THE FIELDS INSTITUTE
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ABSTRACTS 1.2

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A Comparison of the Rate of Convergence, Efficiency and Condition Number of Chebyshev and Legendre Polynomial Series with Prolate Spheroidal, Kosloff/Tal-Ezer and Theta-Mapped Fourier Basis Sets

Chebyshev and Legendre polynomials are not the optimum spectral basis sets for most functions. A braod class of non-polynomial basis functions, here dubbed "pseudo-Fourier non-periodic" (PFNP) functions, are superior in both accuracy and condition number. Through asymptotic analysis and numerical experiments, we compare Chebyshev and Legendre polynomials with three PFNPs: prolate spheroidal wavefunctions, Kosloff/Tal-Ezer functions and the theta-mapped cosine functions, which are introduced here. All PFNPs can be written as $\cos(n\tau(x))$ where τ is *linear* over most of the expansion interval, $x \in [-1, 1]$, so that the basis functions are a Fourier basis over most of the domain. There is, however, a boundary layer where $\tau(x)$ curls up to a square root at $x = \pm 1$. We explain the theorems that show that these singularities are the minimum necessary to defeat Gibbs' Phenomenon, which wrecks the convergence of a pure Fourier series (i. e., τ linear *everywhere*.) We also explain the advantages of the theta-mapped Fourier basis (simplicity, lack of spurious singularities) over the prolate spheroidal and Kosloff/Tal-Ezer bases. All PNFPs contain an adjustable parameter. With an optimum value of this parameter, the PFNPs are able to resolve a typical function with about $(2/\pi)$ fewer unknowns than with a Chebyshev or Legendre series.