Inverse problems and hyperbolic manifolds

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I am going to talk about recent results on inverse spectral problems related with hyperbolic manifolds.

[1] *Equivalence of inverse boundary value problems in euclidean and hyperbolic spaces.* The inverse boundary value problem for the Schrödinger operator in $\mathbf{R}^n$ is equivalent to that in $\mathbf{H}^n$. Moreover the inverse boundary value problem and the inverse scattering problem are equivalent in $\mathbf{H}^n$. Using this fact, one can solve the inverse spectral problems by utilizing spectral properties of Laplacians on hyperbolic spaces.

[2] *Hyperbolic geometry and local DN map.* Consider the equation of conductivity $\nabla \cdot (\gamma \nabla u) = 0$ in a bounded domain $\Omega \subset \mathbf{R}^3$. Let Λ_γ be the associated Dirichlet-Neumann map, $\Lambda_\gamma : u|_{\partial\Omega} \to \gamma \partial u/\partial n|_{\partial\Omega}$, n being the unit outer normal to $\partial\Omega$. Take x_0 from the boundary of the convex hull of Ω, and let $B(x_0, R) = \{y; |y - x_0| < R\}$. If $\Lambda_{\gamma_1} = \Lambda_{\gamma_2}$ on $\partial\Omega \cap B(x_0, R)$ for some $R > 0$, then one can show that $\gamma_1 = \gamma_2$ on $\Omega \cap B(x_0, R)$. This means that the local knowledge of the DN map determines γ locally. This theorem has important applications in practical problems and is proved by using isometries of Möbius transformations in $\mathbf{H}^3$ (a joint work with G. Uhlmann).

[3] *Local conformal deformations of hyperbolic metric.* Take a bounded contractible open set in any hyperbolic manifolds. Then one can solve the associated inverse boundary value problem. If the manifold is non- compact, one can introduce some spectral data at infinity of this manifold to reconstruct the local conformal deformation of the hyperbolic metric. As an example, we can deal with the inverse scattering at the cusp neighborhood at infinity.