

G. DEBS**Universite Paris VI***Undecidable Regularity Properties for Borel Sets*

If \mathcal{P} is a hereditary property of pointsets and Γ is a descriptive class let $\mathcal{P}(\Gamma)$ denote the following “inner regularity” statement: “ $\forall A \in \Gamma, ((\mathcal{P}(K), \forall K \text{ compact } \subset A) \Rightarrow \mathcal{P}(A))$ ”.

We exhibit natural Σ_2^1 hereditary properties \mathcal{P} such that $\mathcal{P}(\Delta_1^1) \Leftrightarrow (\forall \alpha \in \omega^\omega, \aleph_1^{L(\alpha)} < \aleph_1)$. More precisely for any recursive ordinal ξ there exist properties \mathcal{P}_ξ such that $\mathcal{P}_\xi(\Pi_{1+\xi+1}^0) \Leftrightarrow \aleph_\xi^L < \aleph_1$, when ξ is a successor ordinal, and a slightly different but similar equivalence when ξ is limit.