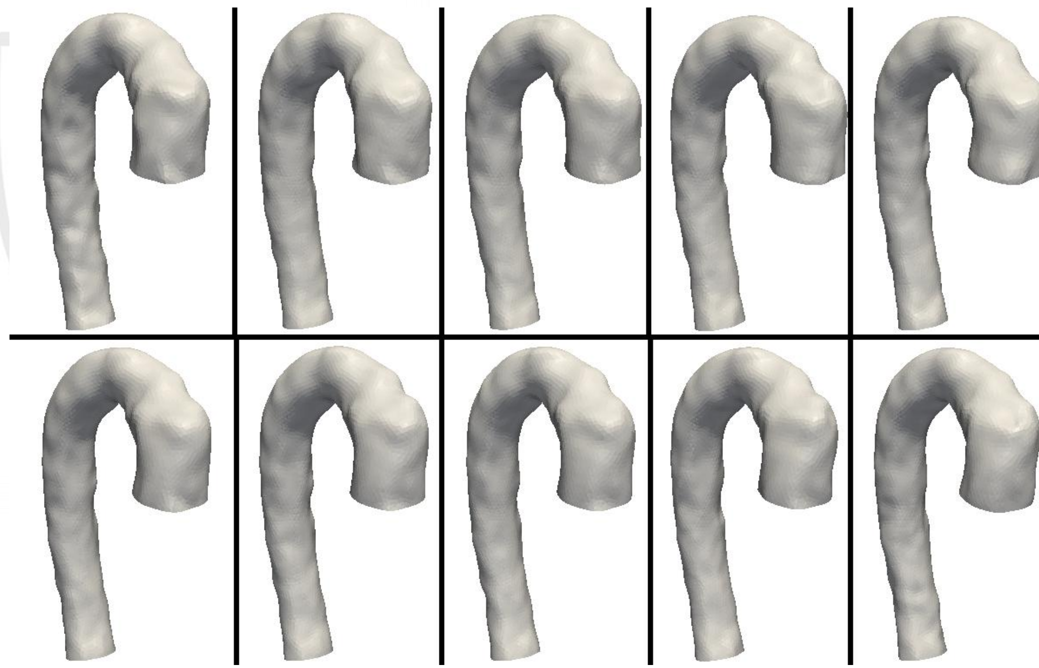
- FACT: New imaging devices provide time sequences of the vascular movement

- Simplified solution: image-based tracking

(see e.g. Zeng 2003, Nicoud 2005, Cebral 2010)





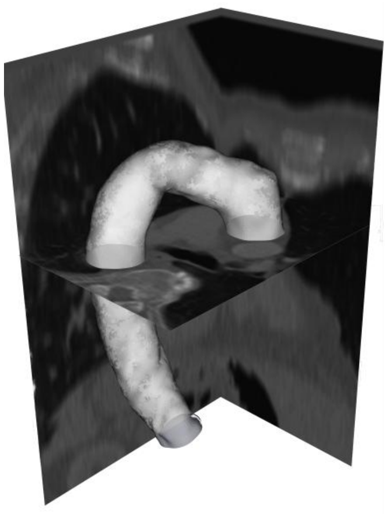


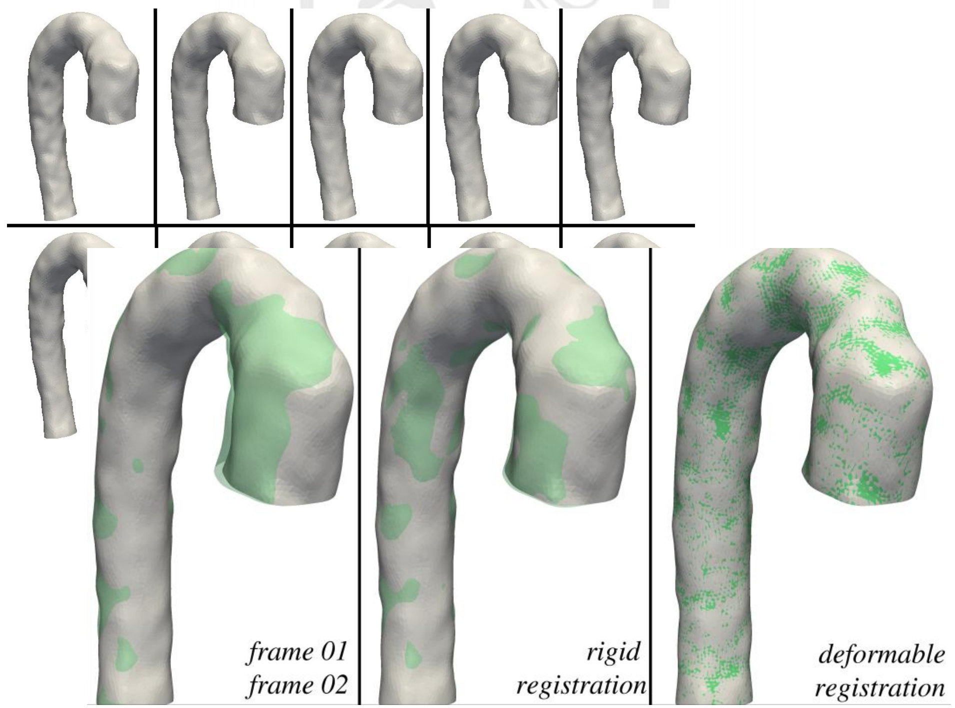
Workflow

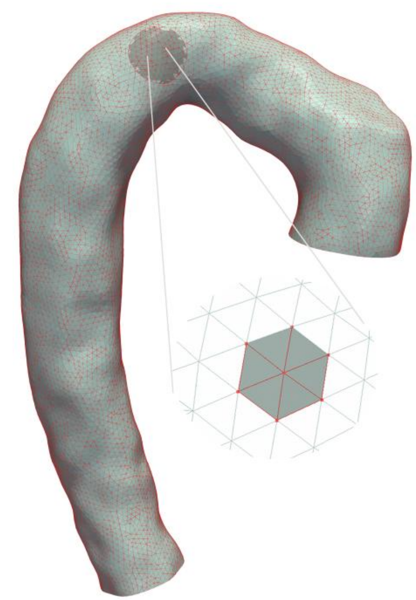
Starting point: a set of time frames of the heart or of a vessel

- Segment the images & Reconstruct the 3D structure
- Register the images for movement tracking
- Formulate the problem at hand on a moving domain framework

End of the story: simulate







Geometry Reconstruction

VMTK (www.vmtk.org): Level Set Method

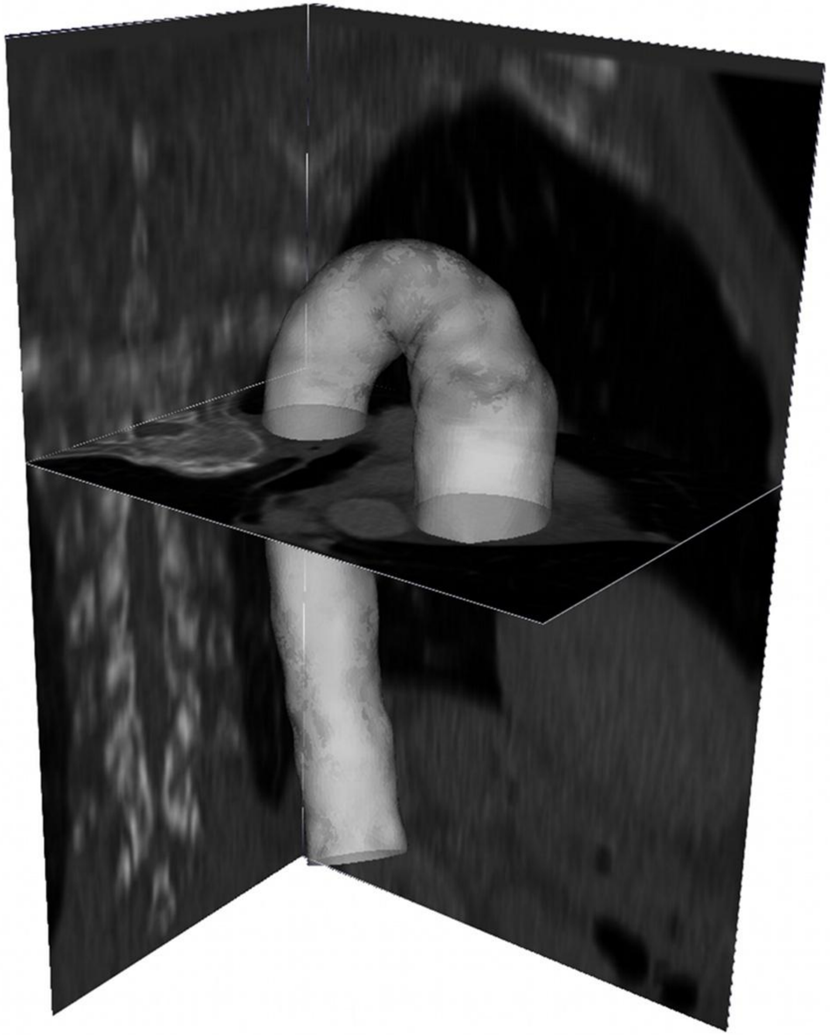
In the level set, the surface of the region of interest (ROI) is the isosurface of a function Φ - mimicking a balloon inflated inside

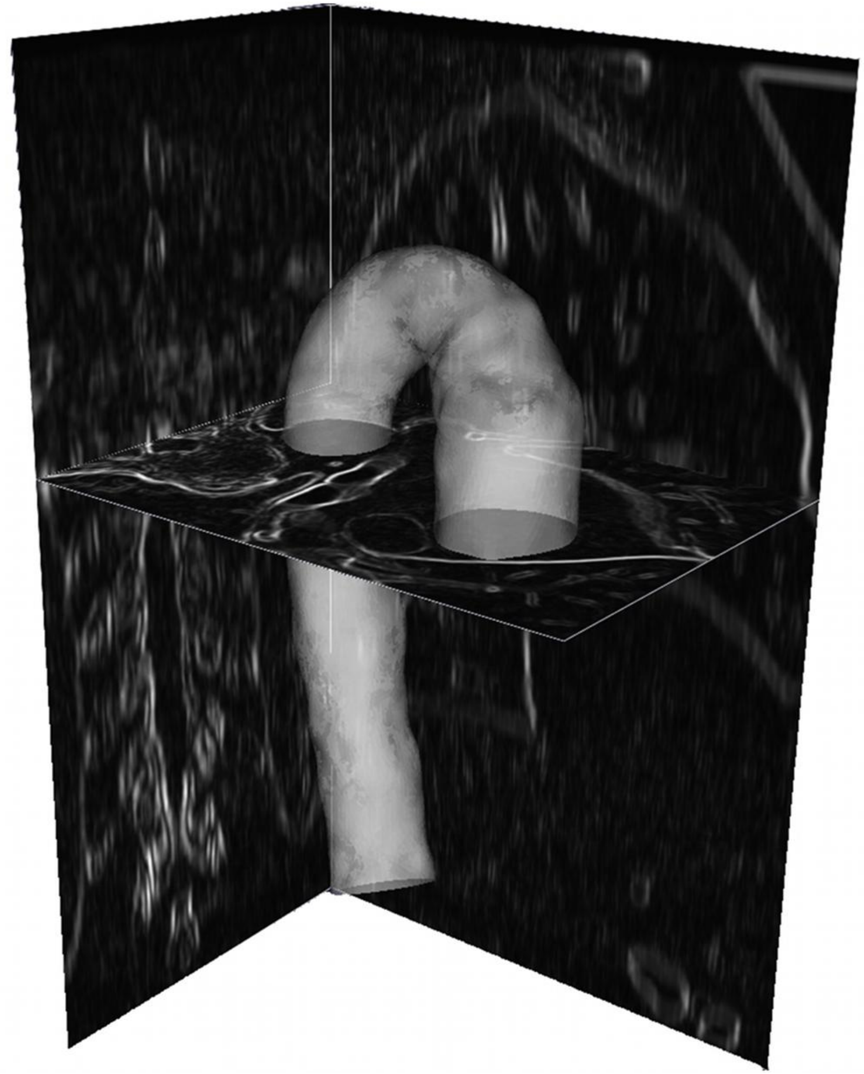
$$\frac{\partial \Phi(\mathbf{x},t)}{\partial t} = w_1 \nabla \cdot \left(\frac{\nabla \Phi}{|\nabla \Phi|} \right) |\nabla \Phi| - w_2 \nabla (|\nabla I(\mathbf{x})|) \cdot \nabla \Phi$$

Smoothing Term

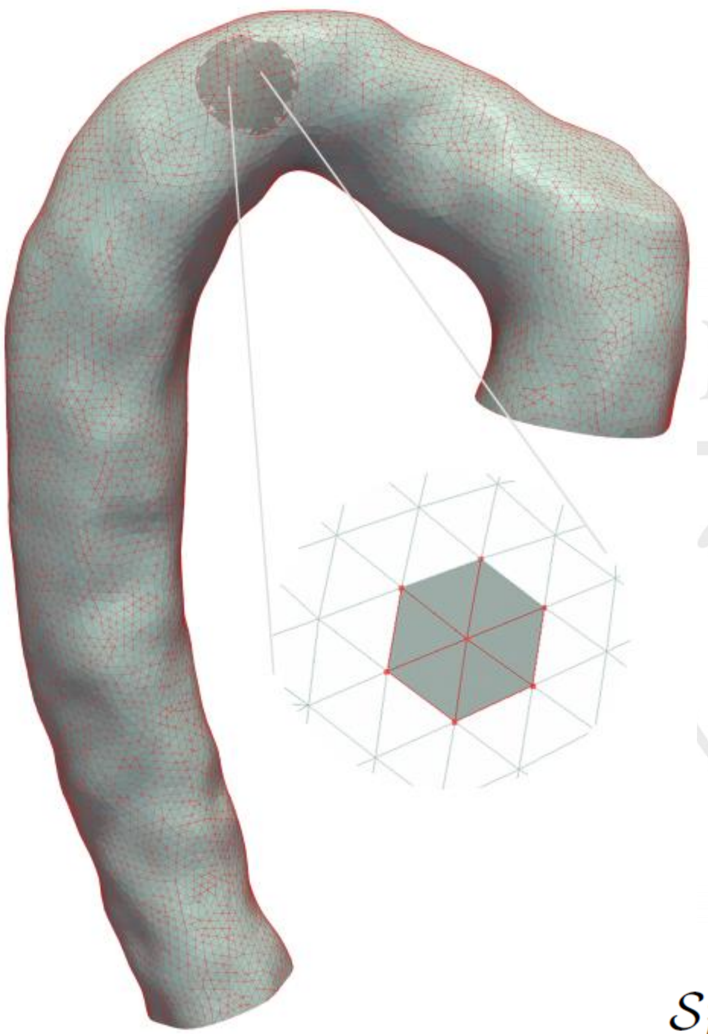
Zero-Level Set attracted to the ridges of the magnitude of ∇I (I is the image intensity)

ADVANTAGE: Robustness





For each time frame *k* we have:



Points

$$\mathcal{P}_k \equiv \left\{ \mathbf{x}_k^j \in \mathbb{R}^3, \quad j = 1, \ldots, N_P \right\},$$

Triangles

$$\mathcal{T}_k \equiv \{(i,j,l)_{k,n} \in \{1, \ldots, N_P\}^3,$$

Surface

$$\mathcal{S}_k^h \equiv \bigcup_{(i,j,l)\in\mathcal{T}_k, \mathbf{x}_k^i, \mathbf{x}_k^j, \mathbf{x}_k^l \in \mathcal{P}_k} \mathrm{tri}(\mathbf{x}_k^i, \mathbf{x}_k^j, \mathbf{x}_k^l)$$

Registration

Goal: find a map that aligns a template surface S_T (**x**) into a target surface S_R(**x**)

Several volume and surface registration algorithms in literature

see e.g. I. Moderistzky et al., 2006 Inv Probl

1. at the first time frame we compute the map φ_0 s.t.

$$\varphi_0(\mathcal{S}_0^h) \approx \mathcal{S}_1^h. \tag{6}$$

Let us denote by $\hat{\mathcal{S}}_1^h$ the resulting surface $\varphi_0(\mathcal{S}_0^h)$.

2. For the generic time frame $k+1$ we compute φ_k s.t.

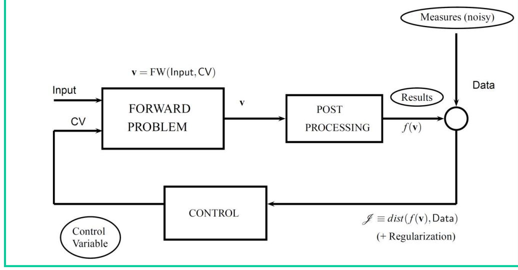
$$\varphi_k(\mathcal{S}_k^h) \approx \mathcal{S}_{k+1}^h, \tag{7}$$

resulting surface $\varphi_0(\mathcal{S}_0^h)$ is denoted by $\hat{\mathcal{S}}_{k+1}^h$.

$$\hat{\mathcal{S}}_{k+1}^h = \Phi(\mathcal{S}_0^h, \tau_k) = \varphi_k(\hat{\mathcal{S}}_k^h) = \varphi_k(\varphi_{k-1}(\hat{\mathcal{S}}_{k-1}^h)) = \varphi_k(\varphi_{k-1}(\ldots \varphi_0(\hat{\mathcal{S}}_0^h)\ldots)).$$

Notice: $\mathcal{S}_0^h = \Phi(\mathcal{S}_0^h, \tau_0 + T)$

Registration



Single step:

surface registration as a non-linear regularized optimization problem

$$\text{Find } \varphi : \mathbb{R}^3 \to \mathbb{R}^3 : \min \mathcal{D}\left(\varphi(T(\mathbf{x})), S(\mathbf{x})\right) + \alpha\varepsilon(\varphi)$$

Iterative Closest Point (modified)

mass-spring model

Map is implicitly collocated at the nodes
then extended with a piecewise interpolation

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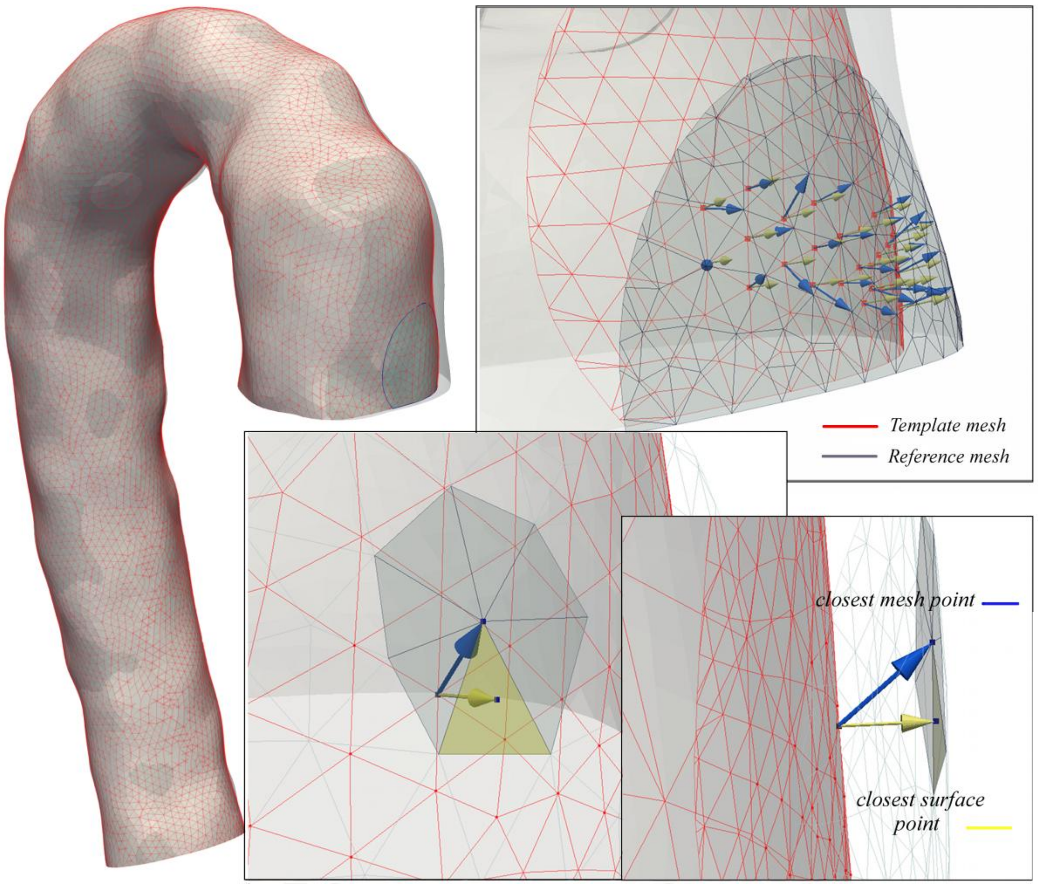
A.Veneziani

FIELDS

Classical vs Modified ICP

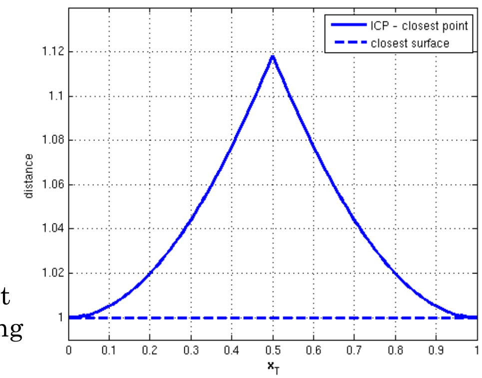


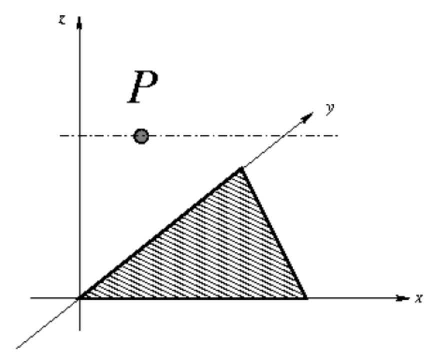
Classical (vertex based)

$$\mathrm{ICP}(\varphi(S_T^h), S_R^h) = \left(\frac{1}{N_T} \sum_{j=1}^{N_T} \min_{k=1,\dots,N_R} \|\varphi(\mathbf{x}_T^j) - \mathbf{x}_R^k\|^2 \right)^{1/2}$$

1. Suboptimal
2. Poorly regular

Distance from the unit simplex of a point along the x axis

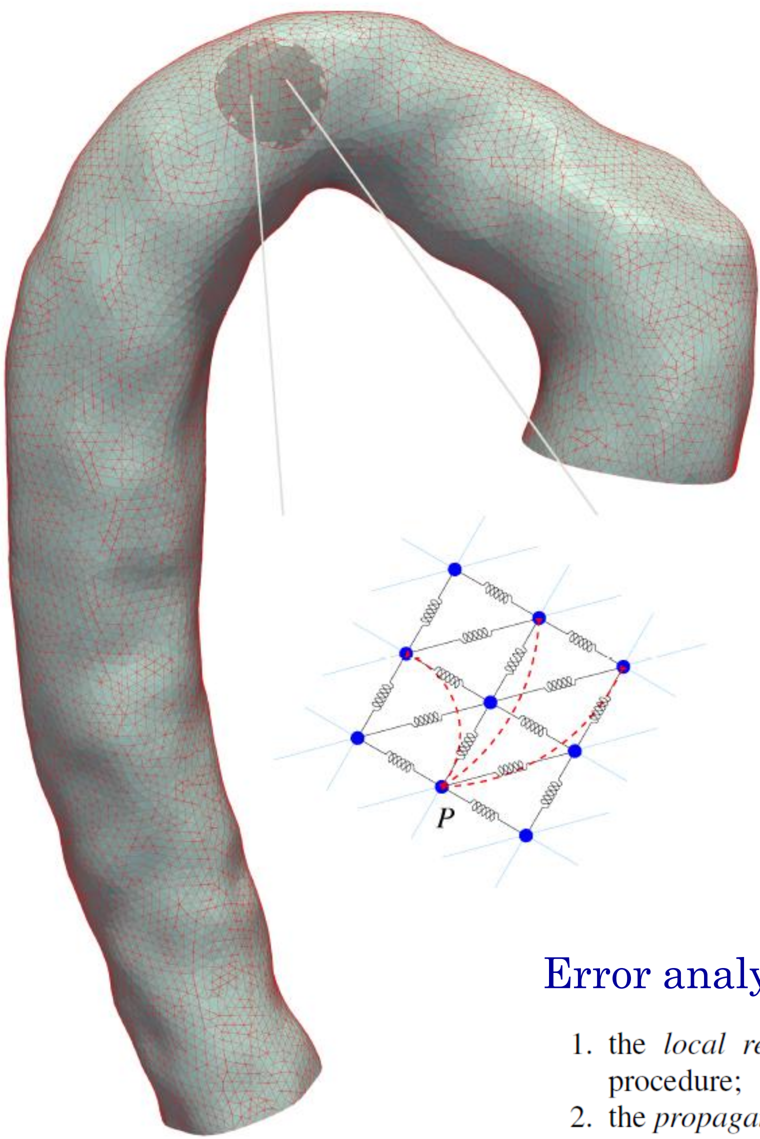




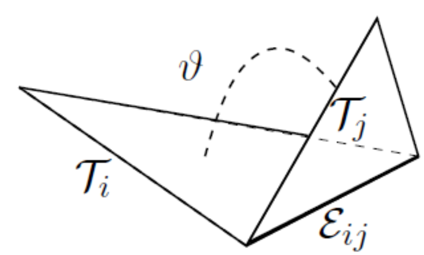
Modified (surface based)

$$\mathcal{D}^h(\varphi(\mathcal{S}_T^h), \mathcal{S}_R^h) \equiv \left(\frac{1}{N_T} \sum_{j=1}^{N_T} \mathrm{dist}^2(\varphi(\mathbf{x}_T^j), \mathcal{S}_R^h) \right)^{1/2}$$

Regularizing term



$$\mathcal{R}^h(\varphi) = \frac{1}{n}\sum_{ij}\kappa_{ij}\left(\frac{|\varphi(\mathbf{x}_T^i) - \varphi(\mathbf{x}_T^j)|}{\ell_{ij}} - 1\right)^2.$$



Error analysis (open)

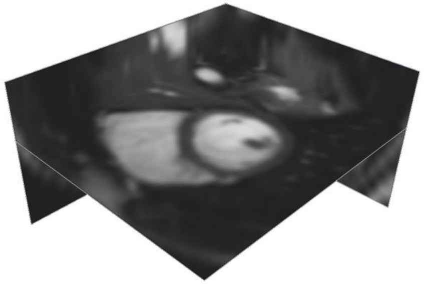
1. the *local registration error*, i.e. the error introduced at each step by the optimization procedure;
2. the *propagated error*, i.e. the error propagated by the previous iterations.

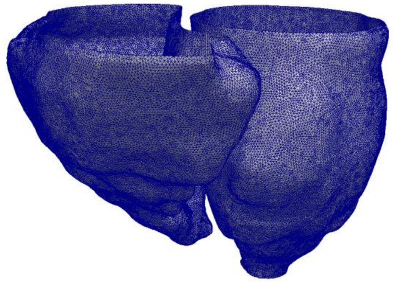
More precisely, we have

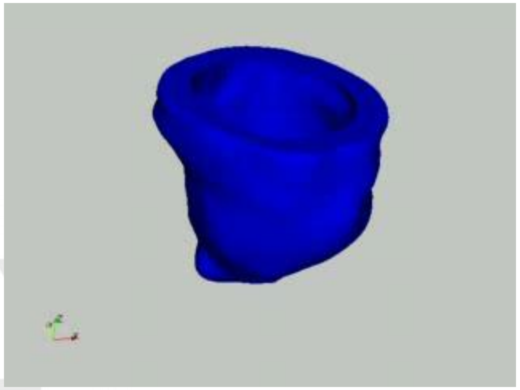
$$|\mathcal{S}_{k+1}^h - \hat{\mathcal{S}}_{k+1}^h| \le |\mathcal{S}_{k+1}^h - \varphi(\mathcal{S}_k^h)| + |\varphi(\mathcal{S}_k^h) - \varphi(\hat{\mathcal{S}}_k^h)| = \underbrace{|\mathcal{S}_{k+1}^h - \varphi(\mathcal{S}_k^h)|}_{\text{local}} + \underbrace{|\varphi(\mathcal{S}_k^h) - \hat{\mathcal{S}}_{k+1}^h|}_{\text{propagated}}.$$

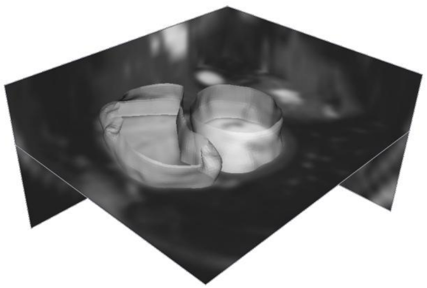
Limits of this method:

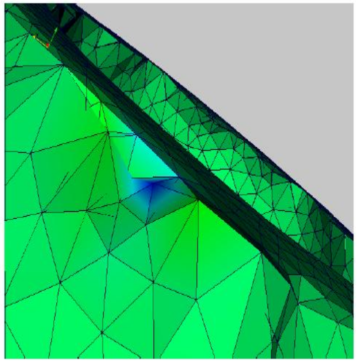
does not manage *large displacements* (like in the heart)











Simulations carried out with **LifeV** (www.lifev.org)

P1-P1 Finite **Elements** + **semi-implicit** scheme for ionic model

mesh and FE space **updated** at each time step

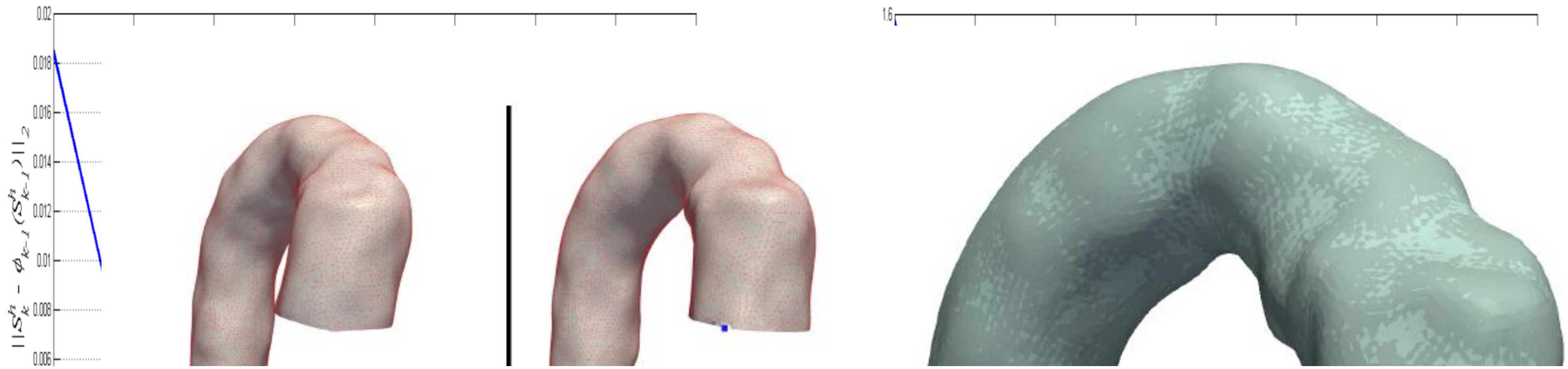
Medical **images**:

4D MRI

20 frames per cardiac cycle

possible inaccuracies at the *extrema sections*

Registration errors (step by step)



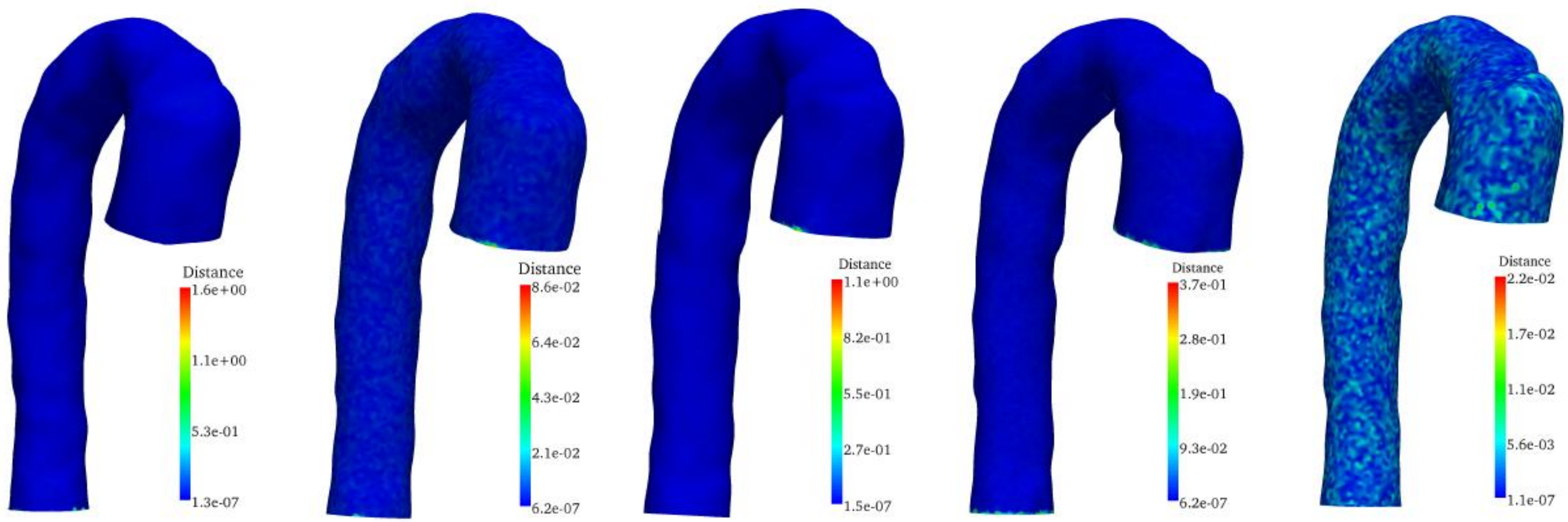


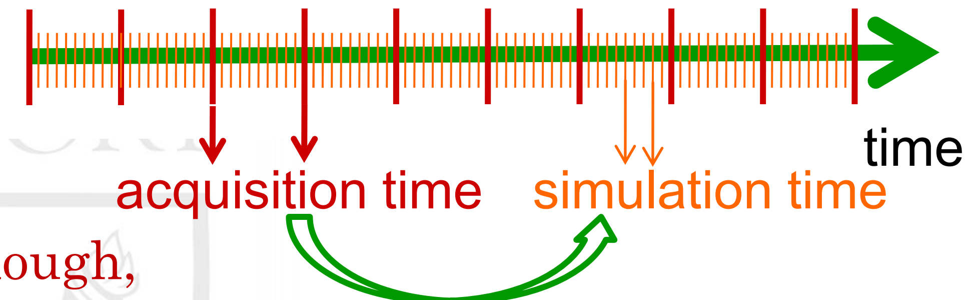
Figure 9. Registration error in the first 5 time frames.

Form frames to time advancing, from displacement to velocity

1. *interpolation*: find the (vector) function $\mathbf{x}(t)$ s.t.

$$\mathbf{x}(\tau_k) = \hat{\mathcal{S}}_k^h = \Phi(\mathcal{S}_0^h, \tau_k);$$

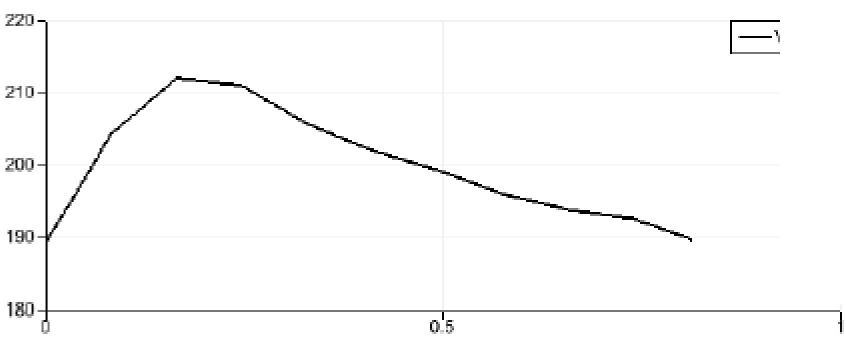
2. *differentiation*: compute $\boldsymbol{w} = \frac{d\mathbf{x}}{dt}$.



Spline interpolation

Interpolation needs to be regular enough,
Differentiation needs to be accurate enough

Cubic Splines!!!



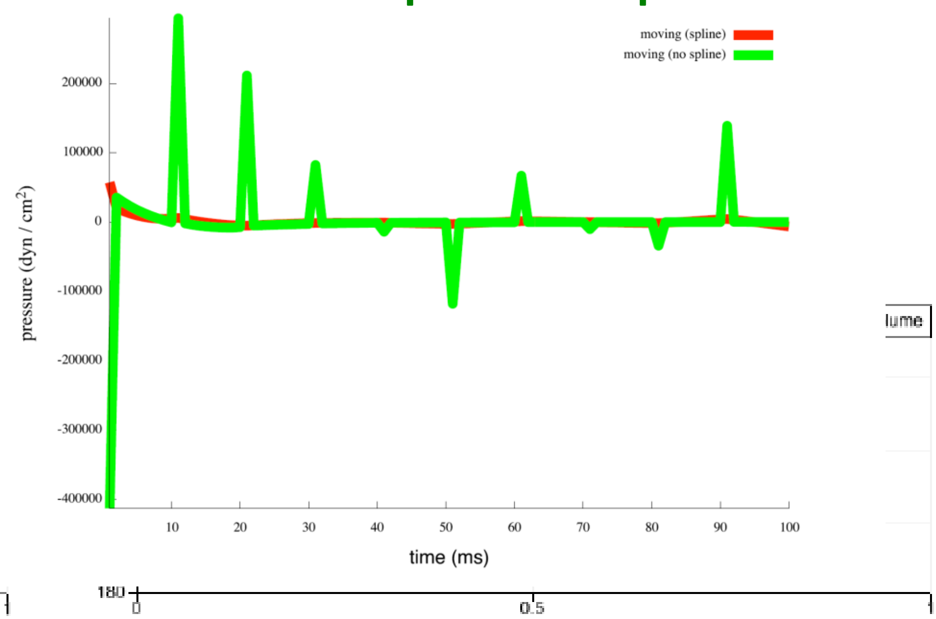


Image-Based Moving Domain CFD

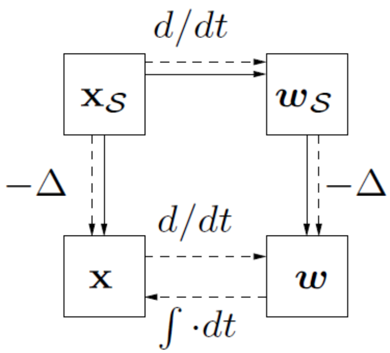
Navier-Stokes equations to describe blood flow in the vascular network
moving domain

⇒ **A**rbitrary **L**agrangian **E**ulerian formulation of **NS** equations

Mesh velocity satisfies:

$$\begin{cases} -\Delta \boldsymbol{w} = 0 & \boldsymbol{x} \in \Omega(t),\ t \in (0, T] \\ \boldsymbol{w}|_{\Gamma\text{wall}} = \boldsymbol{w}_{\Gamma\text{wall}} \\ \nabla \boldsymbol{w}|_{\Gamma\text{in}} \cdot \mathbf{n} = 0 \\ \nabla \boldsymbol{w}|_{\Gamma\text{out}} \cdot \mathbf{n} = 0 \end{cases}$$

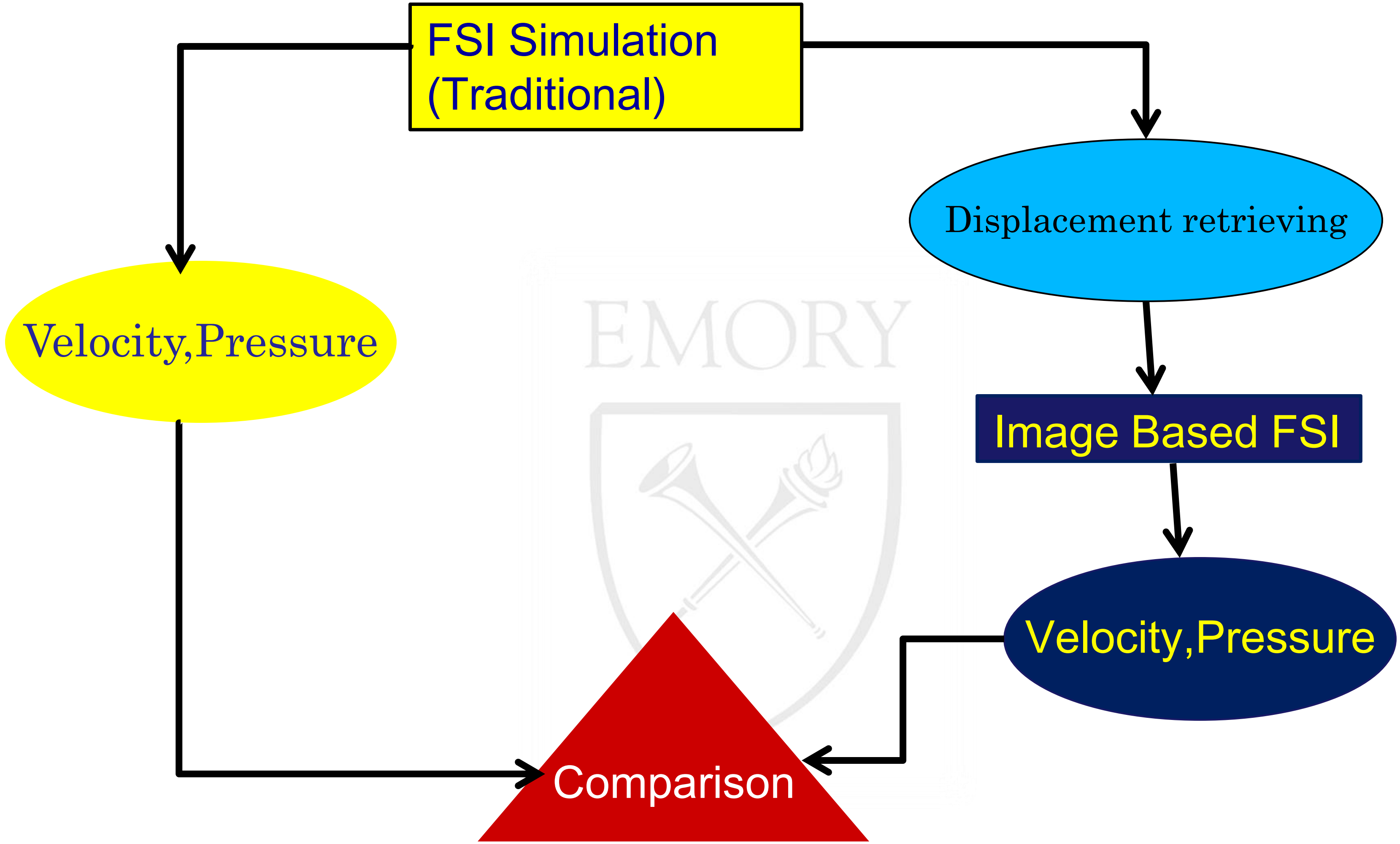
$\boldsymbol{w}_{\Gamma\text{wall}} \to \boldsymbol{w}_{\Gamma\text{wall}}(t^n)$

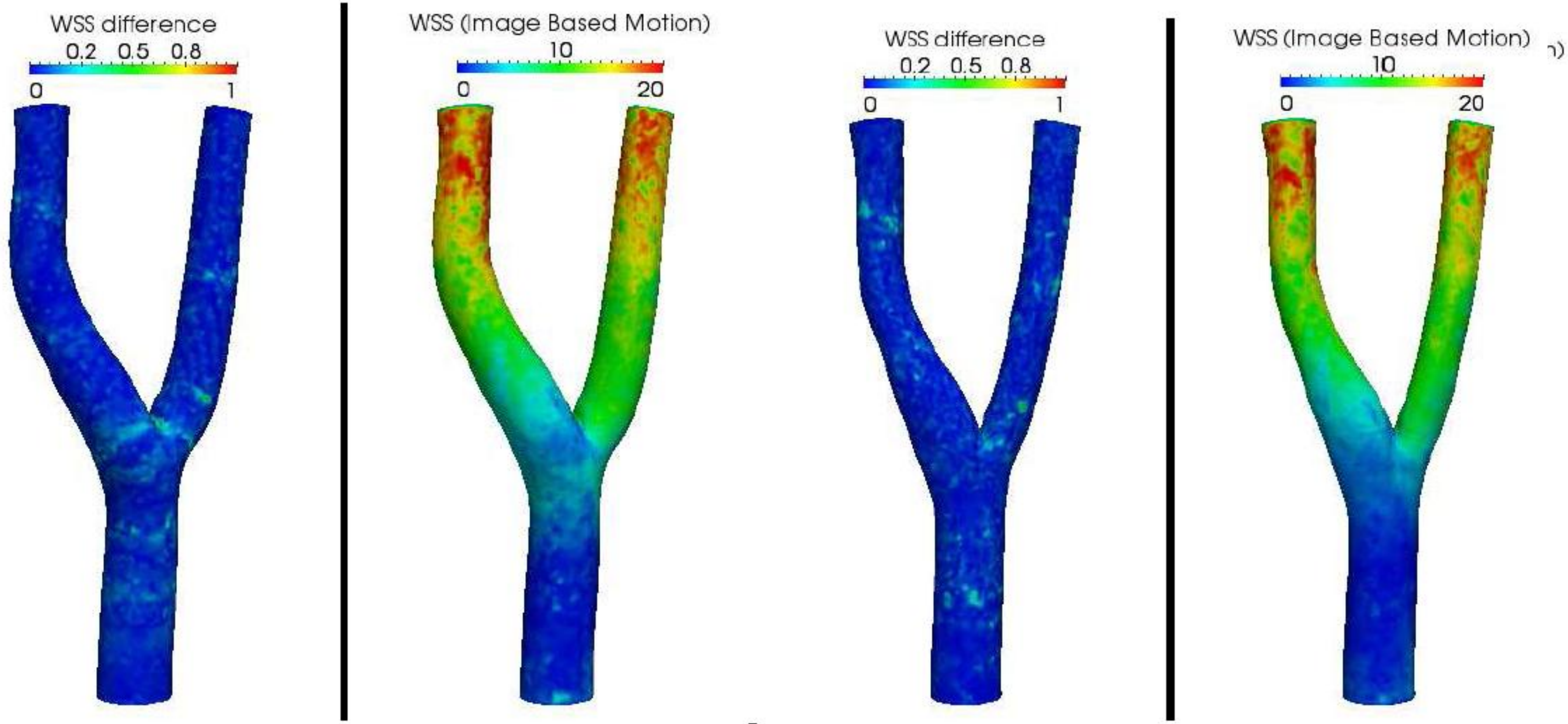


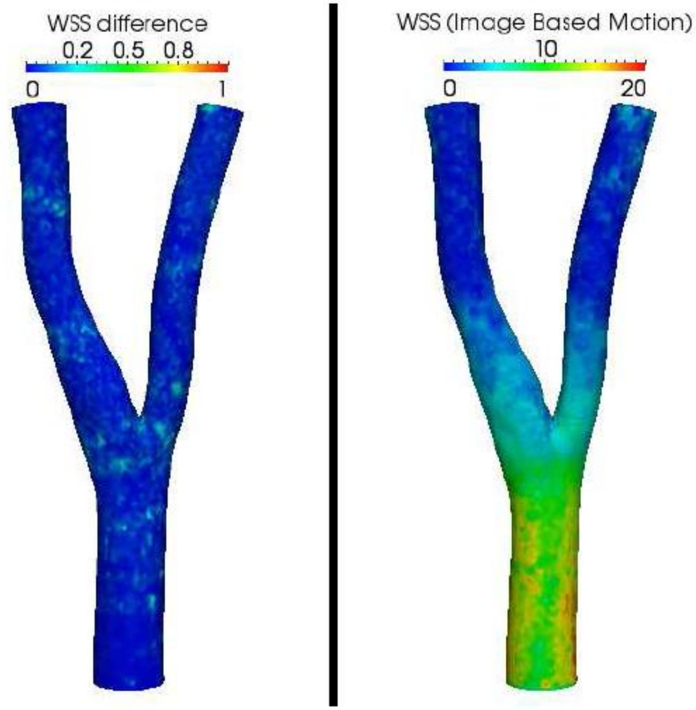
Fluid velocity and pressure satisfy:

$$\begin{cases} \rho \dfrac{\partial \boldsymbol{u}}{\partial t} + \rho((\boldsymbol{u} - \boldsymbol{w}) \cdot \nabla)\boldsymbol{u} - \mu \Delta \boldsymbol{u} + \nabla p = \boldsymbol{f} \\ \nabla \cdot \boldsymbol{u} = 0 & \boldsymbol{x} \in \Omega(t),\ t \in (0, T] \\ \boldsymbol{u}|_{\Gamma\text{wall}} = \boldsymbol{w}_{\Gamma\text{wall}} & + \text{initial condition} \\ \boldsymbol{u}|_{\Gamma\text{in}} = \boldsymbol{u}_{\Gamma\text{in}} \\ \left(p - \mu(\nabla \boldsymbol{u}|_{\Gamma\text{out}} + \nabla \boldsymbol{u}^T|_{\Gamma\text{out}})\right) \cdot \mathbf{n} = 0 \end{cases}$$

Validation: FSI vs Image Based







	max over $t^i \in \mathcal{T}$
$\chi_{\boldsymbol{u},0}$	3.9678×10^{-5}
$\chi_{\boldsymbol{u},1}$	2.1342×10^{-4}
$\chi_{\mathrm{WSS},0}$	4.0177×10^{-5}
$\chi_{\mathrm{WSS},1}$	1.0012×10^{-4}

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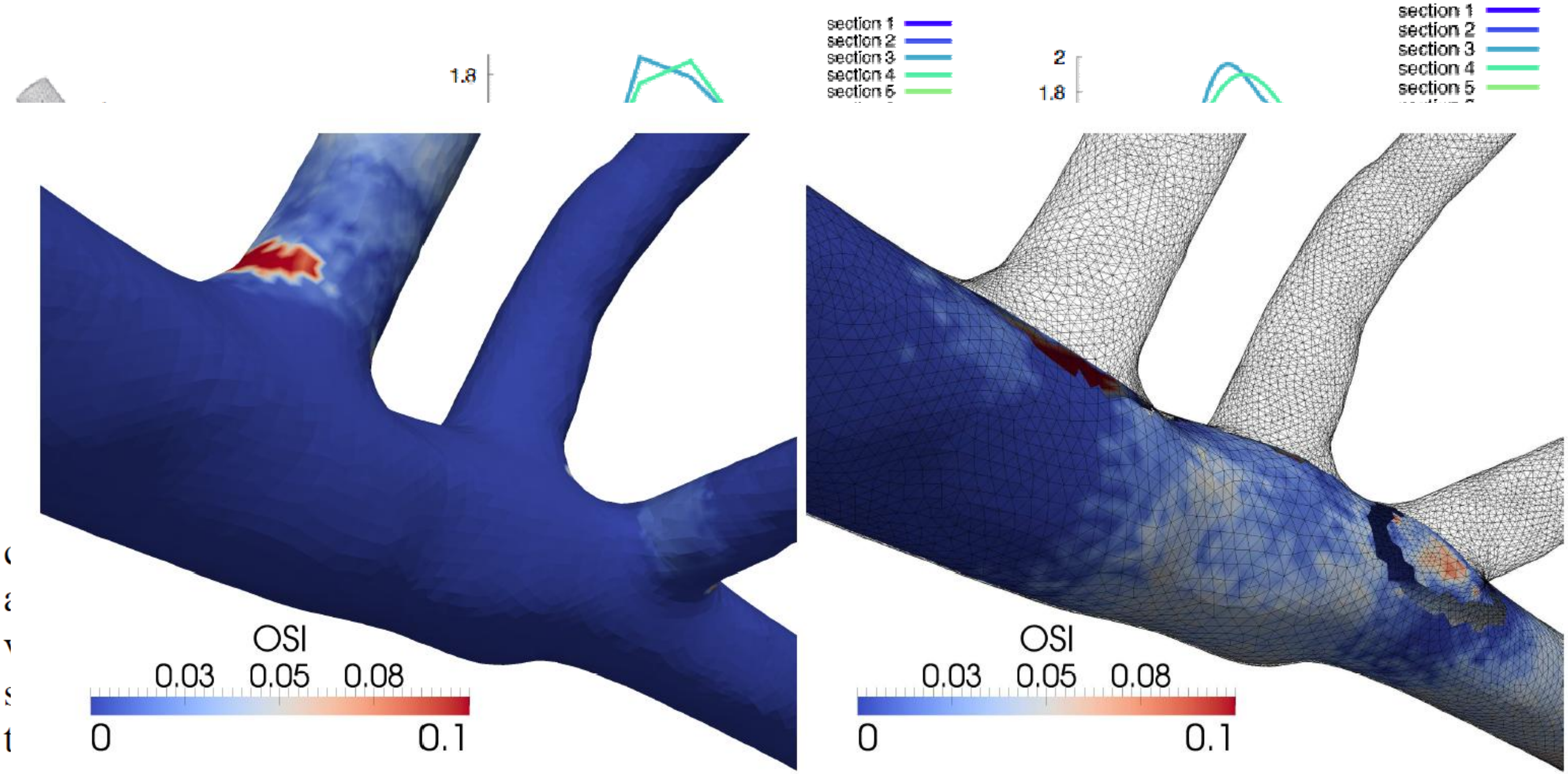
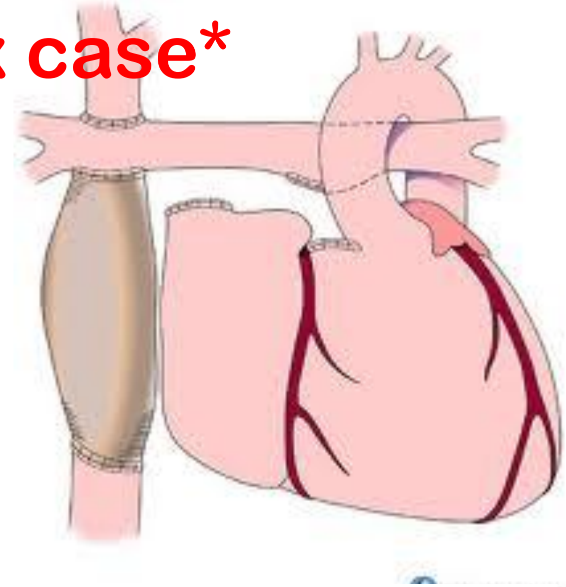
A.

A possible workaround (in absence of complete data)

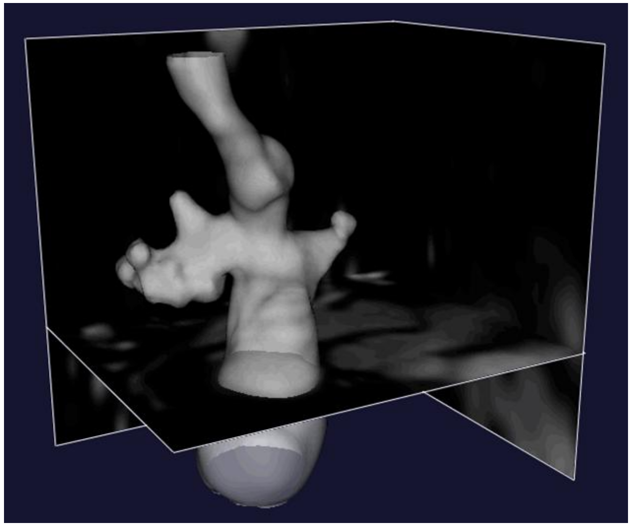


Oscillatory Shear Index (OSI, [43]) on the arterial wall of the proximal abdominal aorta. Left: Results of the rigid wall simulation. Right: Results of the moving wall simulation.

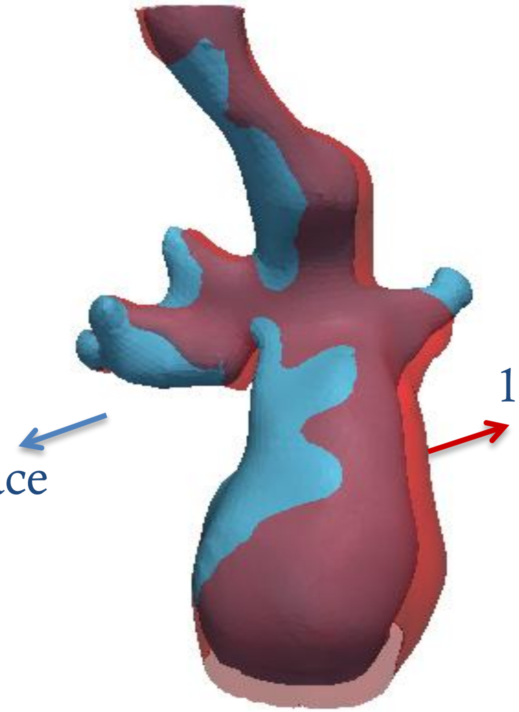
Work in Progress: A more complex case*



- Total Cavo-Pulmonary Connection (**TCPC**), with motion retrieve by a sequence of MRI
- **20 MRI images** per cardiac cycle
- **Resolution: 1.1 x 1.1 x 5.0 mm**



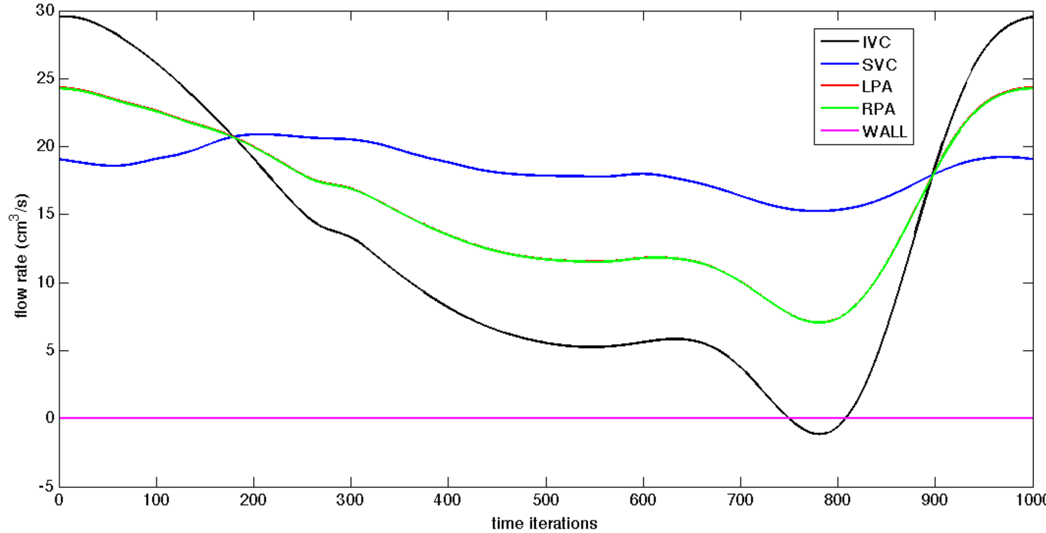
✓Problems induced by segmentation artifacts
✓Complexity of the domain (network)
✓Movement relevant for sedate patients

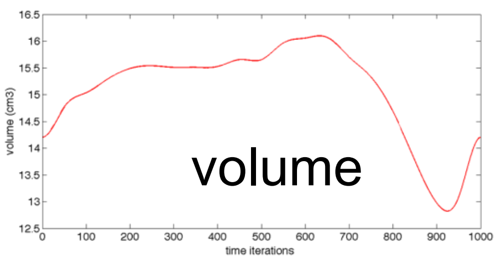


* L. Mirabella, M. Restrepo, A. Yoganathan, GA Tech

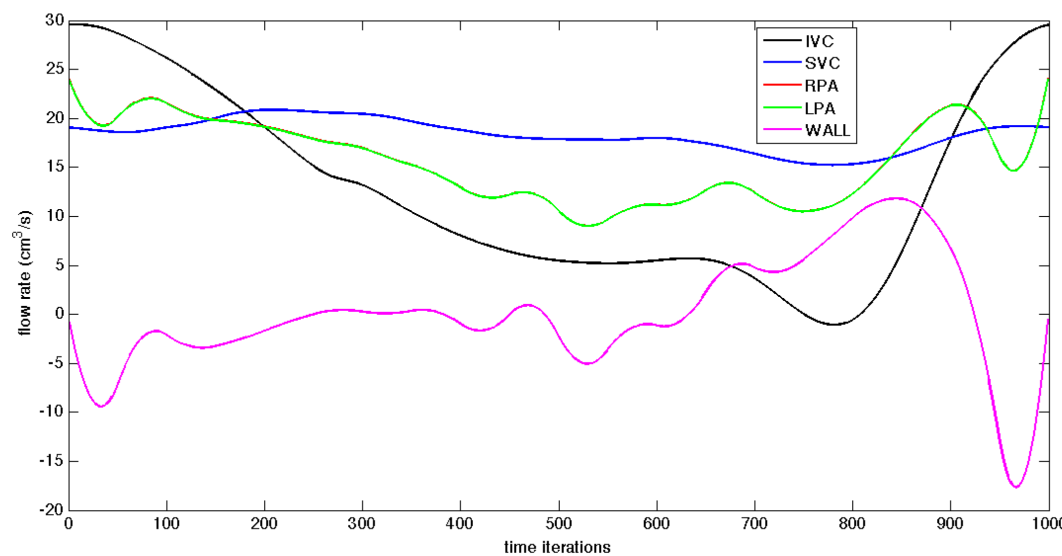
CFD in moving TCPC

Flow rate, rigid





volume

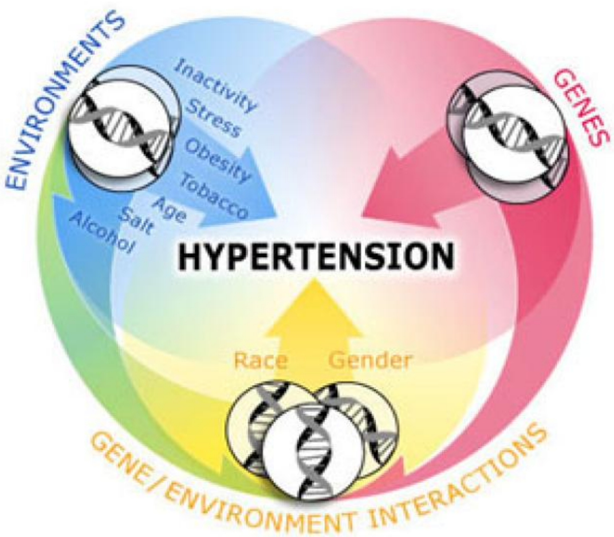


Flow rate, moving

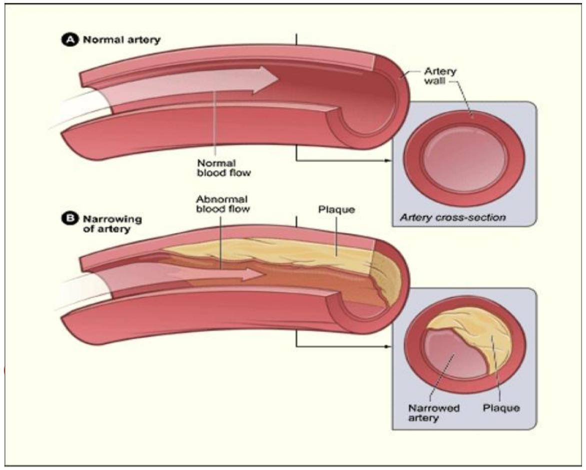
Image-Based Compliance estimation

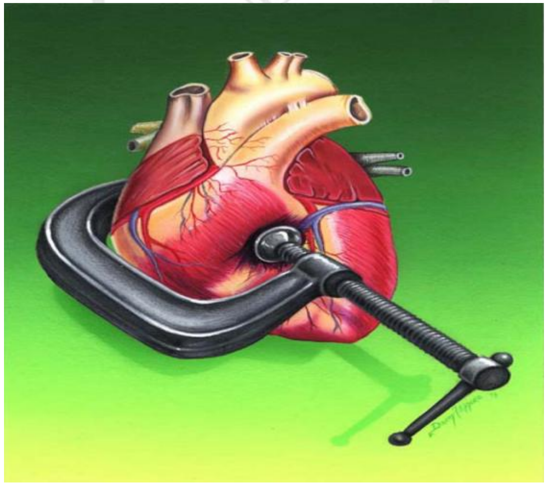
Compliance: tendency of a tissue (artery/heart) to resist the recoil toward the original shape when a compression or distending force is removed.

Physio-pathological interest:



- Atherosclerosis
- Stenting and Vascular Prostheses
- Hypertension
- Atrio-Ventricular deficienci





In vivo measures of compliance are not easy!!!

More in general:
Compliance estimation is useful for tumor detection

Joint work with C. Vergara (UniBg, Italy), M. Perego (FSU)

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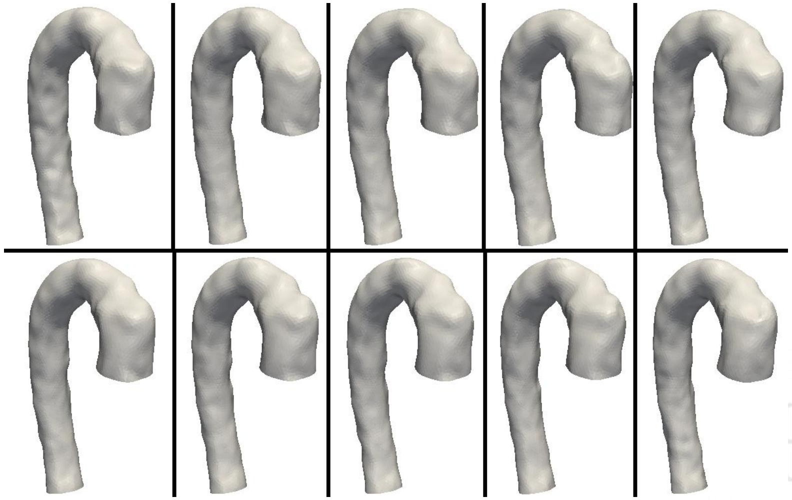
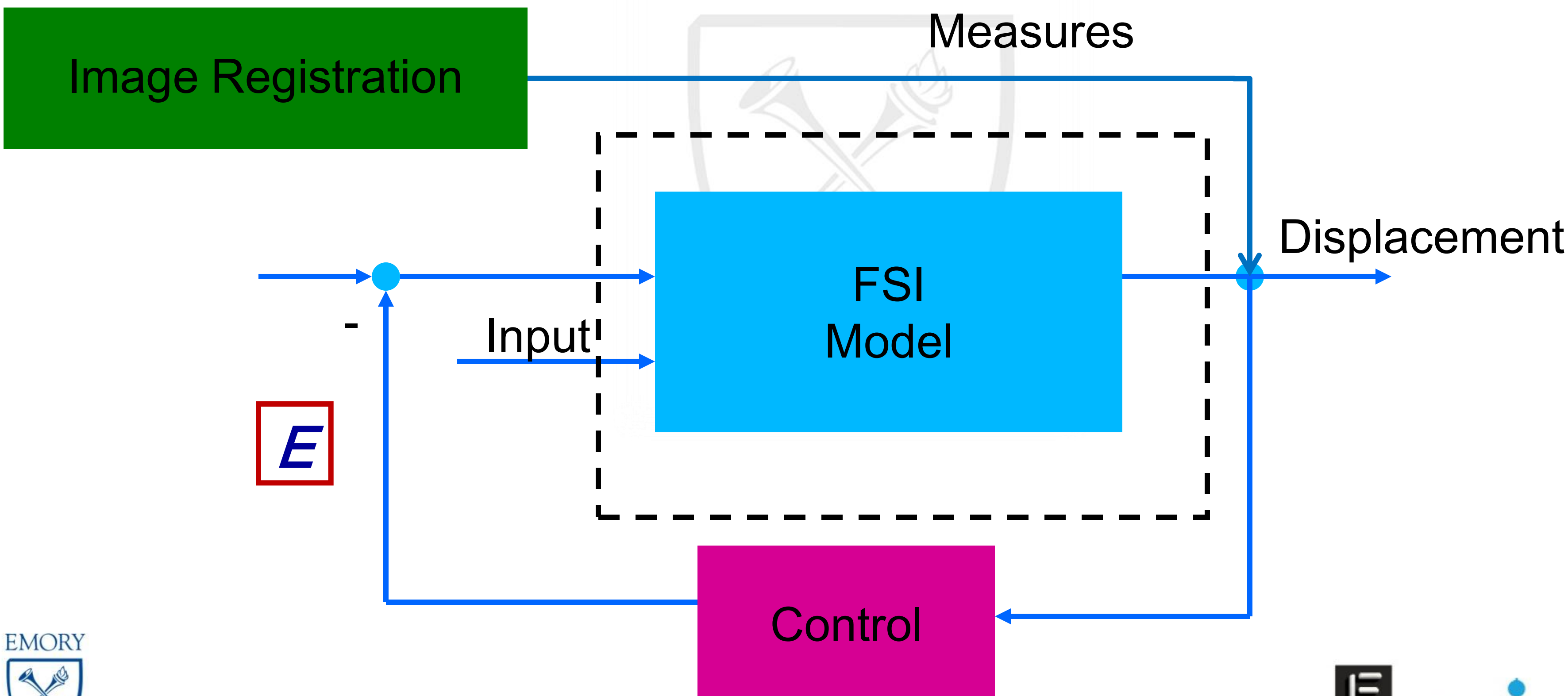
FIELDS - Mitacs

Time frames



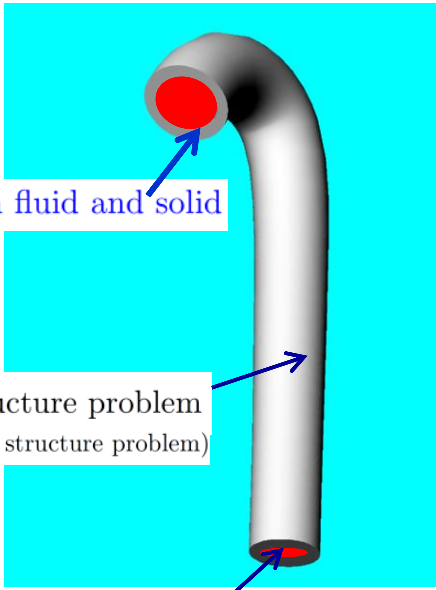
GOAL: to estimate the vessel compliance by means of

1. Image registration
2. Control theory



The mathematical problem

Let $\boldsymbol{\eta}$ be the **computed displacement field** and $J(\boldsymbol{\eta})$ a functional measuring the "distance" data-simulations:



$$J(\boldsymbol{\eta}) = \int_\Sigma (\boldsymbol{\eta} - \boldsymbol{\eta}_{\mathrm{meas}})^2 d\sigma$$

+ possible regularizing terms

Measured displacement (from the registration step)

The displacement field $\boldsymbol{\eta}$ is the solution of a FSI problem, depending on some parameters $\boldsymbol{\beta}$ to be used as control variables (and related to the compliance).

The FSI problem is regarded as a *constraint to the minimization process*, so that $\boldsymbol{\eta} = \boldsymbol{\eta}(\boldsymbol{\beta}^\star)$.

Find the parameters $\boldsymbol{\beta}^\star$ such that

$$J(\boldsymbol{\eta}(\boldsymbol{\beta}^\star)) \leq J(\boldsymbol{\eta}) \qquad \forall \boldsymbol{\eta}$$

under the conditions

$$\begin{cases} \text{Fluid}(\boldsymbol{u}, p) = 0 \\ \text{Structure}(\boldsymbol{\eta}) = 0 \\ \text{Matching conditions}(\boldsymbol{u}, p, \boldsymbol{\eta}) = 0 \end{cases}$$

Incompressible Navier-Stokes

Linear elasticity

Standard

$$\boldsymbol{T}_s = \gamma_1(\nabla\boldsymbol{\eta} + (\nabla\boldsymbol{\eta})^T) + \gamma_2(\nabla\cdot\boldsymbol{\eta})\boldsymbol{I}$$

$$\gamma_1 = \frac{E}{1+\nu}, \qquad \gamma_2 = \frac{E\nu}{(1+\nu)(1-2\nu)} + \gamma_1,$$

Two parameters: E and ν

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FIELDS Mitacs

A few steps into the Theory

Basic Approach: Discretize-in-time/Optimize/Discretize-in-space

Optimization of the time-continuous problem seems unaffordable

Instantaneous optimization: at each instant a value of E is computed

Problem (*) Statement (Time-discrete/Space continuous)

Find a bounded function $E \in [E_{\min}, E_{\max}]$ such that

$$\mathcal{J}_{\mathcal{R}} \equiv \int_{\text{Interface}} (\eta_{\text{meas}} - \eta)^2 + \frac{\xi}{2} \int_{\text{Structure}} (E - E_{\text{ref}})^2$$

is minimized under the constraint of the Fluid-Structure Interaction problem.

At each time step

Theorem

Problem (*) has at least on solution (possibly on $E_{\min}$ or $E_{\max}$). This solution depends "continuously" on the data (in the weak* sense) in the L^∞ topology.

Special cases:

The minimum exists for $E \in (0, \infty)$ in the case of

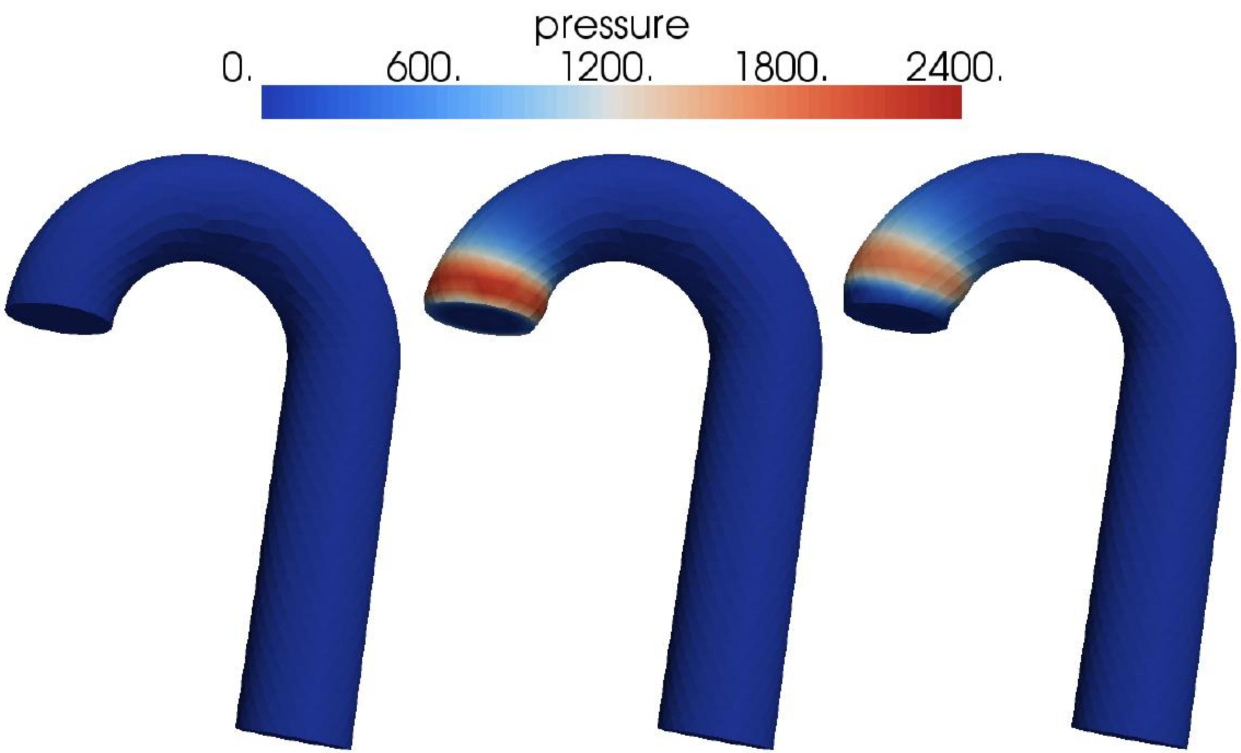
- $E \in W^{1,\infty}(\text{Structure})$
- E piecewise constant or linear.

This solution solves the KKT system.

M. Perego, A. Veneziani, C. Vergara,
To appear in SIAM J Sc Comp (2011)

FIELDS Mitacs

A 3D synthetic case



$\downarrow E_0 \setminus SNR \rightarrow$	20	10	5
$10^7\, dyne/cm^2$	1.32 ± 0.05	1.35 ± 0.12	1.24 ± 0.7
	1.5%	3.8%	9.5%

Perspectives

THEORY

1. Analysis and Improvements of the numerical methods
2. Analysis of the noise impact

PRACTICE

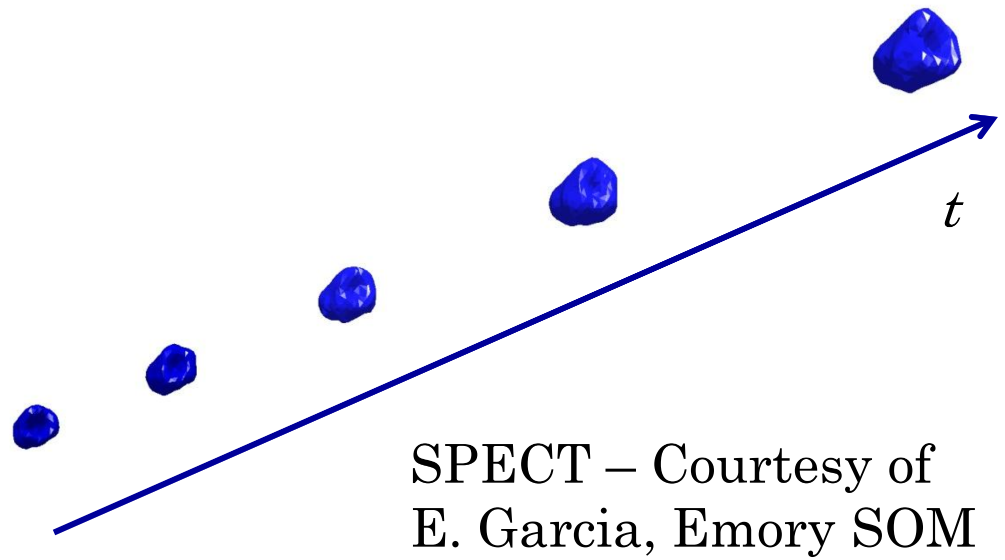
1. 3D testing and validation (Dr Yoganathan @ GA Tech) on real problems

Conclusions & I

- Combination/In
  Methods can i

  ;y)

- Proper numeric
  need to be investi
  constrained minir



- **Reliability of the Data Assimilation:**
  - Error Analysis of Image Registration
  - Impact of the noise of the images on the computed velocity (or pressure, WSS, …)
  - Extension to large displacement cases

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