ON THE WEAK LIPSCHITZIANITY AND DEFINABLE TRIANGULATIONS WITH REGULARITY CONDITIONS

MAŁGORZATA CZAPLA

The main goal of this lecture is to introduce the notion of a weakly Lipschitz mapping on a fixed C^q stratification, discuss its fundamental properties and give some examples. A natural setting for our results is the theory of o-minimal structures on the ordered field of real numbers $\mathbb{R}$. In definable case we have a theorem about existence of a definable, Lipschitz, weakly bi-Lipschitz triangulation of a relatively compact definable set.

We distinguish a class of $\mathcal{WL}$ conditions which are in some sense invariant with respect to definable, locally Lipschitz, weakly bi-Lipschitz homeomorphisms. We also define a class of $\mathfrak{T}$ conditions that involves the $\mathcal{WL}$ conditions with a conical property. In particular, the Whitney (B) condition and the Verdier condition belong to the $\mathfrak{T}$ class. As a final result we have the following triangulation theorem:

Let $\mathcal{Q}$ be a $\mathfrak{T}$ condition of class C^q, $q \in \mathbb{N} \cup \{\infty, \omega\}$. Let $A \subset \mathbb{R}^n$ be a relatively compact, definable set and $A_1, ..., A_r$ be definable subsets of A.

There exists a definable C^q triangulation (K, H) of A, such that the family $\{H(\triangle): \ \triangle \in K\}$ is a definable C^q stratification with the $\mathcal{Q}$ condition of A and is compatible with $A_1, ..., A_r$. Moreover, $H : |K| \longrightarrow A$ is a Lipschitz mapping.