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Asset Management via Risk-Sensitive Stochastic Control

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Risk-sensitive control maximizes $J_{\mathrm{RS}} = -\frac{1}{\theta}\log \mathbb{E}[e^{-\theta F}]$ for some performance functional F rather than the conventional $J = \mathbb{E}[F]$ of stochastic control. For small $\theta > 0$, $J_{\mathrm{RS}} = J - \frac{\theta}{2}\mathrm{var}[F] + O(\theta^2)$, so risk-sensitive control amounts to maximizing the expectation of F with a penalty for variance. In asset management applications, pioneered by Bielecki and Pliska, $F = \log V$ where V is the portfolio value resulting from some investment strategy, and the $\theta = 0$ solution is the Kelly or growth-optimal portfolio. Risk-sensitive optimization is "dynamic Markowitz" in that it maximizes expected return subject to a constraint on return variance.

This talk discusses risk-sensitive portfolio optimization in a setting of jump-diffusion asset price processes whose growth rates depend on a factor process X_t. Jumps are needed so that we can include credit in the mix of assets, while the components of X_t may represent exogenous economic factors or may be unobserved 'latent variables' introduced to model volatility of expected returns. There are two cases, depending on whether or not X_t also has jumps. When X_t is a diffusion process the the problem reduces, using a change of measure idea introduced by Kuroda and Nagai, to a problem of controlled diffusion whose solution is characterized by the classical $C^{1,2}$ solution of the corresponding HJB equation. Thus a problem in controlled jump-diffusions turns out to be characterized analytically by a PDE with no non-local terms. We can also handle the case of unobserved factor process by a Kalman filtering algorithm (even though the log asset price processes, which are the 'observations' for filtering, contain jumps). When X_t also has jumps the HJB equation has a non-local term and we proceed via viscosity solutions. Ultimately, however, we are able to establish classical $C^{1,2}$ regularity of the value function in this case also, under some restrictions on the jump measure. These results are joint work with Sébastien Lleo.