



# Application of Temporally Constrained Compressed Sensing for High Spatial and Temporal Resolution MRI

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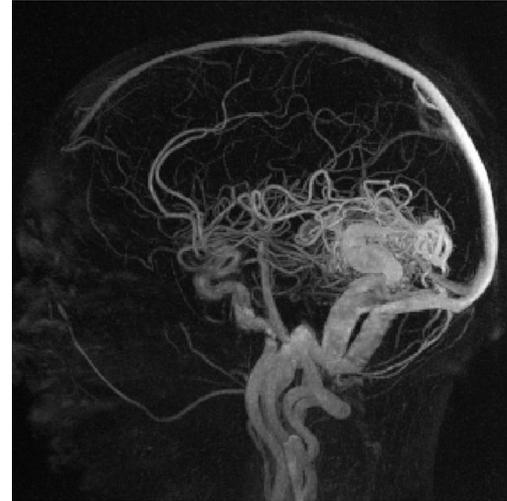
# Accelerated Imaging: Motivation

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- MRI is slower compared to other imaging modalities



X-ray DSA:  
0.4x0.4 mm  
30 fps

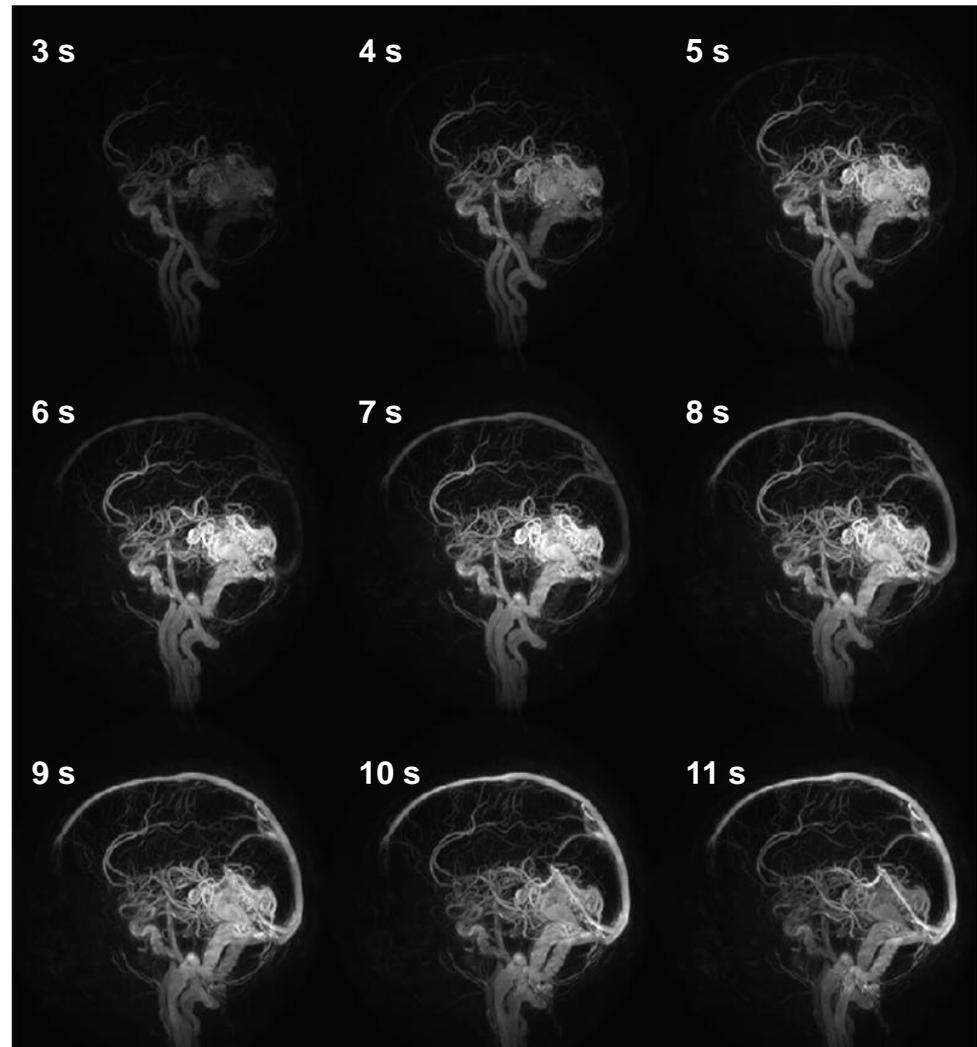


2D SPGR, TR 4ms:  
1.0x1.0 mm  
1 fps

- In MRI data are acquired sequentially, so exam time is proportional to the number of samples
- Many MR applications have to be done within a limited scan time (breath hold, passage of contrast, etc.)

# Dynamic Contrast-Enhanced Imaging

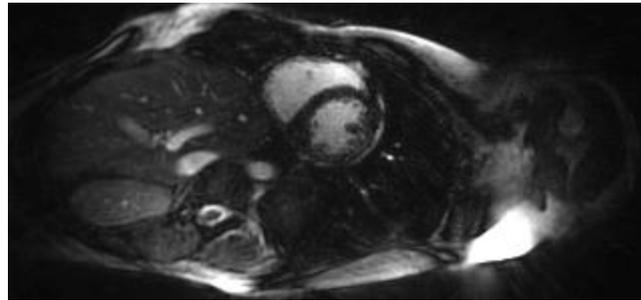
- Contrast uptake and wash-out in neuroangiography may take less than 10 s
- For some pathologies desired temporal resolution is 0.5-1s with sub-millimeter spatial resolution



# Cardiac Imaging

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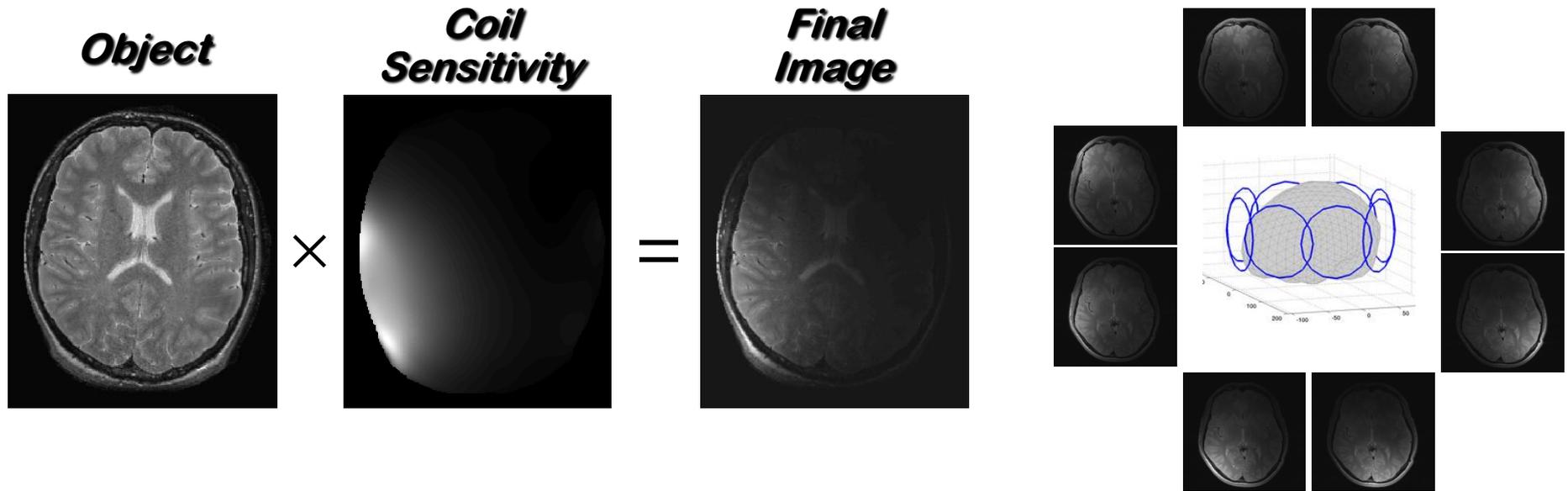
- In cardiac imaging a breath hold is often required to avoid respiratory motion (< 20 s)
- Desirable temporal resolution is 50 ms



- Other considerations: patient discomfort, likelihood of motion, etc.

# Acceleration mechanisms

- Novel acquisition strategies (non-Cartesian imaging,  $R = 2-4$ )
- Parallel imaging ( $R = 2-4$ )



- Advanced reconstruction algorithms – reconstruction from incomplete data

# MRI Signal Modeling

$$s_{\gamma} \mathbf{k} = \int_{VOI} e^{i\mathbf{k}\cdot\mathbf{r}} c_{\gamma}(\mathbf{r}) f(\mathbf{r}) d\mathbf{r}, \quad \gamma = 1, \dots, N_C$$

*Data Vector*  $\mathbf{s}$  *Image Vector*  $\mathbf{f}$

*The Encoding Matrix*  $\mathbf{E}$

$\mathbf{s} = \mathbf{E}\mathbf{f}$

- Incomplete data  $\longrightarrow$  underdetermined problem
  - # of unknowns (pixels) > # of equations (measurements)
  - infinite number of solutions
  - need a way to pick a single solution

# Prior Information

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- Prior information about the underlying image can **constrain** reconstruction

$$Ef = s \text{ such that } f \text{ is ...}$$

- Can use different types of prior information:
  - **theoretical assumptions** (smoothness, sparsity)
  - **image model** (arterial input function, dependence on control variables)
  - **image-specific assumptions** (low resolution or time-averaged image)
- Prior information provides **regularization** of the underdetermined problem

# Compressed Sensing

- CS uses sparsity model of prior info:

$Ef = s$  s. t.  $f$  has smallest possible # of non-zero pixels

- Mathematically sparsity is measured with  $\ell_0$  norm

$$\min_f \|f\|_0 \text{ s.t. } Ef = s$$

- $\ell_0$  norm is computationally challenging

- If  $N \times N$  signal  $f$  is **sparse** (has only  $K$  non-zero entries), and  $E$  satisfies **Restricted Isometry Principle**, then solutions of  $\ell_0$  and  $\ell_1$  problems coincide and can be reconstructed exactly from  $O(K \log N)$  samples by solving

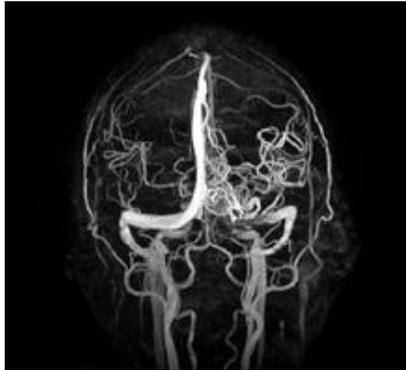
$$\min_f \|f\|_1 \text{ s.t. } Ef = s$$

# Does CS Work in MRI?

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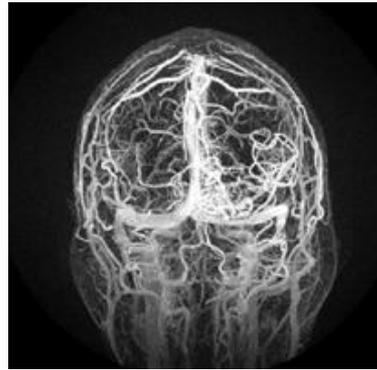
- CS seems a good match for MRI
  - many MRI images appear intrinsically sparse
  - random phase encodes or non-Cartesian acquisitions (radial, spiral) provide incoherent sampling
- Applications of CS in MR demonstrated good results with acceleration factors  $< 4-6$ .
- Higher accelerations typically lead to loss of resolution (blurry images, blocky artifacts)
- Acceleration factors have to correspond to the level of sparsity

# Validity of Sparsity Assumption



Angio images  
should be sparse

Imaging  
imperfections



Lack of sparsity



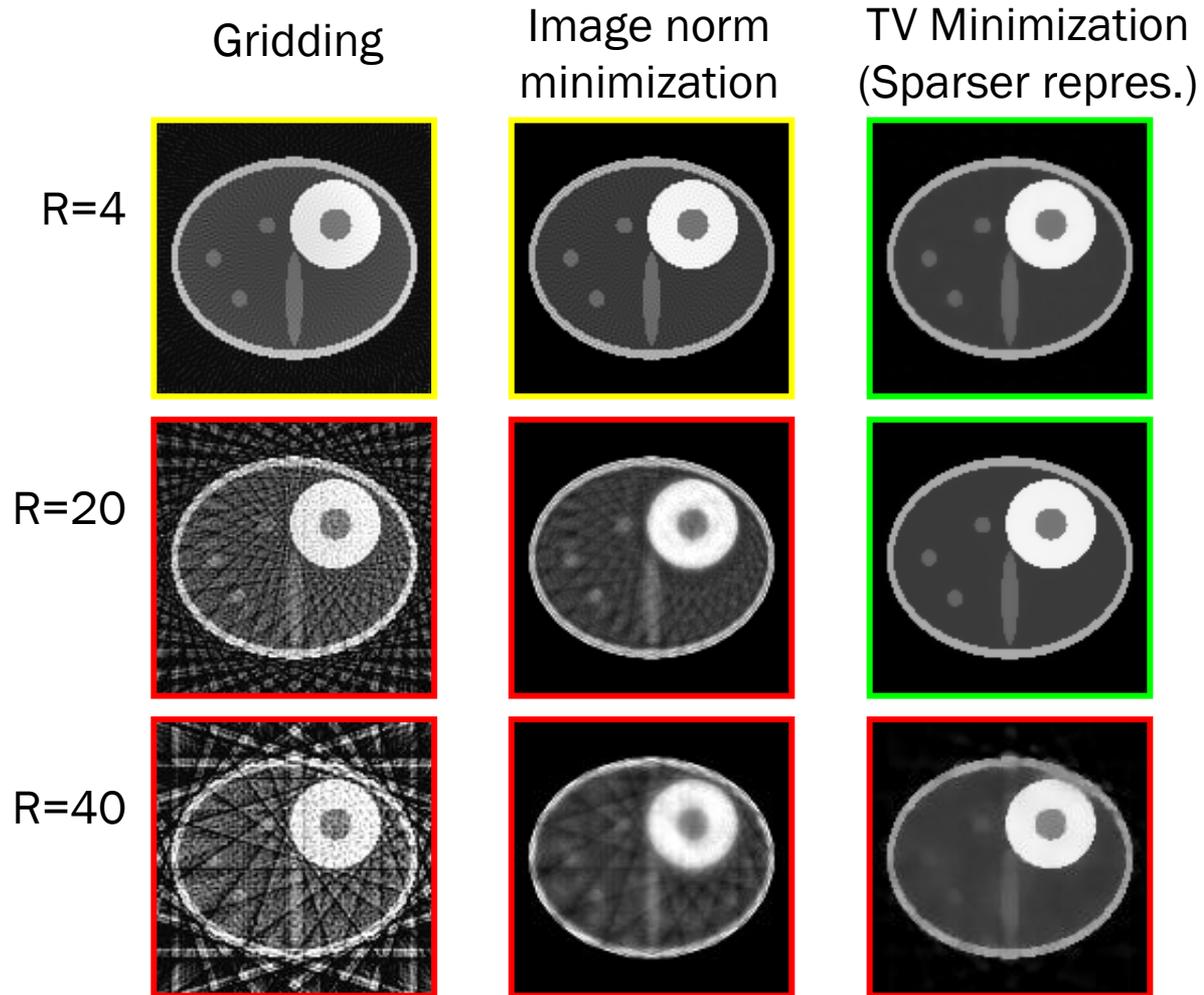
Limit on  
achievable  
acceleration  
factors

- A properly designed sparsifying transform and switching to unconstrained problem may improve reconstruction

$$\min_f \left\| Ef - s \right\|_2^2 + \lambda \left\| \Phi f \right\|_1$$

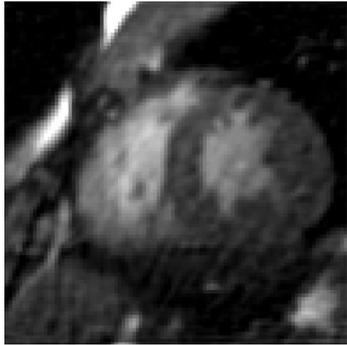
- Typically,  $\Phi$  is a discrete gradient (TV) or a wavelet transform

# Sparsity and Acceleration



Higher acceleration is possible with sparser representation

# Sparsity and Acceleration



Gridding, R=4

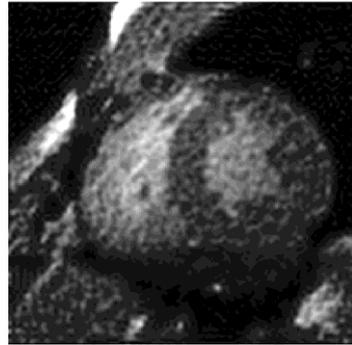
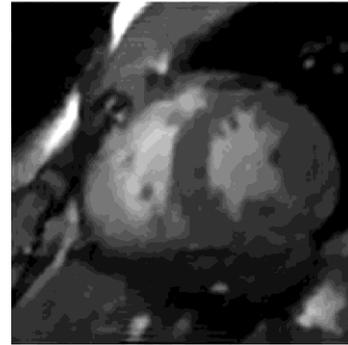
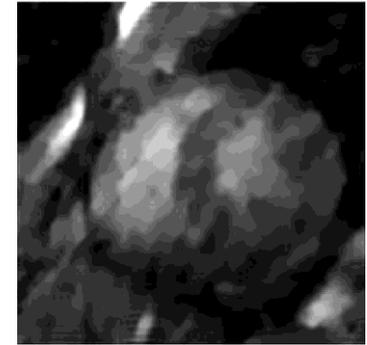


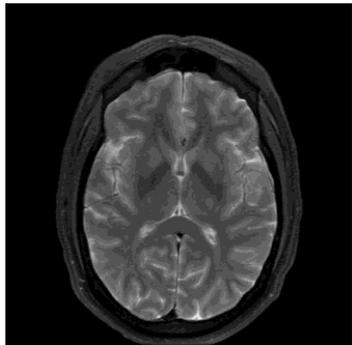
Image Norm  
Minimization, R=4



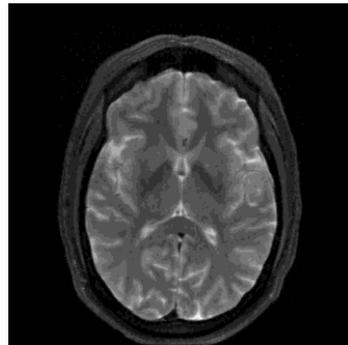
TV Minimization  
R=4



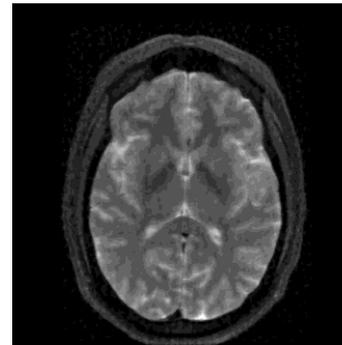
TV Minimization  
R=8



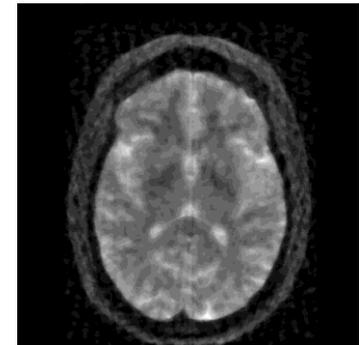
Original Image



TV, R=6



TV, R=8

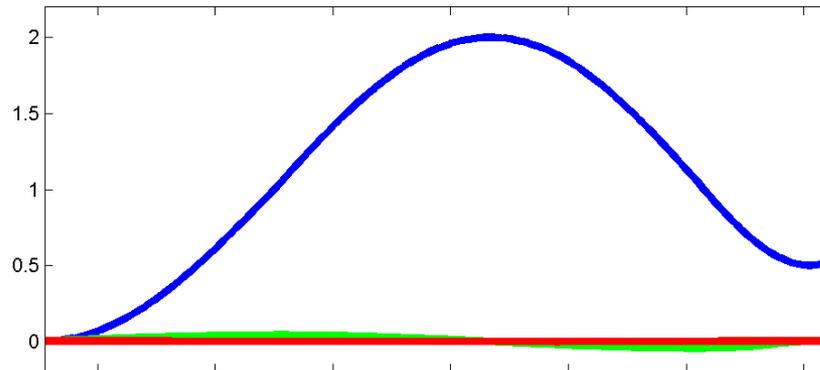


TV, R=16

- If sparsity level is insufficient to support acceleration factors, reconstructed image is biased towards model assumptions

# Temporally Constrained CS

- Sparsity may be enhanced by taking into account spatio-temporal correlations of an image series
- Prior information about dynamic contrast-enhanced image series – temporal waveform of each pixel is smoothly varying



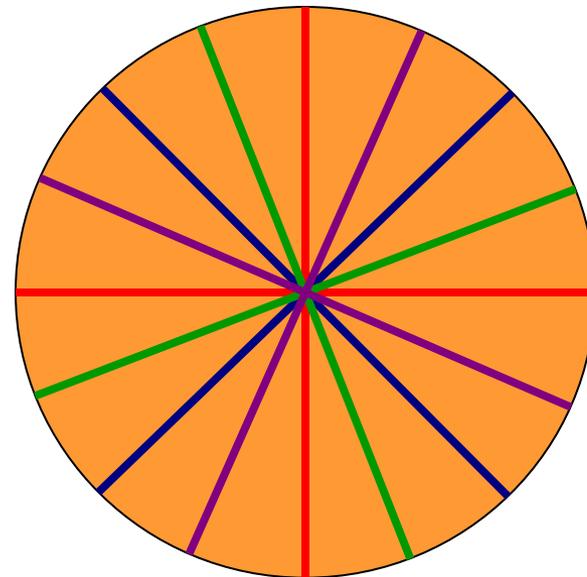
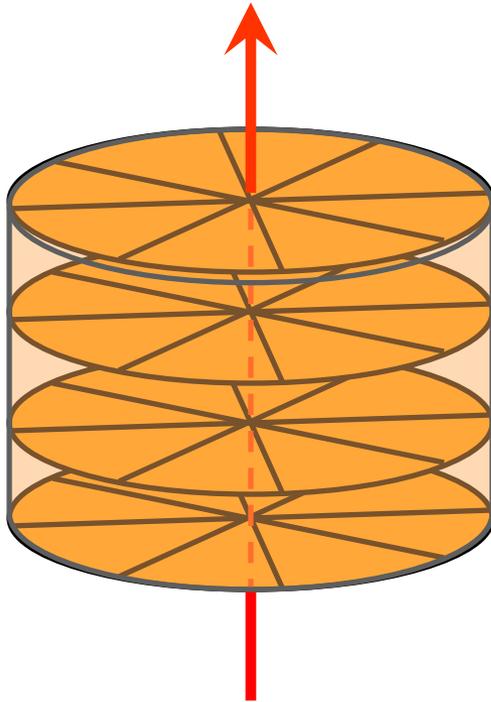
- We regularize the problem with temporal constraint

$$\min_{\bar{\mathbf{f}}} \left( \left\| \mathbf{E}\bar{\mathbf{f}} - \bar{\mathbf{s}} \right\|_2^2 + \lambda \left\| \Delta^2 \bar{\mathbf{f}} \right\|_{\ell_1/\ell_2} \right)$$

# Acquisition scheme

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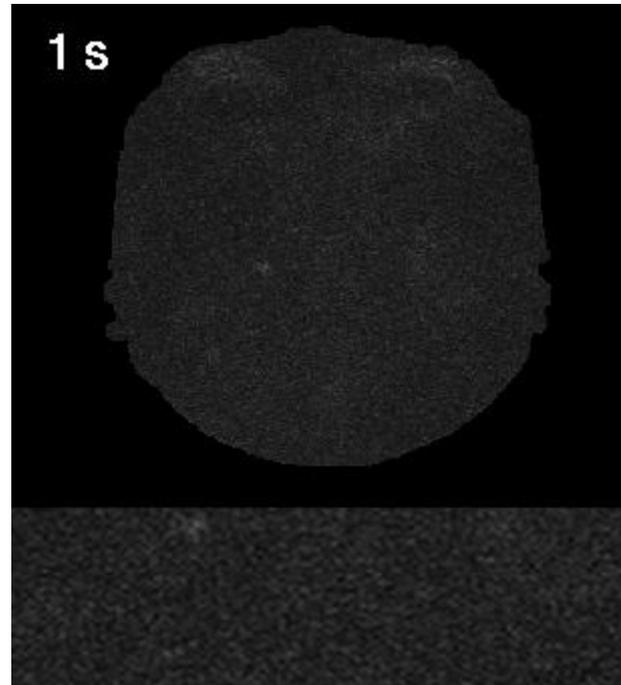
- The data are acquired using “stack-of-stars” radial sampling
- Projections in neighboring frames are interleaved to increase coverage and disperse artifacts



# Aneurysm patient

## ■ 3.0 T GE Discovery™ MR750, 8-channel head coil

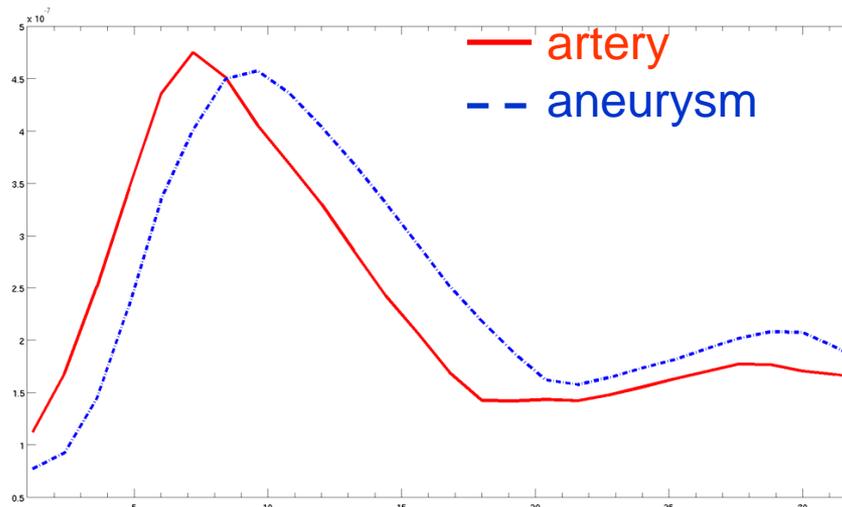
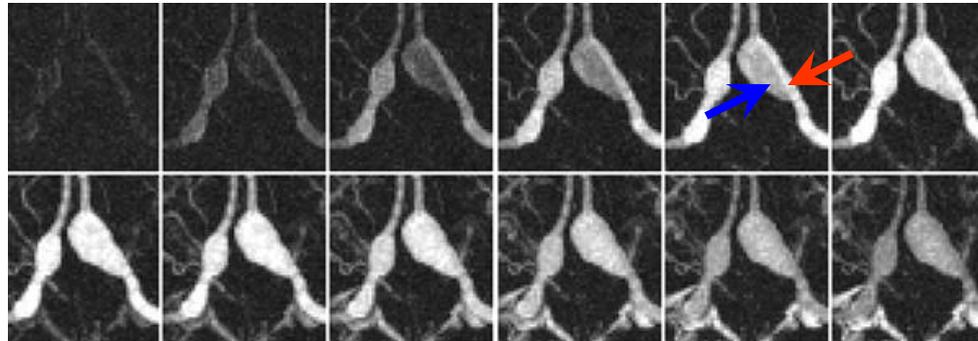
- 0.86x0.86x2 mm<sup>3</sup>
- 20 slices
- 1.2 s / frame
- 15 projections/frame



- R = 27
- TE/TR=1.5/4 ms
- FA=25°,
- BW=125 kHz

# Filling of Aneurysm

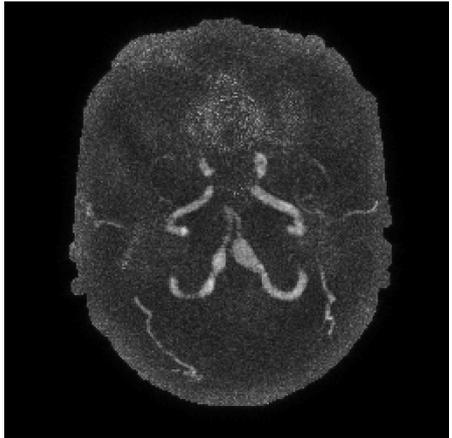
- Temporal resolution is sufficient to demonstrate delayed filling of aneurysm



# What about spatial constraints?

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- At high acceleration factors, “standard CS” produces images of inferior quality



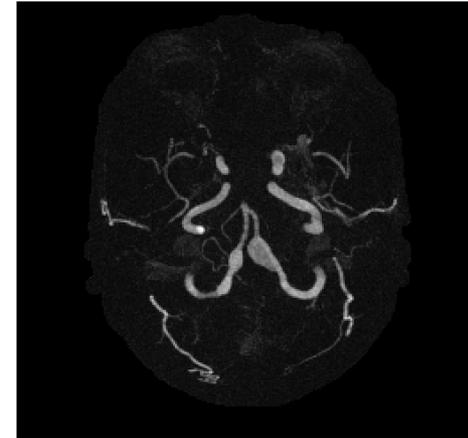
SENSE



Image norm  
minimization



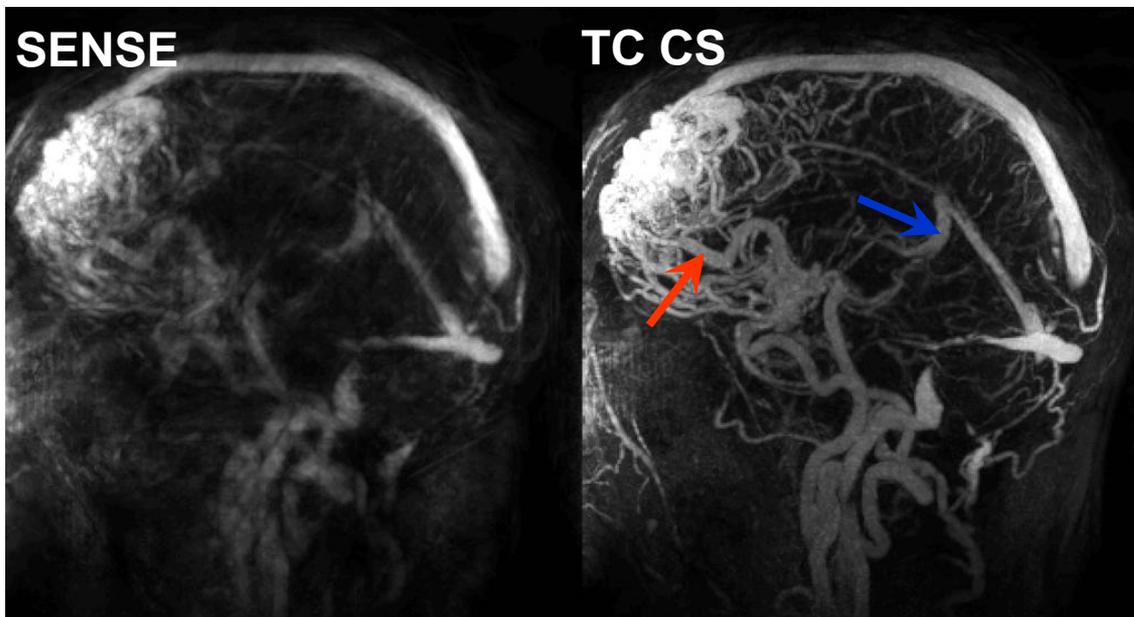
TV minimization



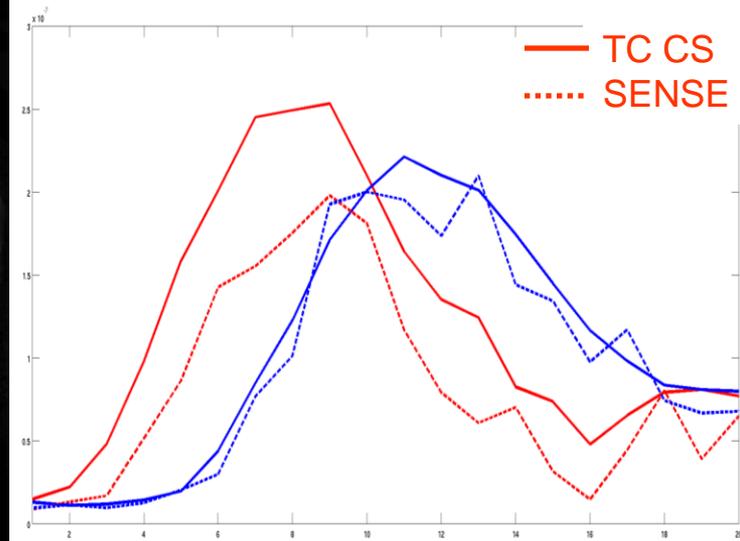
TC CS

# AVM Patient

- 3.0 T GE Discovery™ MR750, 32-channel coil
  - 0.68x0.68x1.5 mm<sup>3</sup>
  - 114 slices (57 acquired , GRAPPA with R=2)
  - 1.2 s / frame
  - 6 projections/frame, R = 84



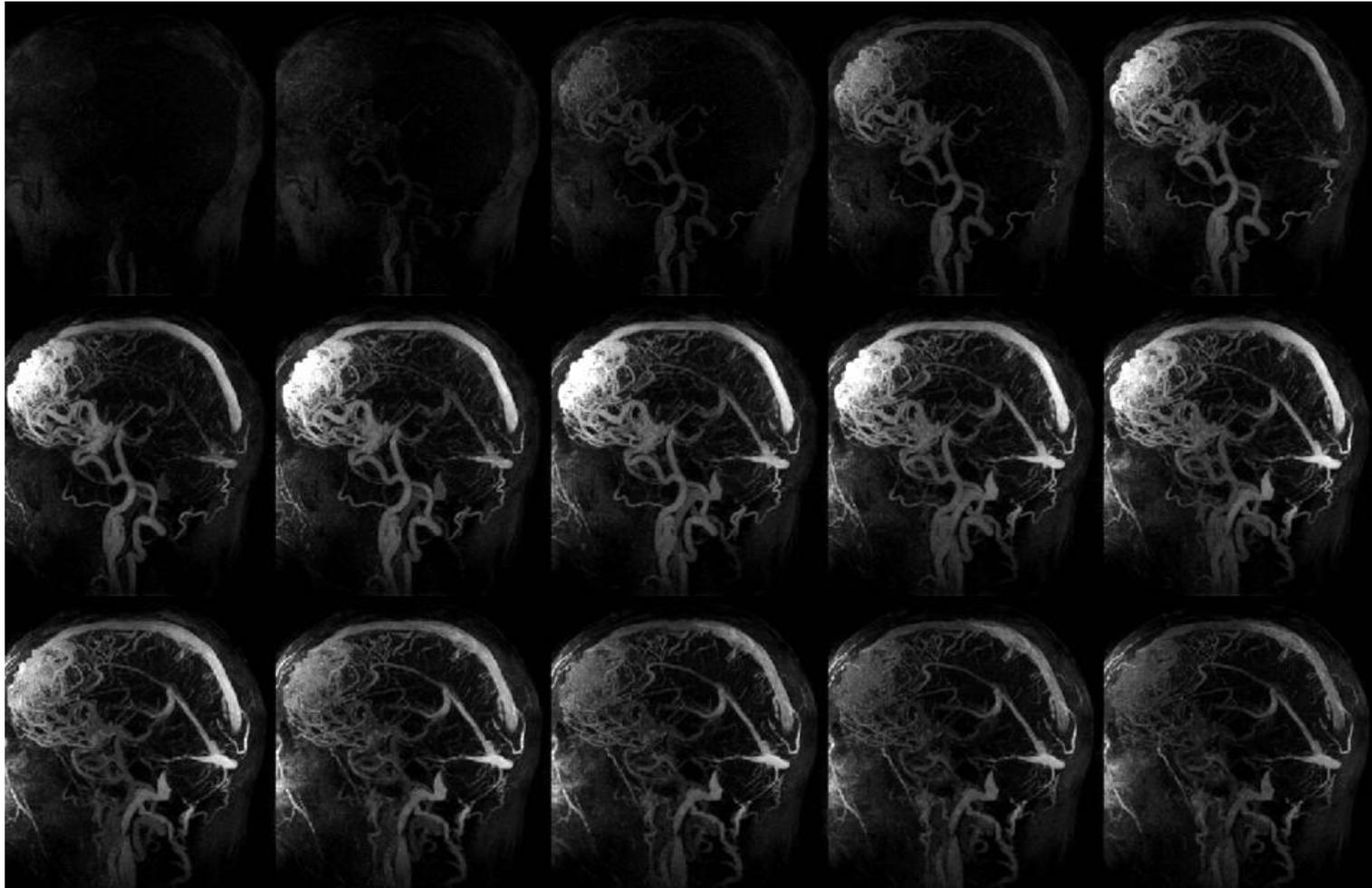
Limited MIPs



# AVM patient

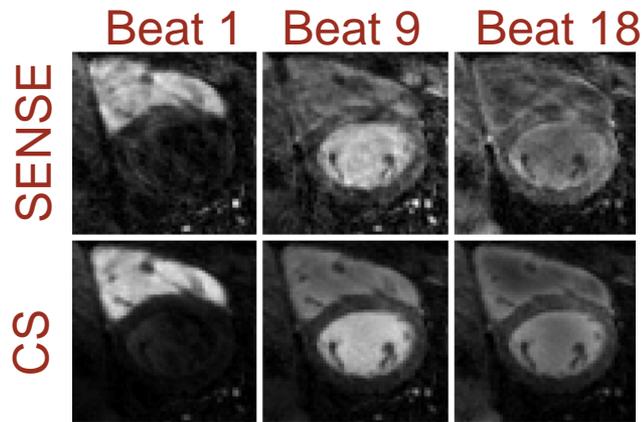
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- TC CS provides good A/V separation and spatial resolution

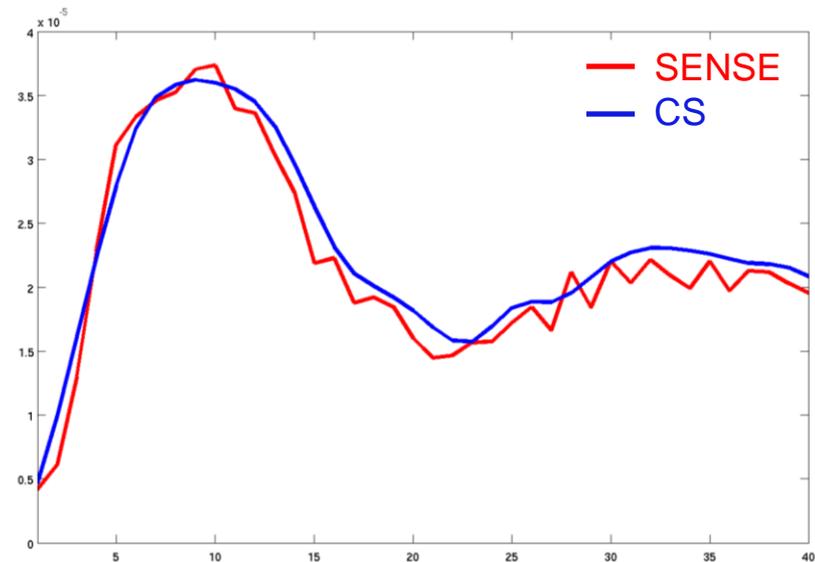


# 3D Cardiac Perfusion

- Imaging of entire left ventricle (FOV = 350 x 350 x 80 mm)
- High spatial resolution: 1.8 x 1.8 x 8 mm
- High temporal resolution: 174 ms
- Acceleration factor: 50 (8 projections per frame)
- Total exam time: 48 s (breath hold 10 s into exam, shallow breathing in the end)



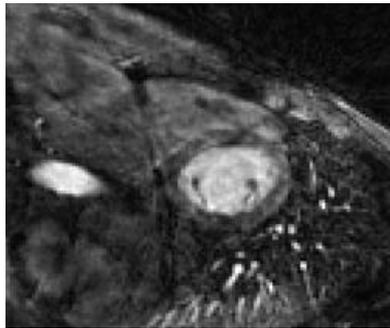
Perfusion measurements in left ventricle



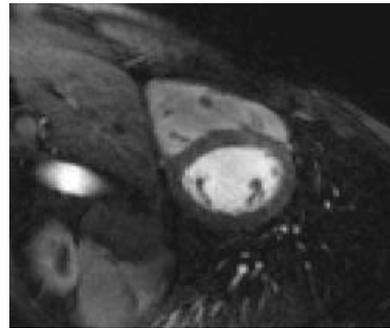
# Why 2<sup>nd</sup> difference?

- Can we constrain 1<sup>st</sup> temporal difference instead?

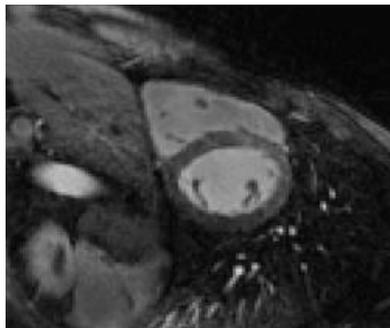
$$\min_{\bar{\mathbf{f}}} \left( \left\| \mathbf{E}\bar{\mathbf{f}} - \bar{\mathbf{s}} \right\|_2^2 + \lambda \left\| \Delta^1 \bar{\mathbf{f}} \right\|_{l_1/l_2} \right)$$



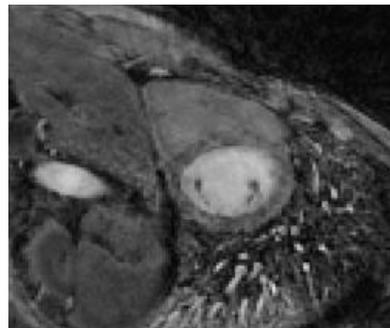
SENSE



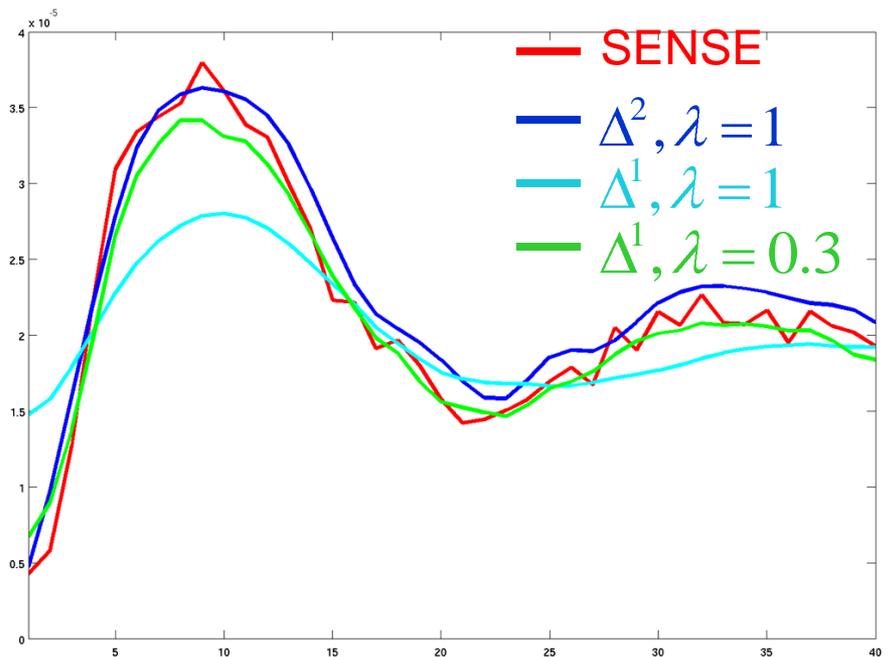
TC CS  $\Delta^2, \lambda = 1$



TC CS  $\Delta^1, \lambda = 1$



TC CS  $\Delta^1, \lambda = 0.3$



# Conclusions

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- Sparsity is necessary for CS but spatial sparsity is usually limited in MRI, allowing only mild acceleration factors
- Sparsity can be achieved by exploiting inter-image dependencies in an image series
- Careful design is needed based on required acceleration and available sparsity
- 2<sup>nd</sup> difference operator in temporal dimension is a novel way to sparsify image series
- The use of 2<sup>nd</sup> difference operator allows acceleration factors 25-85 in contrast-enhanced applications to depict contrast dynamics
- The concept of regularization in temporal (or parametric) dimension was also shown feasible for acceleration of quantitative MRI techniques

# Acknowledgements

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- We thank GE Healthcare for research support
- Thank you for your attention!

