ABSTRACTS 1.2

MICHAEL BATEMAN
Indiana-Bloomington

Maximal averages along one-variable vector fields

We prove L^p estimates for a maximal operator along rectangles pointed in the direction of a vector field in R^2 depending on one variable. These estimates are related to the work of Lacey and Li on the Hilbert transform along vector fields.

VITALY BERGELSON
Ohio State University

10 open problems on positive definite functions

We will formulate and discuss some open problems involving positive definite functions and motivated by applications of ergodic theory to combinatorics.

JIM COLLIANDER
University of Toronto

Weak turbulence for cubic NLS on the two dimensional torus

This talk will motivate and briefly describe the construction of a weakly turbulent solution of the cubic nonlinear Schrodinger equation on the two dimensional torus. In particular, the solution will start off small in a high Sobolev norm and eventually the solution will grow larger than any target threshold in the same norm. This talk reports on joint work with M. Keel, G. Staffilani, H. Takaoka and T. Tao.

CIRPRIAN DEMETER
IAS and Indiana

Bounds for the two dimensional bilinear Hilbert transform

We investigate the bilinear Hilbert transform in the plane. Our analysis, while of independent interest, is also motivated by questions from Ergodic theory. This is joint work with Christoph Thiele.

BURAK ERDOGAN
University of Illinois at Urbana-Champaign

Near-linear evolution for 1D periodic NLS

In a joint work with V. Zharnitsky, we prove that the evolution of 1D cubic NLS on the torus is close to a linear Schrodinger evolution if the Fourier sequence of the initial data has small ℓ^∞ norm and not very large ℓ^1 norm. This result was suggested by the numerical experiments performed by engineers working on fiber optic telecommunication systems.

SUSHRUT GAUTAM
University of California, Los Angeles

A critical-exponent Balian-Low theorem

Given a lattice in $\mathbb{R}^2$ and a function f in $L^2(\mathbb{R})$, the associated Gabor system is a collection of functions obtained by taking modulations and translations of f associated to points in the lattice. The classical Balian-Low theorem is a manifestation of the uncertainty principle in the setting of Gabor systems; it states that if both f and its Fourier transform are in the Sobolev space $H^1(\mathbb{R})$, then the Gabor system associated to f and the integer lattice is not an orthonormal basis (or, more generally, a frame) for $L^2(\mathbb{R})$. We generalize this result by showing that if f is in $H^{p/2}$ and its Fourier transform is in $H^{q/2}$ with p and q conjugate exponents, then the associated Gabor system is not a frame. We accomplish this by proving a variant of the endpoint Sobolev embedding into VMO.

MICHAEL GOLDBERG
Johns Hopkins University

Strichartz estimates in the presence of an oscillating electromagnetic field

We prove Strichartz estimates for the Schrödinger operator

$$H = -\Delta + V(t,x) + i\mathbf{A}(t,x)\cdot\nabla$$

with time-periodic short range potentials in dimensions $n \geq 3$. This is done directly from estimates on the resolvent of the Floquet operator $i\partial_t + H$ rather than using dispersive bounds, as the latter are unknown even in the time-independent case. In typical fashion, we project onto the continuous spectrum of the operator and must assume an absence of resonances. Eigenvalues are permissible at any location within the spectrum, including at threshold energies, provided that the associated eigenfunction decays sufficiently rapidly.

LOUKAS GRAFAKOS
University of Missouri at Columbia

Rough and rougher singular integrals

This talk focuses on properties of rough multilinear singular integrals of Calderón-Zygmund type and other ones that may not have the smoothness and decay to fall under the scheme of a general theory. Such operators include the commutators of Calderón, multilinear multipliers given by characteristic functions of sets, and rough homogeneous singular integrals.

ALLAN GREENLEAF
University of Rochester

Estimates for Fourier integral operators with both singular symbols and folds

Operators which exhibit both "singular Radon transform"- like singularities and degeneracy in the underlying geometry arise in various applications. I will discuss recent work motivated by seismic imaging, where the presence of caustics or conjugate points for the background wave propagator may result in artifacts in the images. The most prevalent type of caustics, folds, produces imaging operators in a class that extends both singular Radon transforms and the folding FIOs of Melrose and Taylor, and estimates for these, which include a loss of derivatives in some cases, can be obtained by an elaboration of the Phong-Stein-Cuccagna decomposition. This is joint work with Raluca Felea and Malabika Pramanik.

DERRICK HART
University of Missouri-Columbia

Sum-Product Theory in Finite Fields

Let A be subset of a finite field. In recent years there has been considerable interest in sums versus products problems, i.e. the fact that either the product set or the sum set of A is large. We use estimates on various operator bounds in finite fields to yield several results of this flavor.

ALEXANDRU IONESCU
University of Wisconsin, Madison

Semilinear Schrodinger flows on hyperbolic spaces: scattering in H^1

I will talk about some recent work (joint with G. Staffilani) on the global well-posedness and scattering theory of semilinear Schrodinger equations on hyperbolic spaces.

NETS KATZ
Indiana, Bloomington

Sums, Products, and other things

We discuss some recent developments in the sum product theory and mention some applications.

DOOWON KOH
University of Missouri-Columbia

Extension theorems for paraboloids in the finite field setting

We study the $L^p - L^r$ boundedness of the extension operators associated with paraboloids in vector spaces over finite fields. We estimate the number of additive quadruples in the subset of the paraboloids. As a result, in higher dimensions, we improve upon the standard Stein-Tomas exponents.The discrete Fourier analytic machinery and Gauss sum estimates make an important role in the proof.

IZABELLA LABA
University of British Columbia

Arithmetic progressions in sets of fractional dimension

Let $E \subset \mathbf{R}$ be a closed set of Hausdorff dimension α. We prove that if α is sufficiently close to 1, and if E supports a probabilistic measure obeying appropriate dimensionality and Fourier decay conditions, then E contains non-trivial 3-term arithmetic progressions.

MICHAEL LACEY
Georgia Tech

The Small Ball Inequality in all Dimensions

The Small Ball Inequality concerns a lower bound on the L norm of sums of Haar functions adapted to rectangles of a fixed volume. The relevant conjecture is improvement of the average case lower bound by an amount that is the square-root log of the volume of the rectangles. We obtain the first non-trivial improvement over the average case bound in dimensions four and higher. The conjecture is known in dimension 2, a result due to Wolfgang Schmidt and Michel Talagrand, with important contributions from Halasz and Temlyakov. Jozef Beck established a prior result in three dimensions, which argument we extend and simplify.

This question is related to (1) Irregularities of Distribution, (2) Probability and (3) Approximation Theory.

XIAOCHUN LI
University of Illinois, Urbana-Champaign

Some problems on bilinear oscillatory integrals along curves

Let P be a polynomial and $T_{P,\beta}$ be defined by

$$T_{P,\beta}(f,g)(x) = \int_{-1}^{1} f(x-t)\, g(x-P(t))\, e^{i|t|^{-\beta}} \frac{dt}{|t|}\,.$$

If $\beta > 1$ and P is a homogeneous polynomial, then $T_{P,\beta}$ maps $L^\infty \times L^2$ to L^2. This is a joint work with D. Fan.

This problem was motivated by Hilbert transform along curves and the bilinear Hilbert transform. One crucial point in the proof is the stability of the critical points of the phase function $a\xi t + b\eta t^2 + f(t)$ for some $a, b \in \mathbb{R}$ and C^∞ function. The proof is also based on a TT^* argument and an uniform estimate of a bilinear oscillatory integral proved by Phong and Stein.

VICTOR LIE
University of California, Los Angeles

The (weak-L^2) Boundedness of The Quadratic Carleson Operator

We prove that the generalized Carleson operator with polynomial phase function of degree two is of weak type (2,2). For this, we introduce a new approach to the time-frequency analysis of the quadratic phase.

NEIL LYALL
University of Georgia, Athens

Polynomial configurations in difference sets

Suppose A is a subset of the integers of positive upper density. We prove a quantitative result on the existence of linearly independent polynomial configurations in the difference set of A. Our approach is to first establish a higher dimensional analogue of a theorem of Sárközy and Furstenberg, and then apply a simple lifting argument. This is joint work with Ákos Magyar.

AKOS MAGYAR
University of Georgia, Athens

A coloring problem for squares

One of the earliest results in Ramsey theory due to Schur, says that if the natural numbers are finitely colored, then there is a monochromatic solution of the equation: $x+y-z=0$. This was generalized by Rado, to equations $a_1x_1+\ldots+a_sx_s=0$, for which there is a subset of the coefficients which adds up to 0. We consider an inhomogeneous version of Rado's equation, when only the squares of the natural numbers are finitely colored, that is the existence of monochromatic solutions $x_1,\ldots,x_s$ to the equation: $a_1x_1^2+\ldots+a_sx_s^2=P(x)$, for a family of integral polynomials P satisfying a natural condition. The proof is inspired by a result of Khalfalah and Szemerédi on monochromatic solutions x_1,x_2 of the equation: $x_1+x_2=x^2$.

VLADIMIR MAZYA
Ohio State University

The talk is based on a new joint work with S.Mayboroda

One of results is boundedness of derivatives of order m-1 of solutions to the Dirichlet problem for the m-harmonic equation in an arbitrary 3-dimensional domain. The question of continuity of the derivatives is studied as well.

DETLEF MULLER
Christian-Albrechts-Universitt zu Kiel

Sharp L^p-estimates for maximal operators for $p>2$, oscillation indices and a Fourier restriction theorem associated to hypersurfaces in $\mathbb{R}$

We study the boundedness problem for maximal operators M associated to smooth hypersurfaces S in 3-dimensional Euclidean space. For $p>2$, we prove that if no affine tangent plane to S passes through the origin and S is analytic, then the associated maximal operator is bounded on $L^p(RR^3)$ if and only if $p>h(S)$, where $h(S)$ denotes the so-called height of the surface S. For non-analytic finite type S we obtain the same statement with the exception of the exponent $p=h(S)$. Our notion of height $h(S)$ is closely related to A. N. Varchenko's notion of height $h(\phi)$ for functions ϕ such that S can be locally represented as the graph of ϕ after a rotation of coordinates.

Several consequences of this result are discussed. In particular we verify a conjecture by E. M. Stein and its generalization by A. Iosevich and E. Sawyer on the connection between the decay rate of the Fourier transform of the surface measure on S and the L^p-boundedness of the associated maximal operator M, and a conjecture by Iosevich and

Sawyer which relates the L^p-boundedness of M to an integrability condition on S for the distance function to tangential hyperplanes, in dimension three.

In particular, we also give essentially sharp uniform estimates for the Fourier transform of the surface measure on S, thus extending a result by V. N. Karpushkin from the analytic to the smooth setting and implicitly verifying a conjecture by V. I. Arnol'd in our context.

As an immediate application, we obtain an $(L^p, L^2(S))$- Fourier restriction theorem for S, which improves on a related result by A. Magyar.

CAMIL MUSCALU
Cornell University

On an interesting multi-parameter structure in harmonic analysis and its connection to the theory of differential equations

We shall present two quite distinct problems coming from the general theory of differential equations and show that their analysis is deeply related to the understanding of some very interesting multi-parameter objects

ALEXANDER NAGEL
University of Wisconsin, Madison

A covering lemma for certain monomial polyhedra

In joint work with Malabika Pramanik, we obtain weak-type estimates for certain maximal averages over monomial polyhedra by establishing a covering lemma. The proof uses ideas from the work of A. Cordoba and R. Fefferman on a geometric proof of the strong maximal theorem.

RICHARD OBERLIN
University of California, Los Angeles

Some estimates for the X-ray transform

Given a suitable function f on $\mathbb{R}^d$ and a line l in $\mathbb{R}^d$, the X-ray transform of f at l is defined to be the integral of f over l. We discuss various mixed-norm estimates for this operator.

CARLOS PEREZ
University of Seville

Sharp weighted end-point estimates for Calderón-Zygmund Singular Integral Operators

In this talk we will present recent results about a sharp weighted weak type $(1,1)$ estimate for any Calderón-Zygmund singular integral operator assuming that the weight satisfy the A_1 condition. This result is related to a problem of Muckenhoupt-Wheeden that we will discuss. We will show that the endpoint result follows by proving first a corresponding sharp weighted L^p estimate both sharp on p and the A_1 constant of the weight. We will end by showing the connection of this result with the A_2 conjecture for Singular Integrals Operators.

This is joint work with Andrei Lerner and Sheldy Ombrosi.

LILLIAN PIERCE
Princeton University

Discrete fractional integral operators

We consider higher dimensional versions of discrete fractional integral operators first investigated by Stein and Wainger. Specifically, we define operators I_λ and J_λ for $0 < \lambda < 1$ by

$$I_\lambda f(n) = \sum_{m \in \mathbb{Z}^k_+} \frac{f(n - |m|^2)}{|m|^{k\lambda}}, \qquad J_\lambda f(n,t) = \sum_{m \in \mathbb{Z}^k_+} \frac{f(n-m, t-|m|^2)}{|m|^{k\lambda}};$$

here I_λ acts on functions of $\mathbb{Z}$, J_λ on functions of $\mathbb{Z}^{k+1}$. Our interest is in proving the boundedness of these operators from ℓ^p to ℓ^q for appropriate p, q, λ. We show how this can be done using complex interpolation and ideas originating from the circle method in number theory; furthermore we consider the case where $|m|^2$ is replaced by a more general quadratic form.

JILL PIPHER
Brown University

Some remarks on absolute continuity of elliptic measure

We recall some techniques to prove mutual absolute continuity of the measure of second order elliptic operators (in divergence or nondivergence form), in the absence of L^2 identities. We give some applications, and discuss some recent connections to boundary value problems in BMO.

KEITH ROGERS
Universidad Autonoma de Madrid

Mixed-norm estimates for the free Schrödinger equation

We consider when the Schrödinger operator $e^{it\Delta}$ is bounded from $\dot{H}^s(\mathbb{R}^n)$ to $L^q_x(\mathbb{R}^n, L^r_t(\mathbb{R}))$. When $q > r$, the Sobolev index s can be negative. For $n \geq 5$, we find the sharp range of such estimates up to endpoints. When $q < r$, we prove that the sharp estimates would follow if the maximal operator $\sup_{0<t<1} |e^{it\Delta} f|$ were bounded from $H^{1/4}(\mathbb{R}^n)$ to $L^2_{loc}(\mathbb{R}^n)$.

ERIC SAWYER
McMaster University

Nehari theorems for the Dirichlet space

We discuss boundedness of two different Hankel operators associated to a given symbol on the Dirichlet space. This is joint work with Richard Rochberg and Brett Wick.

RAANAN SCHUL
University of California, Los Angeles

Towards uniform rectifiability in a general metric space

There is a very well developed theory of uniform rectifiability in R^n (due to David, Semmes, Jones and others). There are now several results of similar nature for metric spaces. We will recall/explain several Euclidean results and then go on to their counterparts for a general metric space. We will also discuss some parts of the Euclidean theory we have been unable (thus far) to transfer to the category of metric spaces and explain what the obstacles seem to be.

ANDREAS SEEGER
University of Wisconsin, Madison

Radial Fourier multipliers and a local smoothing inequality

We shall discuss recent work on two problems on radial FL^p multipliers, for suitable $p < 2$, and give some partial answers.

Consider the convolution operator T_m defined via the Fourier transform by

$$\widehat{T_m f}(\xi) = m(|\xi|)\widehat{f}(\xi).$$

I. Can one characterize those m for which $\|T_m f\|_p \lesssim \|f\|_p$ holds for all *radial* functions in $L^p(\mathbb{R}^d)$?

The results are joint work with **G. Garrigós**.

II. Can one characterize those m for which $\|T_m f\|_p \lesssim \|f\|_p$ holds for *all* functions in $L^p(\mathbb{R}^d)$?

Related to this question is the local smoothing problem for the wave equation as formulated by Sogge: Does the inequality

$$\Big(\int_0^1 \|e^{it\sqrt{-\Delta}} f\|_q^q \, dt \Big)^{1/q} \lesssim \|f\|_{L^q_\beta}$$

hold for $\beta = (d-1)(1/2 - 1/q) - 1/q$, for suitable $q \gg 2$? Here L^q_β is the usual Sobolev space.

The results are joint work with **F. Nazarov**.

HART SMITH
Washington, Seattle

L^p bounds for eigenfunctions for Lipschitz metrics

I will discuss recent joint work with H. Koch and D. Tataru towards establishing L^p bounds on spectrally localized functions on compact manifolds with low-regularity metrics. It is known by examples that the spectral cluster estimates of Sogge fail for metrics that are less regular than C^2. In our work we obtain estimates for the case of Lipschitz metrics which, while likely not sharp, improve substantially on known bounds.

ALEXANDER STOKOLOS
DePaul University

On the Tauberian condition for geometric maximal operators

It is shown that if a maximal operator associated with a homothecy invariant collection of convex sets R^n satisfies Cordoba-Fefferman Tauberian condition at some fixed level, then it must satisfy the same condition at all levels and moreover the maximal operator is $L^p - bounded$ for sufficiently large p. As a corollary of these results it is shown that any density basis that is a homothecy invariant collection of convex sets in R^n must differentiate integrals of the functions from L^p for sufficiently large p. This is a joint result with Paul Hagelstein.

RODOLFO H. TORRES
University of Kansas

New maximal functions and commutator and weighted estimates for the multlinear Caldern-Zygmund theory

We will present recent results about a multi(sub)linear maximal operator smaller that the m-fold product of the Hardy-Littlewood maximal function. This operator can be used to obtain a precise control on multilinear singular integral operators of Caldern-Zygmund type. This allows us to build a theory of weights intrinsically adapted to multilinear operators. Also, a (log) variant of the operator can be employed to study certain commutators of multilinear Calderon-Zygmund operators with BMO functions. As a consequence we obtain the optimal range of strong type estimates, a sharp end-point estimate, and weighted norm inequalities involving both the clas- sical Muckenhoupt weights and the new multilinear ones for these commutators too.

This is joint work with A. Lerner, S. Ombrosi, C. Perez and R. Trujillo-Gonzalez.

ANA VARGAS
Universidad Autnoma de Madrid

Null form estimates for the wave equation

This is a joint work with Sanghyuk Lee and Keith Rogers.

Null form estimates for the wave equation have been considered by several authors, as Foschi, Klainerman, Machedon, Selberg, Tao, Tataru. They are inequalities of the form

$$\begin{aligned} \|D_0^{\beta_0} D_+^{\beta_+} D_-^{\beta_-}(\phi\psi)\|_{L_t^q L_x^r} \le C(\|\phi(0)\|_{\dot H^{\alpha_1}} + \|\phi_t'(0)\|_{\dot H^{\alpha_1 - 1}}) \\ \times (\|\psi(0)\|_{\dot H^{\alpha_2}} + \|\psi_t'(0)\|_{\dot H^{\alpha_2 - 1}}), \end{aligned} \tag{1}$$

where ϕ and ψ are solutions of the wave equation. We obtain sharp estimates in dimension $n \ge 3$, except for the endpoints.

ALEXANDER VOLBERG
Michigan State University, East Lansing

Electrostatic field with finitely many charges: the sharp estimates of the size of the level sets

There are many interesting problems about the electrostatic potential of finitely many charges. We (Eiderman, Nazarov and myself) consider one of them concerning the magnitude of the field. We want to give a sharp estimate of the size of the set of points where this field is large. Of course we want the estimate to be sharp in number N of charges. The size will be measured by the Hausdorff content with various gauge functions. Such a setting allows us to consider a wide class of measures (not necessarily with finitely many charges). The main technique will be Calderón-Zygmund capacities and nonhomogeneous Calderón-Zygmund operators. We establish a relationship between various types of capacities with singular kernels (e. g. analytic capacity, lipschitz harmonic capacity, etc) and non-linear capacity from the theory of potential á la Adams, Hedberg, Havin, Maz'ya, Wolff. "Capacitary" part of the talk extends the theorem of Mateu, Prat and Verdera [J. reine und angew. Math., 578 (2005), 201–223]. "Size estimates" part of the talk extends the theorem of M. Anderson and V. Eiderman [Annals of Math., 163 (2005), 1057–1076]. The difficulty lies in the fact that we cannot use Menger's curvature anymore because we are working in spaces of dimension bigger than two.

XIANGJIN XU
Binghamton University

Upper and lower bounds for normal derivatives of spectral clusters of Dirichlet Laplacian

In this talk, we study the upper bounds and lower bounds for normal derivatives of the spectral clusters of Dirichlet Laplacian, which generalizes a result of A. Hassell and T. Tao for single Dirichlet eigenfunction.