Wald-type tests when rank conditions fail: a smooth regularization approach

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ABSTRACT

This paper examines Wald-type tests in presence of (possibly) singular covariance matrices. Two different types of singularity are addressed: *first*, the sample matrix has *full rank* but converges to a *singular* covariance matrix; in this case, the Wald statistic is still computable, but usual regularity conditions do not hold anymore, which modifies its asymptotic distribution. This asymptotic singularity causes the rank condition of Andrews (1987) to be violated at the limit due to isolated values of the parameter. *Second*, the sample matrix does not have full rank, but converges to a possibly nonsingular population matrix. This finite sample singularity may be due to redundant restrictions. To address such difficulties, we introduce a class of *regularized* inverses that exploits *total eigenprojection* techniques, [Kato (1966), Tyler (1981)], together with a *variance regularizing function* (VRF) that modifies the small eigenvalues that fall below a certain threshold c so that their inverse is well defined. Under specific regularity conditions, the new regularized inverse converges to its regularized counterpart. This class of regularized inverses nests the spectral cut-off type inverse used by Lutkepohl and Burda (1997), and the Tikhonov-type inverse. We define *three* regularized Wald statistics: the first statistic admits a nonstandard asymptotic distribution, which corresponds to a linear combination of χ^2 variables if the restrictions are Gaussian. An *upper bound* is derived that corresponds to a χ^2 variable with *full rank*. The second regularized statistic relies on a *superconsistent* estimator of the eigenvalues at the threshold c whose distribution can be simulated. The third statistic lets the threshold vary with the sample size leading to the spectral cut-off modified Wald statistic of Lutkepohl and Burda (1997). The regularized statistics are consistent against global alternatives, with a loss of power for the spectral cut-off Wald statistic relative to the other statistics, as illustrated in a simulation exercise.

Key words: Regularized Wald test; Moore-Penrose inverse; spectral cut-off and Tikhonov regularizations; superconsistent estimator.
JEL classification: C1, C13, C12, C32, C15