Diffeomorphic Shape Momentum, Computational Anatomy, & Neuroinformatics at 1mm Scale

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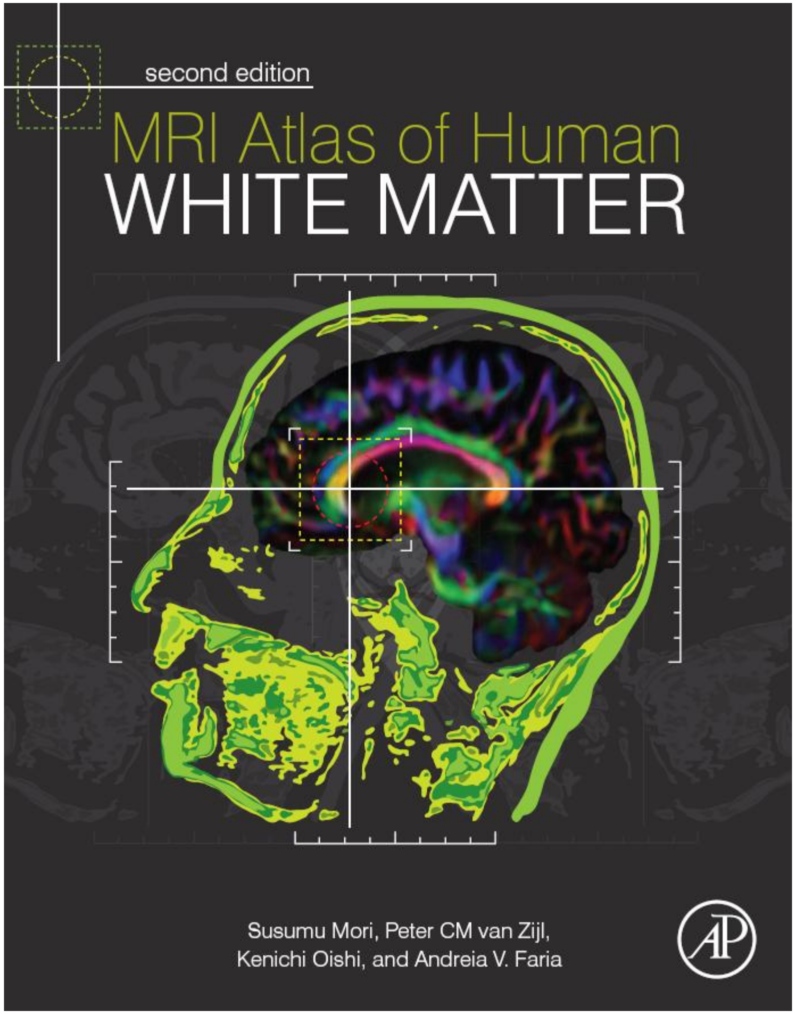
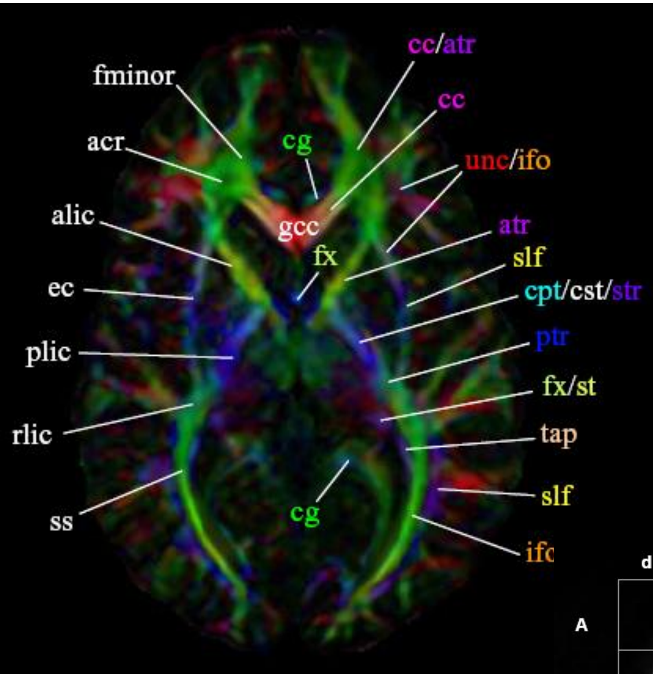
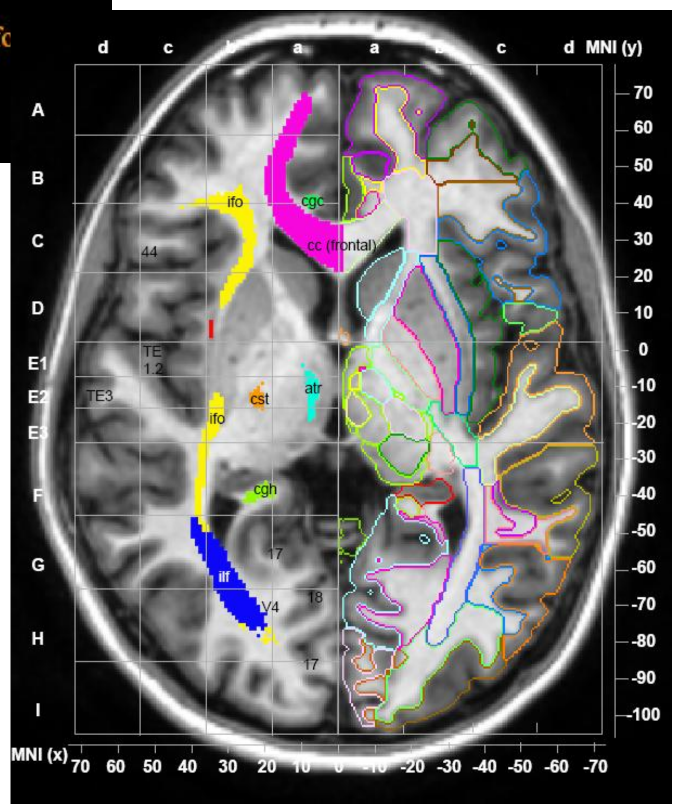
Neuroinformatics at 1mm Scale

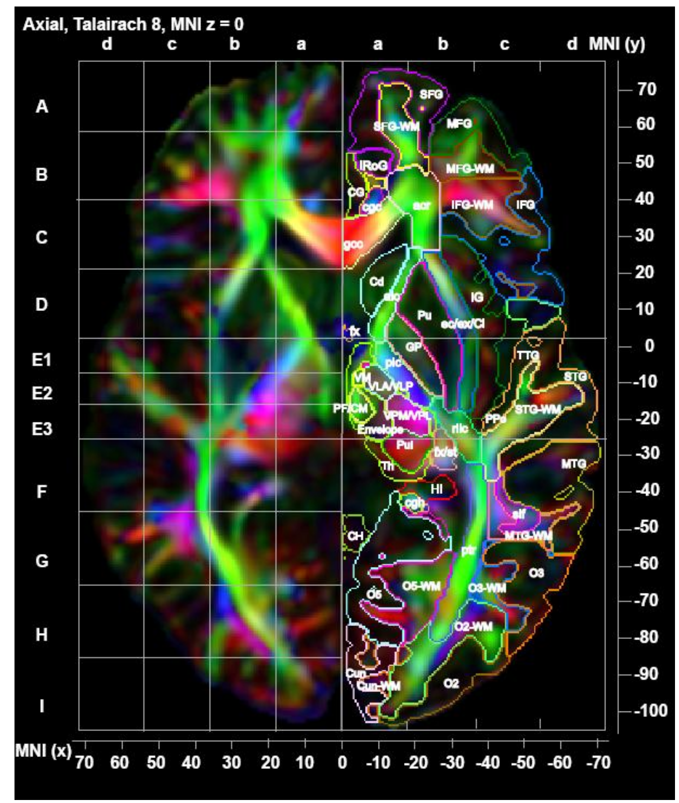




- Subcortical and Cortical Structures
- Stereotaxic (MNI and Talairach)
- Brodmann's maps
- White matter tracts
- fMRI bold response networks
- Spectroscopy

Diffeomorphic Mapping





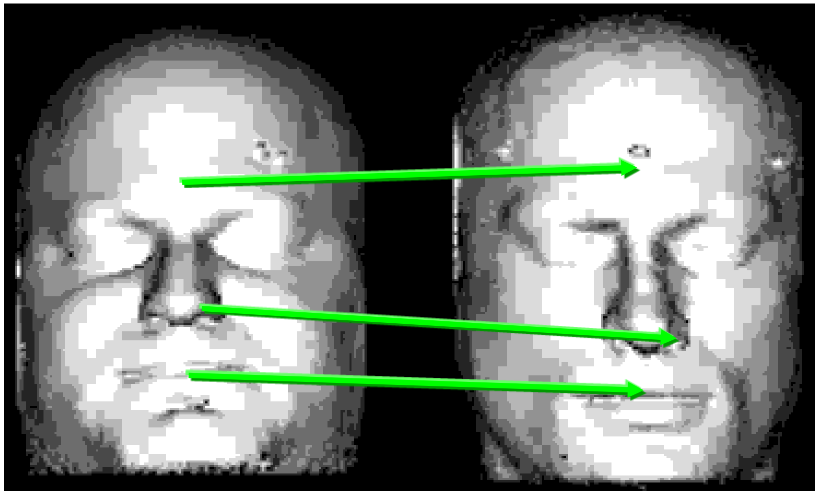
Computational Anatomy models human anatomy as an orbit of exemplars under the "diffeomorphism group".

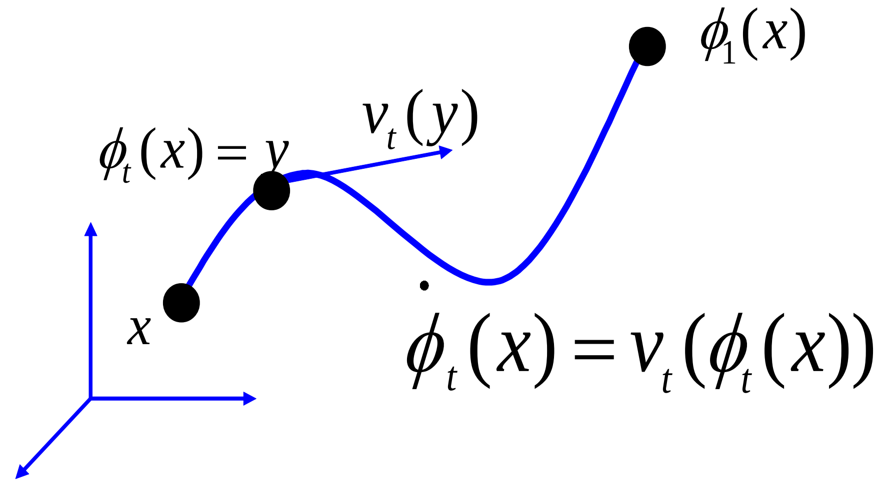
Populations are studied via templates with statistics encoded in template coordinates.

Diffeomorphic correspondences are used to carry the information from population coordinate systems to template coordinates.

Computational Anatomy: An Emerging Discipline, Quart. Applied Math.
Grenander,Miller, 1997

We generate the diffeomorphism group as solutions of the ODE's.





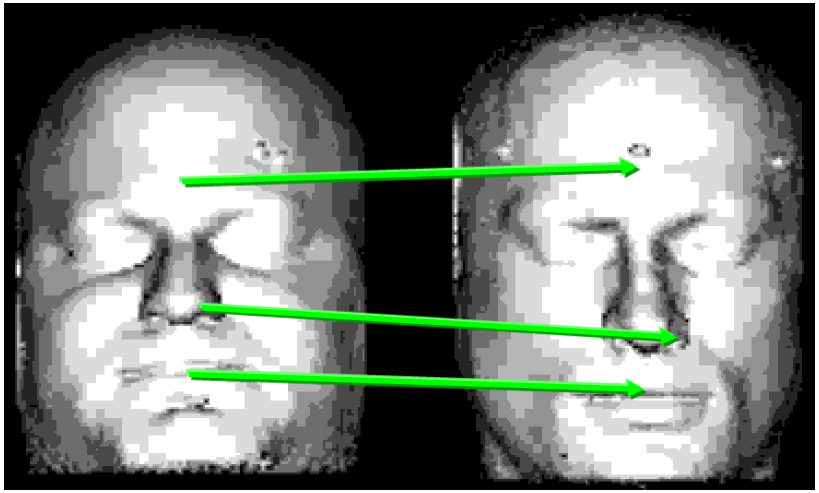
Lagrangian $\dot{\phi}_t(x) = v_t(\phi_t(x)), \phi_0 = id$

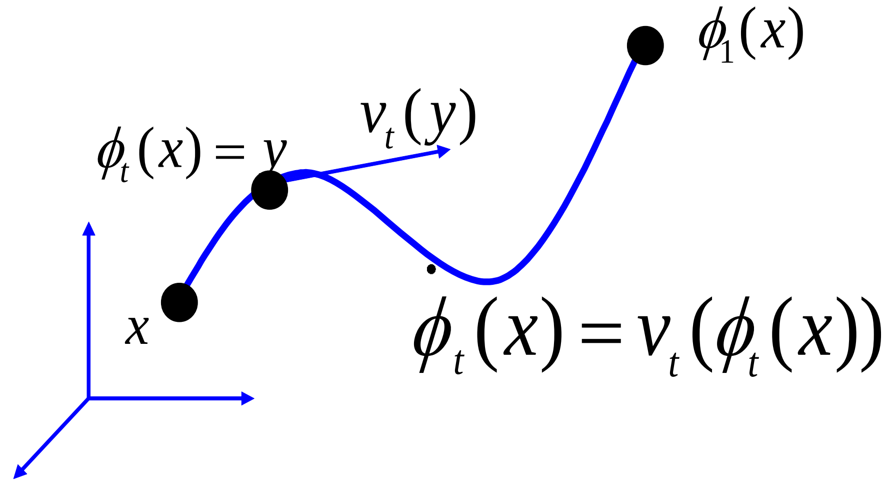
Eulerian $\dot{\phi}_t^{-1}(x) = -D\phi_t^{-1}(x) v_t(x), \phi_0^{-1} = id$

D=Jacobian matrix $\left(\frac{\delta v_i}{\delta x_j} \right)$

Deformable Templates Using Large Deformation Kinematics, IEEE. Trans. on Med. Imaging. Christense, Rabbitt, Miler, 1996.

We require the vector fields to be spatially smooth.





Variational Problem

$$\inf_{v \in V : \dot{\phi} = v(\phi)} \int_0^1 \underbrace{\| v_t \|_V^2}_{\int A v_t^* v_t} \, dt$$

smoothness

$A = \Delta^{2p}, p \geq 1.5 (\mathbb{R}^3)$

subject to

$$\phi_1 \cdot I \sim I'$$

groupaction

Variational Problems on Flows of Diffeomorphisms, Quart. Applied Math. Dupuis Grenander Miller, 1998

Diffeomorphic Shape Momentum Conservation

$$\inf_{\substack{\dot{\phi}=v(\phi) \\ \phi_0=id,\phi_1=\varphi}} \int_0^1 \|v_t\|_V^2 \, dt = \int_0^1 \int_X \underset{M_t}{Av_t^*} v_t dt$$

Force Equation on Geodesics

D=Jacobian matrix $\left(\frac{\delta v_i}{\delta x_j}\right)$

$$\frac{d}{dt} M_t + (Dv_t)^* M_t + (DM_t) v_t + (\mathrm{div}\, v_t) M_t = 0$$

Momentum Conservation : $M = Av$

$$\frac{d}{dt} \int_X M_t^* \, D\phi_t w \Big|_{\phi_t^{-1}} = 0$$

(interpret momentum as a function in action on smooth vector fields w)

$$M_t = | D\phi_t^{-1} | \left(D\phi_t^{-1} \right)^* M_0 \circ \phi_t^{-1}$$

(as a vector function determined by initial condition)

Geodesic Shooting for Computational Anatomy, J. Math. Imaging and Vision Miller, Trouve' Younes 2006

One example: Steering the momentum via dense image matching.

$$\inf_{v \in V : \dot{\phi} = v(\phi)} \int \int_X A v_t^* v_t \, dt \, dM_t + \int_X | I' - \underbrace{I \circ \phi_1^{-1}}_{\phi \cdot I : \text{group action}} |^2 \, dx$$

smooth template
momentum
smooth

Momentum Conservation Law

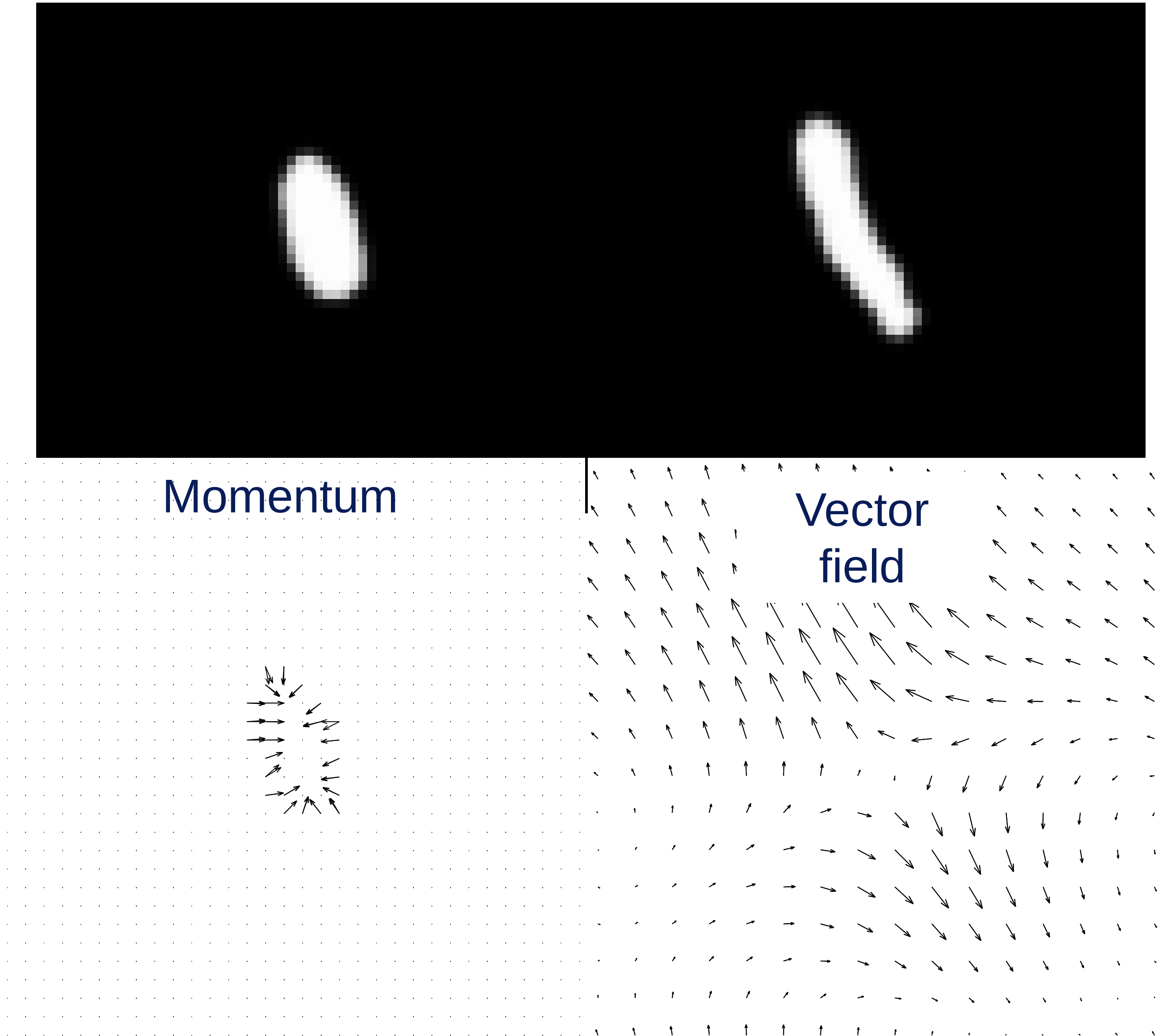
$$M_t = | D\phi_t^{-1} | \left(D\phi_t^{-1} \right)^* M_0 \circ \phi_t^{-1}$$

$$M_t = \nabla \left(I \circ \phi_t^{-1} \right) | D\phi_{t,1} | \left(I' \circ \phi_{t,1} - I \circ \phi_t^{-1} \right)$$

Normal to level lines of image.

Computing LDDMM via Geodesic Flows of Diffeomorphisms Beg, Miller, Trouve' Younes 2005.

The momentum is a highly compressed representation of shape.

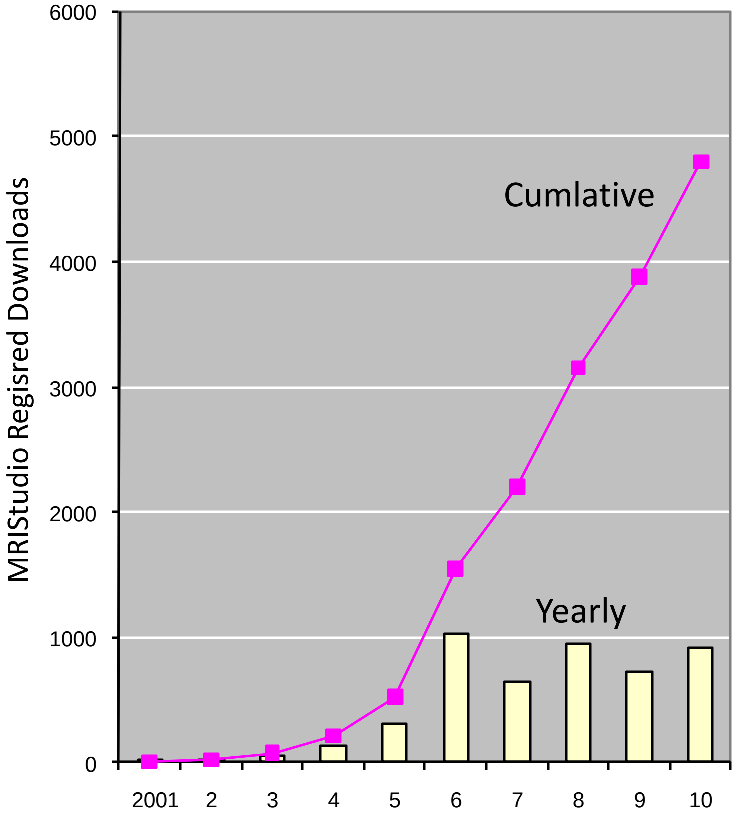


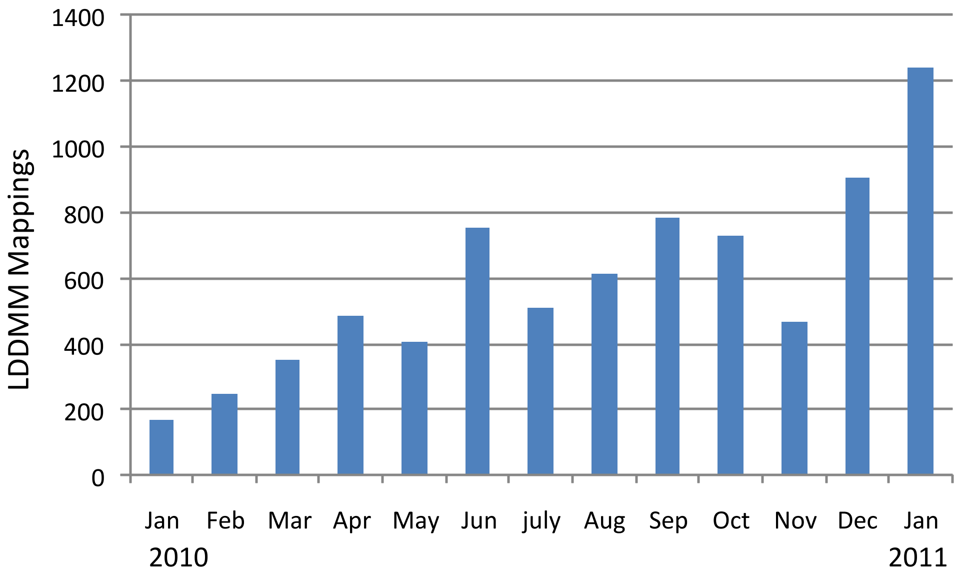
We have many methods of steering anatomical configurations
one onto the other. We call these codes LDDMM.

- pointsets (landmarks, curves, surfaces)
- dense images
- vectors
- tensors

MRIStudio

http://lists.mristudio.org/mailman/listinfo/mristudio-users





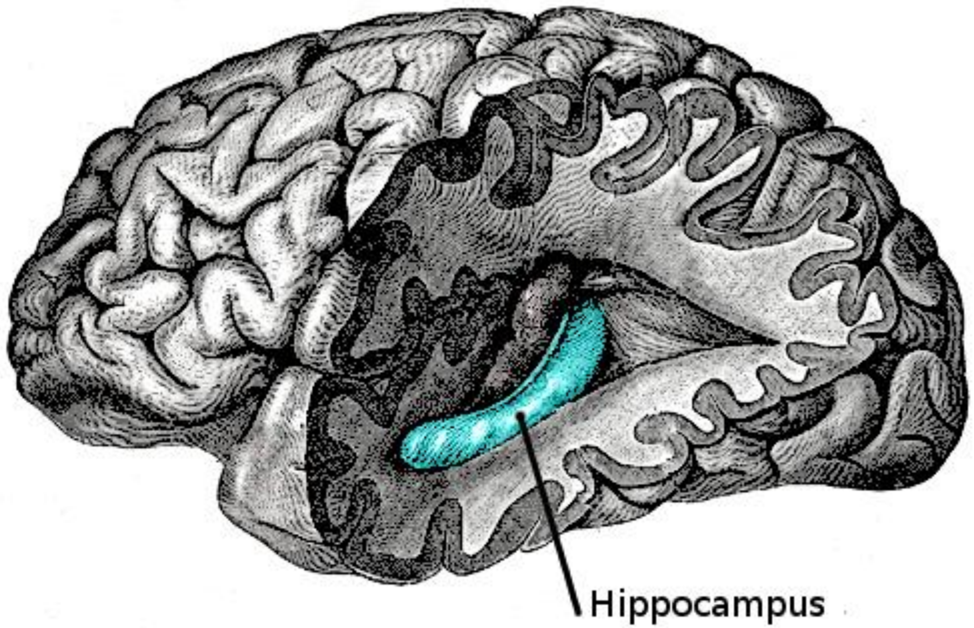
Computational Anatomy of Populations & Statistics on Shape Spaces

An Anatomical Model of Subcortical Human Anatomy
Daniel Tward

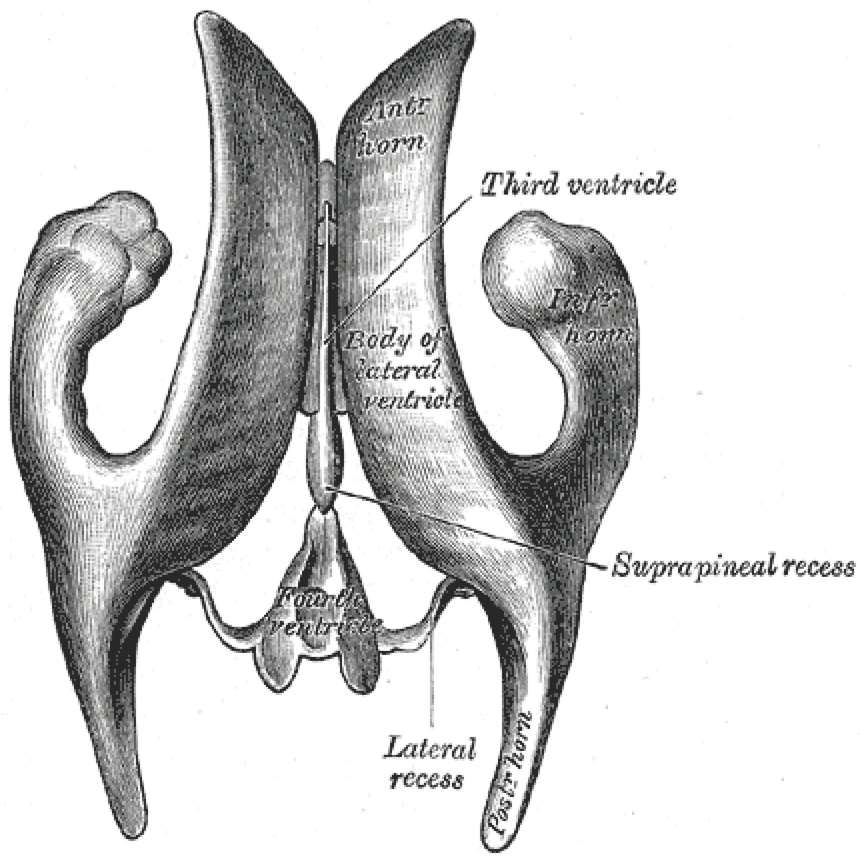
Anatomy at 1mm scale is clumpy. Diffeomorphic Shape Momentum provides a massive data reduction for morphometry and helps with the curse of dimensionality.

Multi-Structure Network Shape Analysis via Normal Surface Momentum Maps, Neuroimage, Qiu, Miller,2008.

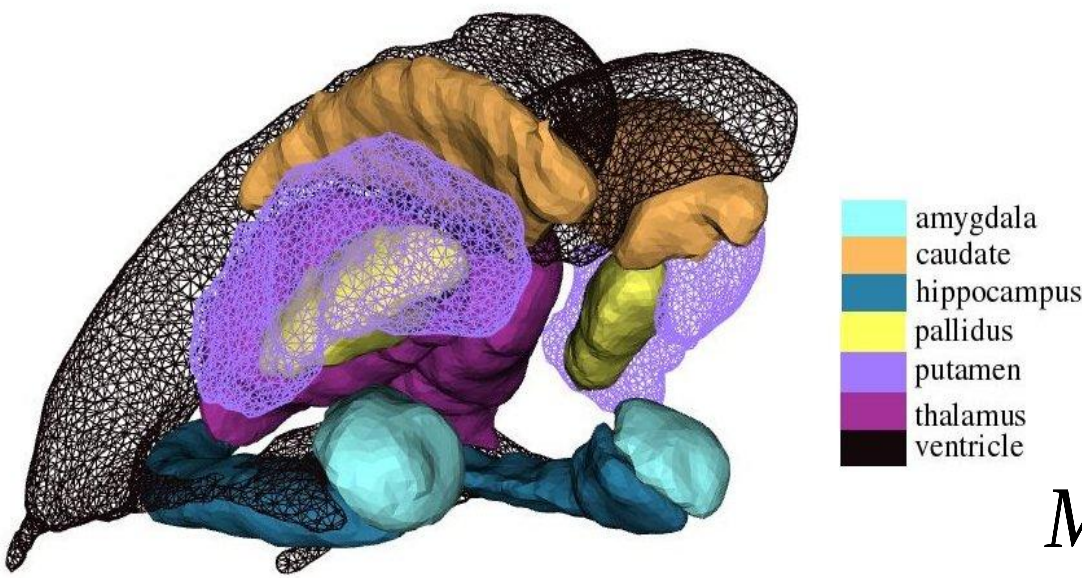
Subcortical Anatomy



Subcortical Anatomy



The Subcortical Random Field Model



$$M_0(dx) = \sum_{S} \underbrace{\alpha(x)}_{\in R^3} \sigma_S(dx)$$

$$V_0(\cdot) = \int_X \underbrace{K(\cdot, x)}_{\text{3x3Green's matrix}} \underbrace{M_0(dx)}_{\text{3x1Vector Measures}}$$

Dimensionality Reduction
1/3 x 3 x 10E7 -> 7 x 2 x 10E3

We use PCA and Surface Harmonics supported on anatomical structures for representing structure and function in curved anatomical coordinates

PCA requires training data.

$$F = \sum_{k} F_k \phi_k$$

structure-function response-variables

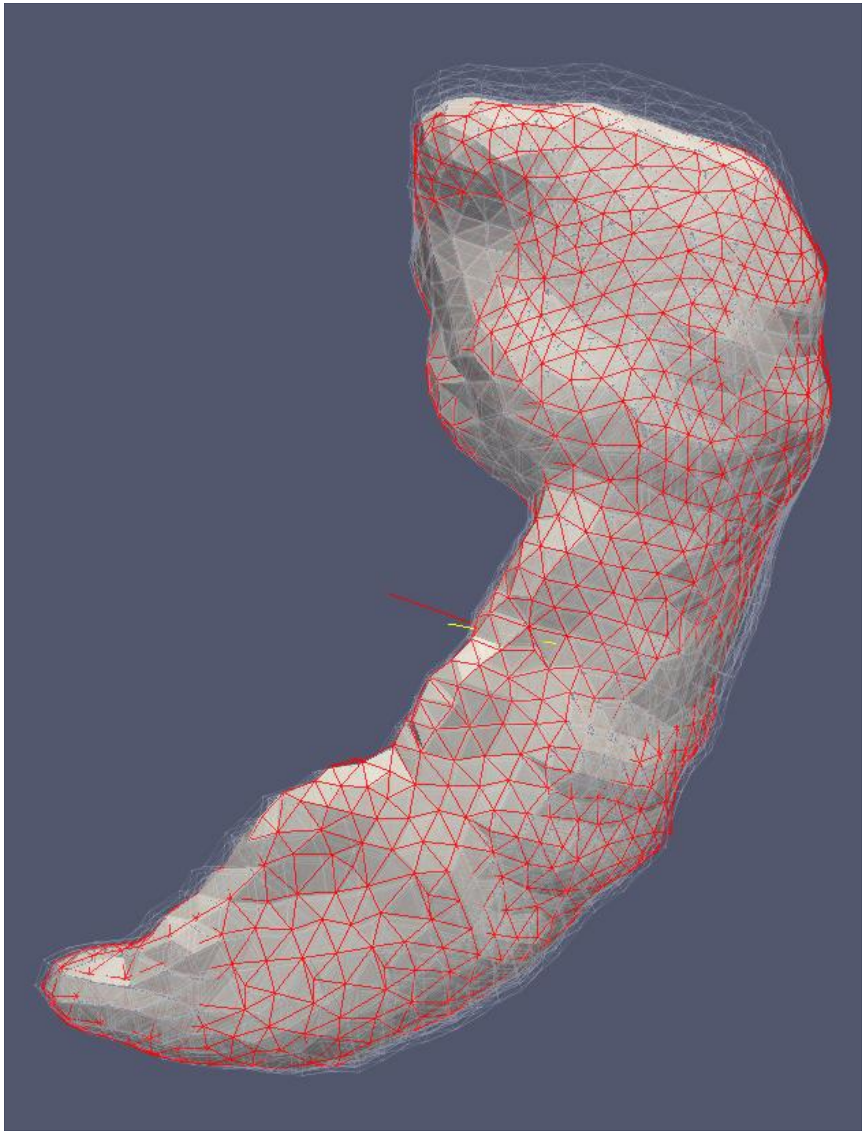
Laplace-Beltrami or PCA Basis

The emerging discipline of Computational Functional Anatomy, Neuroimage, Qiu, Miller,2009.

Representation of Subcortical Neuroanatomy in the Aging Population (Daniel Tward)

- A population of 600 whole brain anatomies which have been segmented (published) into all subcortical structures (ADNI, OASIS).
- A template atlas was generated from the population to which statistics on the Momentum is indexed.
- LDDMM surface mapping calculated the initial momentum carrying the template onto all 600 anatomies with 14 target surfaces.
- PCA analysis on the 600 momentum fields indexed over each of the surfaces taking into account the metric.

1 example: hippocampus template surface mapped to single target



- Surfaces mapped for all 600 left and right structures
- PCA was done on the momentum fields indexed to the template
- Shadow shows the amount of transformation from initial condition for one example.

Thank-You