Credit and Interest **Assignment #3** **March 11, 2010**

Announcement: We will be having the final guest lecturer on Thursday March 25th. Prof. Kay Giesecke will give six hours of lectures that day (9:30-12:15, then 1:30-4:15 pm) on the topic of portfolio credit risk. Please let me know if you will have any difficulty in attending that important event.

This assignment is due in class on Thursday March 25, 2010.

Exercise 1. Course notes Exercise 36.

Exercise 2. Course notes Exercise 37. Choose parameters $c = 2, k = 1, \theta = .3, J = .1, \lambda_0 = 0$, and plot the simulated path over the period $[0, 10]$. Compute the long-term average value of λ.

Exercise 3. Course notes Exercise 43.

Exercise 4. Course notes Exercise 44.

The following two exercises are based on the model described in Exercise 35. We take the following default values of the parameters: $a = 1, c = d = .1, b = 0$, with X, Y both Vasicek processes with $(k_X, \theta_X, \sigma_X) = (.5, 0.05, .05)$ and $(k_Y, \theta_Y, \sigma_Y) = (.1, 0.1, .025)$. In addition, we assume a constant recovery fraction $R = 0.5$.

You will also need to use the corrected Vasicek "master formula" (3.70):

$$
\begin{aligned}
F^{VAS}(T, r; c_1, c_2) &:= E_r[e^{-c_1 \int_0^T r_s ds - c_2 r_T}] = \exp[A(T) + B(T) r] \qquad (1)\\
B(T) &= \frac{c_1}{k}(e^{-kT} - 1) - c_2 e^{-kT}
\end{aligned}
$$

$$
A(T) = \left(\theta - \frac{c_1\sigma^2}{2k^2}\right)[B(T) + c_2] - \frac{\sigma^2(B^2(T) - c_2^2)}{4k} - \frac{c_1}{k^2}\left(\frac{c_1\sigma^2}{2k^2} - \theta\right)\log\left|\frac{B(T) - c_1/k}{-c_2 - c_1/k}\right|
$$

Exercise 5. Suppose a defaultable 20 year coupon bond pays half-yearly coupons at a rate chosen so that the bond trades at par on its issue date. Assuming the recovery of par mechanism, what should the value of this "defaultable par coupon rate" be? Compute the par coupon rate for a similar default free bond. Can you explain the difference? Compute the credit spread for a 20 year zero coupon defaultable bond.

Exercise 6. Compute the defaultable yield curve and credit spread curve for maturities $T \in [0, 30]$ years under the recovery of par method.

Exercise 7. (Construction of continuous time Markov chains) Suppose the probabilities

$$\Pi_{ij}(s, t) = P[Y_t = j | Y_s = i], i, j = 0, \dots, K$$

are well specified (i.e. $\Pi(s, t)$ is a $K+1 \times K+1$ stochastic matrix), and continuous in $s \leq t$. Assume further that $\Pi(s, t) = \Pi(s, u)\Pi(u, t)$ for any $u \in [s, t]$ (and give a probabilistic interpretation of this property) and that for each s the limit

$$\Lambda(s) := \lim_{t \to s+} \frac{\Pi(s, t) - I}{t - s}$$

exists.

1. Show that $\Lambda(s)$ is a "generator" for all s.

2. Show that for each $s \leq t$,

$$\lim_{h \to 0+} \frac{\Pi(s, t+h) - \Pi(s, t)}{h} = \Pi(s, t)\Lambda(t)$$

3. Assuming uniqueness of solutions of this matrix valued ODE, show that if there is a single matrix B such that for all s, $\Lambda(s) = B\mu(s)B^{-1}$ for a diagonal matrix $\mu(s)$, then

$$\Pi(s, t) = Be^{\int_s^t \mu(u)du}B^{-1}.$$

(Hint: differentiate the formula with respect to t and draw a conclusion.)