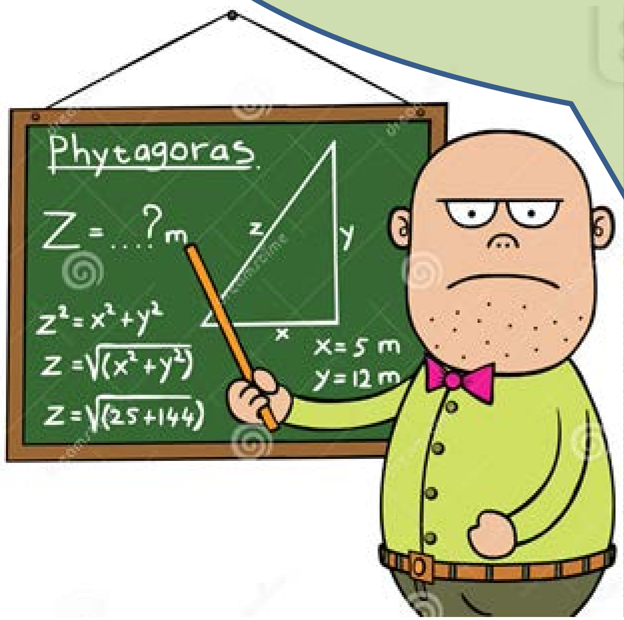
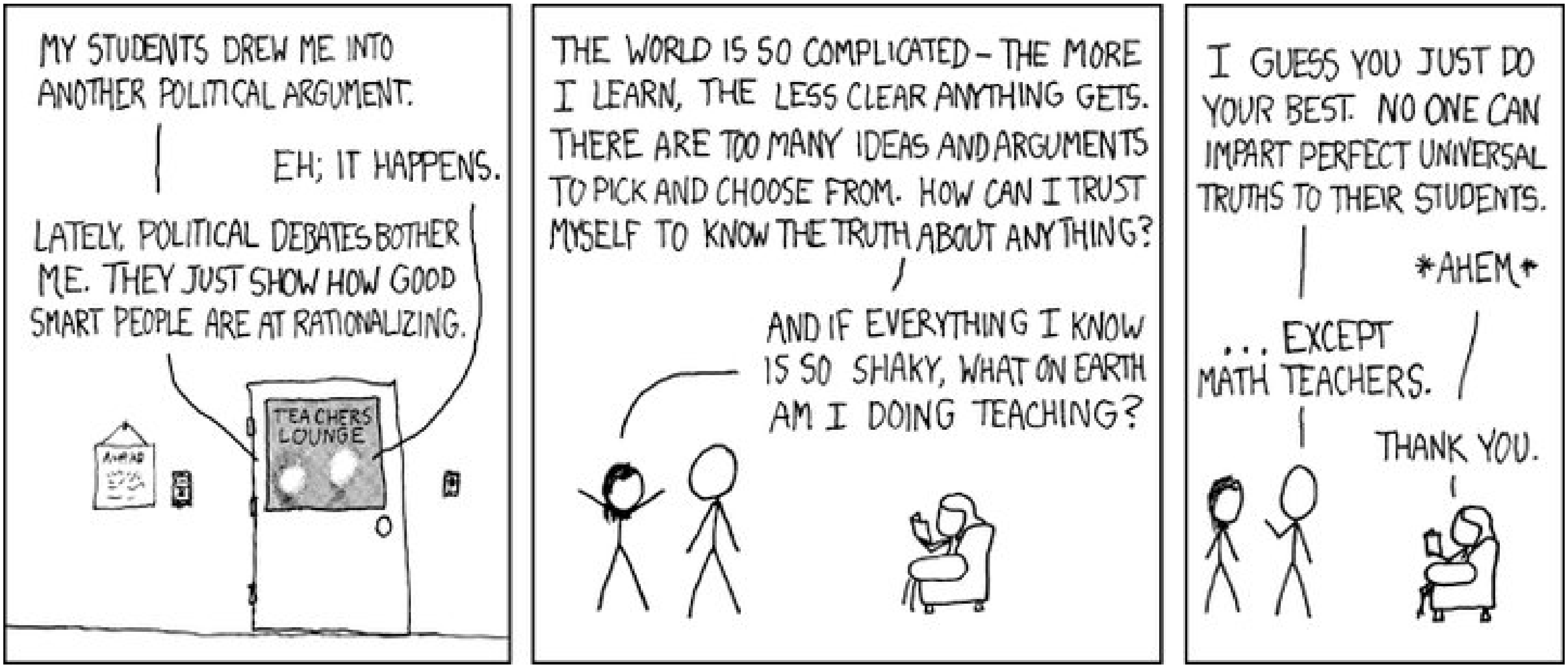
“I can see how it can work in other disciplines but not in a math class”

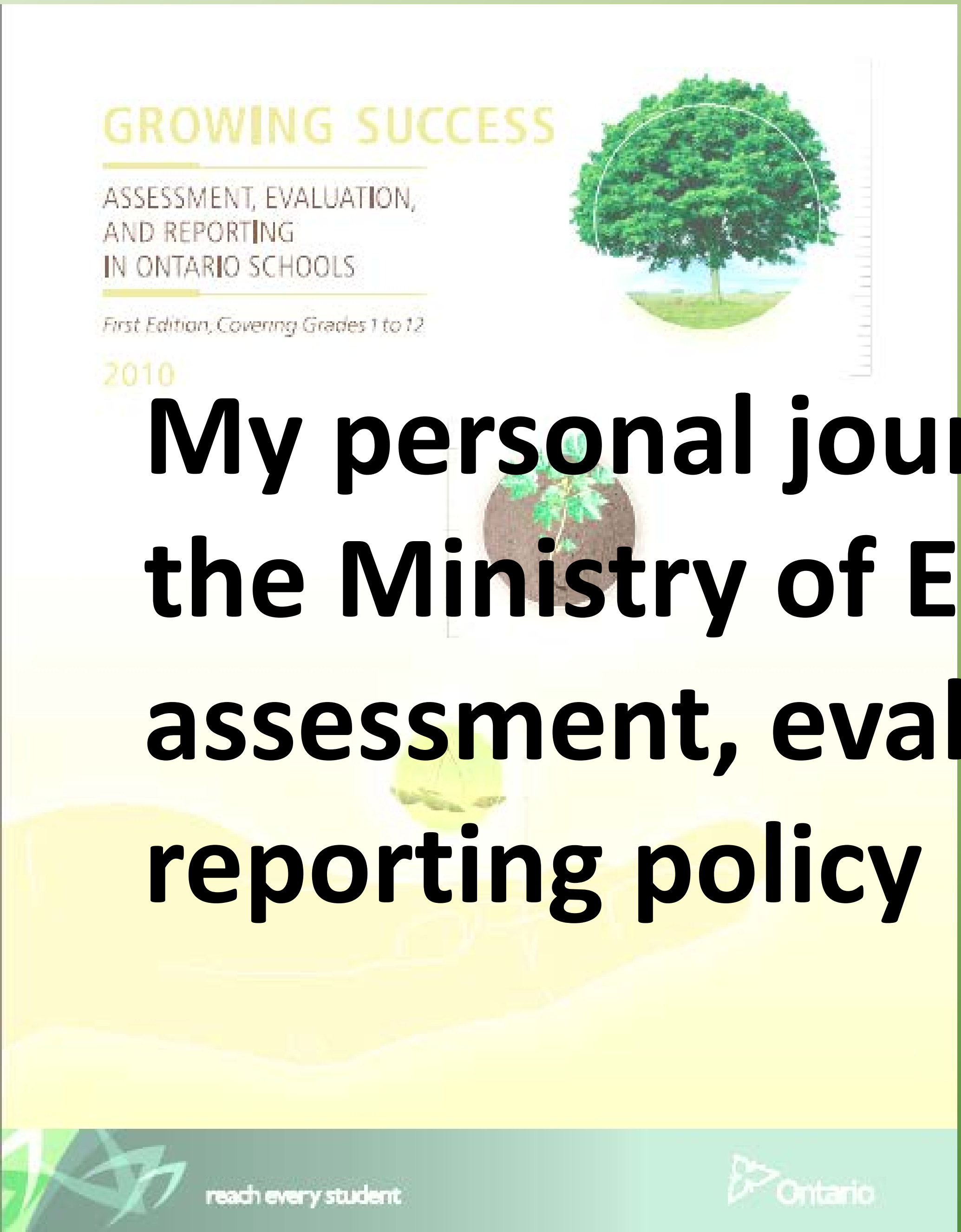


Presentation to the Fields MathEd Forum – April 26, 2014

by Judy Mendaglio
judy.mendaglio@gmail.com

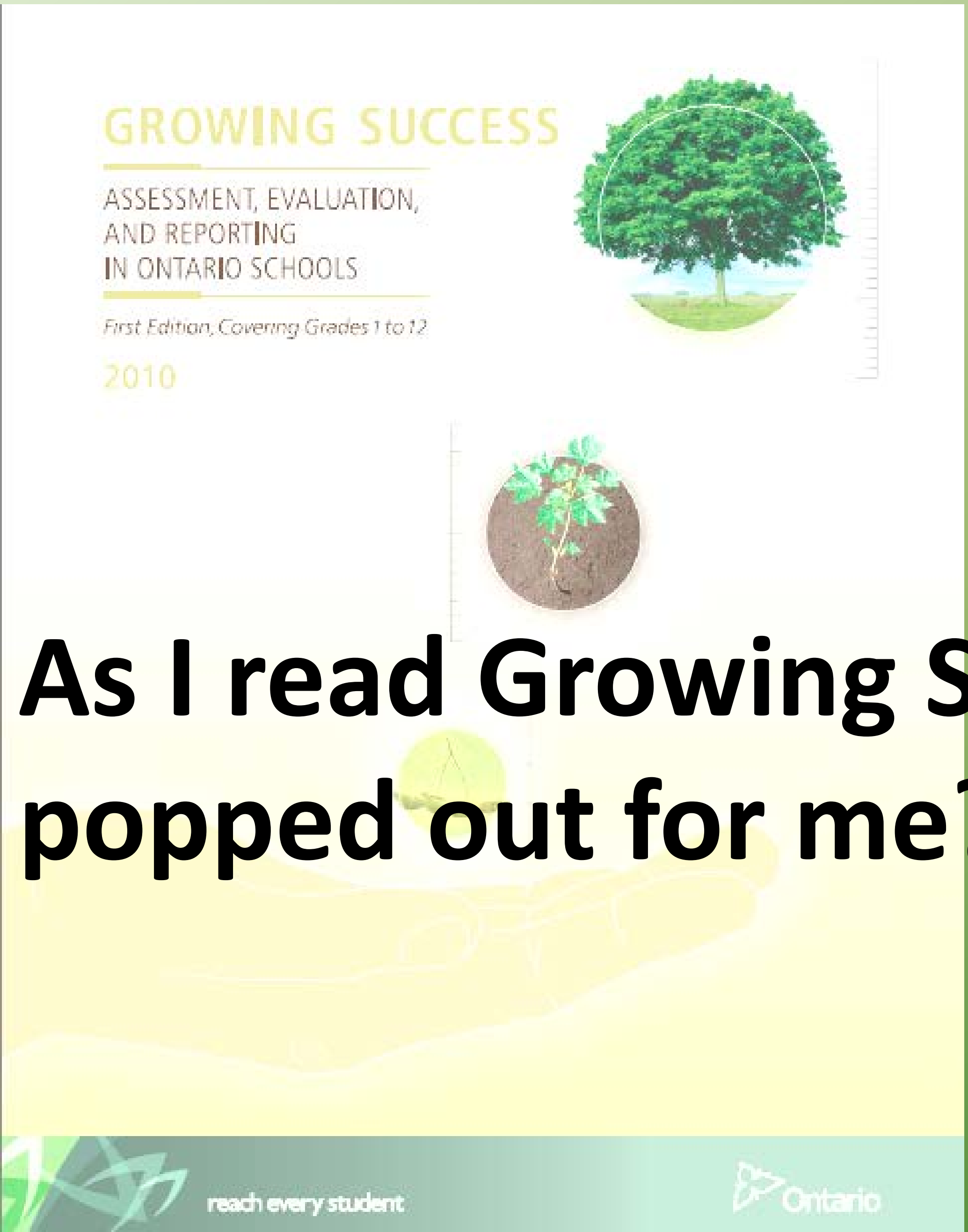

MY STUDENTS DREW ME INTO ANOTHER POLITICAL ARGUMENT.
EH; IT HAPPENS.
LATELY, POLITICAL DEBATES BOTHER ME. THEY JUST SHOW HOW GOOD SMART PEOPLE ARE AT RATIONALIZING.
TEACHERS LOUNGE
THE WORLD IS SO COMPLICATED – THE MORE I LEARN, THE LESS CLEAR ANYTHING GETS. THERE ARE TOO MANY IDEAS AND ARGUMENTS TO PICK AND CHOOSE FROM. HOW CAN I TRUST MYSELF TO KNOW THE TRUTH ABOUT ANYTHING?
AND IF EVERYTHING I KNOW IS SO SHAKY, WHAT ON EARTH AM I DOING TEACHING?
I GUESS YOU JUST DO YOUR BEST. NO ONE CAN IMPART PERFECT UNIVERSAL TRUTHS TO THEIR STUDENTS.
AHEM
... EXCEPT MATH TEACHERS.
THANK YOU.

http://www.edu.gov.on.ca/eng/policyfunding/growsuccess.pdf



My personal journey with the Ministry of Education's assessment, evaluation, and reporting policy

Guiding question: In a "traditional math instruction" model – socratic or lecture based lesson with a chapter quiz/test assessment and evaluation tool – is it possible to meet the policy demands of Growing Success? If not, then what?



As I read Growing Success, what popped out for me?

The Seven Fundamental Principles

To ensure that assessment, evaluation, and reporting are valid and reliable, and that they lead to the improvement of learning for all students, teachers use practices and procedures that:

- are fair, transparent, and equitable for all students;

1

The Seven Fundamental Principles

To ensure that assessment, evaluation, and reporting are valid and reliable, and that they lead to the improvement of learning for all students, teachers use practices and procedures that:

- support all students, including those with special education needs, those who are learning the language of instruction (English or French), and those who are First Nation, Métis, or Inuit;

2

The Seven Fundamental Principles

To ensure that assessment, evaluation, and reporting are valid and reliable, and that they lead to the improvement of learning for all students, teachers use practices and procedures that:

- are carefully planned to relate to the curriculum expectations and learning goals and,

as much as possible,

to the interests, learning styles and preferences, needs, and experiences of all students;

3

The Seven Fundamental Principles

To ensure that assessment, evaluation, and reporting are valid and reliable, and that they lead to the improvement of learning for all students, teachers use practices and procedures that:

- are communicated clearly to students and parents at the beginning of the school year or course and at other appropriate points throughout the school year or course;

4

The Seven Fundamental Principles

To ensure that assessment, evaluation, and reporting are valid and reliable, and that they lead to the improvement of learning for all students, teachers use practices and procedures that:

- are ongoing, varied in nature, and administered over a period of time to provide multiple opportunities for students to demonstrate the full range of their learning;

5

The Seven Fundamental Principles

To ensure that assessment, evaluation, and reporting are valid and reliable, and that they lead to the improvement of learning for all students, teachers use practices and procedures that:

- provide ongoing descriptive feedback that is clear, specific, meaningful, and timely to support improved learning and achievement;

6

The Seven Fundamental Principles

To ensure that assessment, evaluation, and reporting are valid and reliable, and that they lead to the improvement of learning for all students, teachers use practices and procedures that:

- develop students' self-assessment skills to enable them to assess their own learning, set specific goals, and plan next steps for their learning.

7

OUTLINING THE CHALLLENGES

Teachers' professional judgements are at the heart of effective assessment, evaluation, and reporting of student achievement.

Learning Skills different from Demonstrated Achievement

- Responsibility
- Organization
- Independent Work
- Collaboration
- Initiative
- Self -regulation

Learning Skills and Work Habits	Sample Behaviours
Responsibility	The student: • fulfils responsibilities and commitments within the learning environment; • completes and submits class work, homework, and assignments according to agreed-upon timelines; • takes responsibility for and manages own behaviour.
Organization	The student: • devises and follows a plan and process for completing work and tasks; • establishes priorities and manages time to complete tasks and achieve goals; • identifies, gathers, evaluates, and uses information, technology, and resources to complete tasks.
Independent Work	The student: • independently monitors, assesses, and revises plans to complete tasks and meet goals; • uses class time appropriately to complete tasks; • follows instructions with minimal supervision.
Collaboration	The student: • accepts various roles and an equitable share of work in a group; • responds positively to the ideas, opinions, values, and traditions of others; • builds healthy peer-to-peer relationships through personal and media-assisted interactions; • works with others to resolve conflicts and build consensus to achieve group goals; • shares information, resources, and expertise and promotes critical thinking to solve problems and make decisions.
Initiative	The student: • looks for and acts on new ideas and opportunities for learning; • demonstrates the capacity for innovation and a willingness to take risks; • demonstrates curiosity and interest in learning; • approaches new tasks with a positive attitude; • recognizes and advocates appropriately for the rights of self and others.
Self-regulation	The student: • sets own individual goals and monitors progress towards achieving them; • seeks clarification or assistance when needed; • assesses and reflects critically on own strengths, needs, and interests; • identifies learning opportunities, choices, and strategies to meet personal needs and achieve goals;

Learning Skills different from Demonstrated Achievement means changing how we respond to behaviour concerns because they are not tied directly to marks. Ex:

Handing in work late

Skipping classes

Homework not done

Missed quizzes and tests

PERFORMANCE STANDARDS – THE ACHIEVEMENT CHART

- **Knowledge and Understanding:** Subject-specific content acquired in each grade/course (knowledge), and the comprehension of its meaning and significance (understanding)

- **Thinking:** The use of critical and creative thinking skills and/or processes

- **Communication:** The conveying of meaning through various forms

- **Application:** The use of knowledge and skills to make connections within and between various contexts

PERFORMANCE STANDARDS – THE ACHIEVEMENT CHART

We get an "A" in teaching and assessing K/U . . .

✓ **Knowledge and Understanding**

PERFORMANCE STANDARDS – THE ACHIEVEMENT CHART

. . . But maybe we need to work on how we teach and assess some of these categories?

- ✓ **Application**
- ✓ **Communication**

11. A van's gas tank holds 75 L. The van uses 0.125 L/km.

A a) Describe the relation between the distance the van travels and the volume of gas in its tank.

b) How far can the van travel on a full tank of gas?

11. The volume of a rectangular box is $(x^3 + 6x^2 + 11x + 6)$ cm^3. The box is $(x + 3)$ cm long and $(x + 2)$ cm wide. How high is the box?

A

17. Describe each relation in words.

C a) $I = 2.54c$, where I is inches and c is centimetres

b) $F = \frac{9}{5}C + 32$, where F is degrees Fahrenheit and C is degrees Celsius

c) $k = \frac{p}{2.2}$, where p is pounds and k is kilograms

d) $K = C + 273$, where K is degrees Kelvin and C is degrees Celsius

PERFORMANCE STANDARDS – THE ACHIEVEMENT CHART

Are we teaching and assessing Thinking, arguably the most important category, adequately?

Thinking

16. Determine algebraically where the cubic polynomial function that has zeros at 2, 3, and −5 and passes through the point (4, 36) has a value of 120.

Why is Thinking undervalued by teachers and students alike?

PERFORMANCE STANDARDS – THE ACHIEVEMENT CHART

Levels of Achievement

Level 1 represents achievement that falls much below the provincial standard. The student demonstrates the specified knowledge and skills with limited effectiveness. Students must work at significantly improving learning in specific areas, as necessary, if they are to be successful in the next grade/course

Level 2 represents achievement that approaches the provincial standard. The student demonstrates the specified knowledge and skills with some effectiveness. Students performing at this level need to work on identified learning gaps to ensure future success.

Level 3 represents the provincial standard for achievement. The student demonstrates the specified knowledge and skills with considerable effectiveness. Parents of students achieving at level 3 can be confident that their children will be prepared for work in subsequent grades/courses.

Level 4 identifies achievement that surpasses the provincial standard. The student demonstrates the specified knowledge and skills with a high degree of effectiveness. *However, achievement at level 4 does not mean that the student has achieved expectations beyond those specified for the grade/course.*

We still see struggles applying the levels of achievement to student work in mathematics. Can more focus on moderated marking assist?

PERFORMANCE STANDARDS – THE ACHIEVEMENT CHART

Determining a report card grade will involve teachers' professional judgement and interpretation of evidence and should reflect the student's **most consistent level of achievement, with special consideration given to more recent evidence.**

Where should the teacher look to find the best indicator of most recent/most consistent performance? Is this too much data?

	Weighted Average	Weighted Median	Weighted Mode	Blended Mode	Blended Median
Student 1	81	86	90	88	83
Student 2	72	85	90	83	79
	77	80	75	77	77
	77	80	90	77	76
	85	95	90	86	88
	80	80	90	83	80
	78	91	90	90	87
	59	58	25	47	58
	75	77	75	85	80
	79	86	90	86	80
	89	90	90	90	92
	70	71	90	73	72
	89	90	90	90	90
	74	64	90	81	80
	70	60	90	70	70
	87	91	90	90	92
	59	65	75	70	61

EXPECTATIONS

Overall expectations versus specific expectations

general terms

greater detail

EXPECTATIONS

All curriculum expectations must be accounted for in instruction, but **evaluation focuses on students' achievement of the overall expectations**.

Do we test too many "details" but miss the big ideas?

1a b c d

2a b c d

ASSESSMENT *FOR LEARNING AND AS LEARNING*

*"Teachers will obtain assessment information through a **variety of means**, which may include formal and informal observations, discussions, learning conversations, questioning, conferences, homework, tasks done in groups, demonstrations, projects, portfolios, developmental continua, performances, peer and self-assessments, self-reflections, essays, and tests."*

ASSESSMENT *FOR LEARNING AND AS LEARNING*

How do we build a "variety of means" that can meet the Seven Principles of Growing Success?

TRIANGULATION OF ASSESSMENT DATA

Teachers use a variety of assessment strategies to elicit information about student learning. These strategies should be triangulated to include observation, student-teacher conversations, and student products.

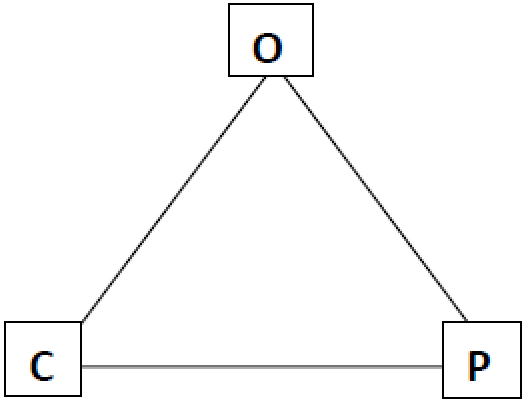
TRIANGULATION OF ASSESSMENT DATA

Observation

Watching students and using checklists to record information that will be considered, when grading students' achievement of outcomes

Conversation

Talking to students to assess their understanding / comprehension and/or to detect areas of concern and **recording** this information for formative or summative purposes



Product

Projects, presentations, demonstrations, tests, quizzes, dances, songs etc. that can be used to assess the achievement of outcomes

TRIANGULATION OF ASSESSMENT DATA will not occur when all assessment and evaluation comes in the form of quizzes and tests

"The Myth of Objectivity in Mathematics Assessment"

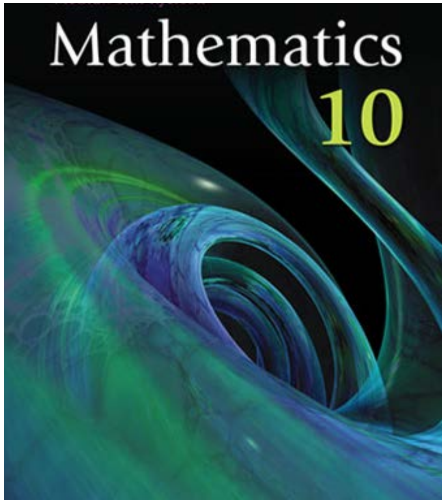
Teachers can gather information about learning by:

- designing tasks that provide students with a variety of ways to demonstrate their learning;
- observing students as they perform tasks;
- posing questions to help students make their thinking explicit;
- engineering classroom and small-group conversations that encourage students to articulate what they are thinking and further develop their thinking.

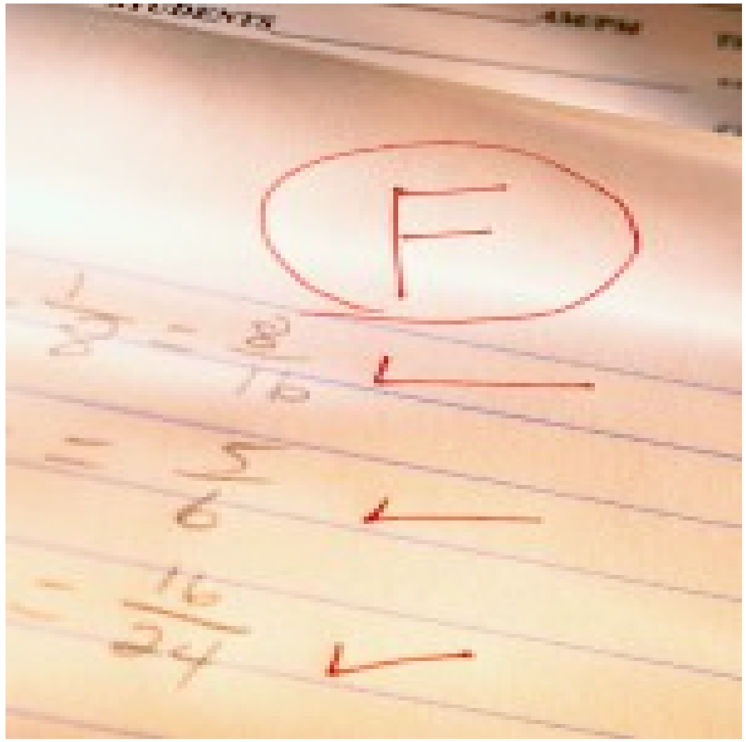
Teachers then use the information gathered to adjust instruction and provide feedback.

A math program designed around " following the textbook" may be impeding the ability of the teacher to "adjust instruction".

Learning and teaching are not linear. They are messy and complicated.



Descriptive feedback and coaching for improvement



This is NOT what descriptive feedback looks like.

Descriptive feedback takes time.

Teachers engage in assessment *as learning by helping all students develop their capacity to be independent, autonomous* learners who are able to set individual goals, monitor their own progress, determine next steps, and reflect on their thinking and learning.

As essential steps in assessment *for learning and as learning, teachers need to:*

• plan assessment concurrently and integrate it seamlessly with instruction;

Tests and quizzes are distinctly separate from instruction. “Test day”.

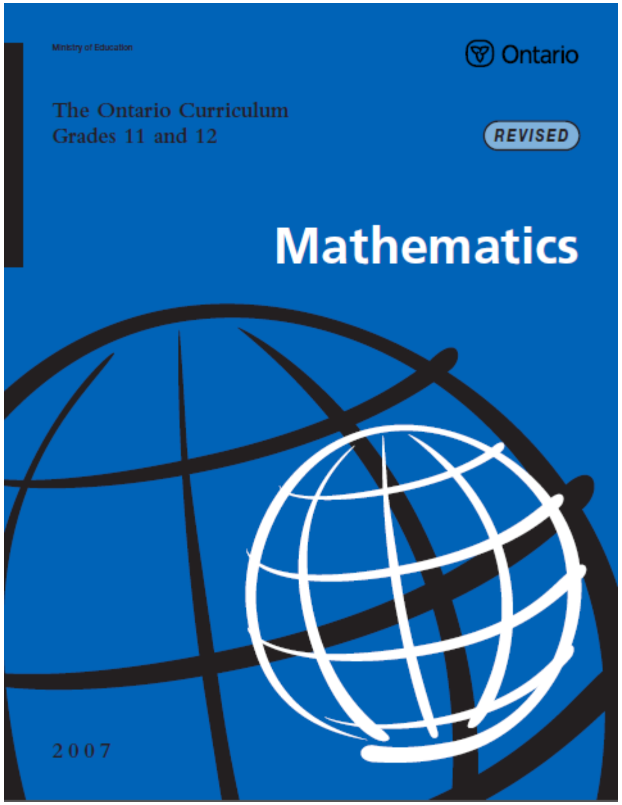
As essential steps in assessment *for learning and as learning, teachers need to:*

- gather information about student learning before, during, and at or near the end of a period of instruction, using a variety of assessment strategies and tools;
- use assessment to inform instruction, guide next steps, and help students monitor their progress towards achieving their learning goals;

As essential steps in assessment *for learning and as learning, teachers need to:*

- analyse and interpret evidence of learning;
- give and receive specific and timely descriptive feedback about student learning;
- help students to develop skills of peer and self-assessment

Most of Growing Success was already to be found in our curriculum documents . . .



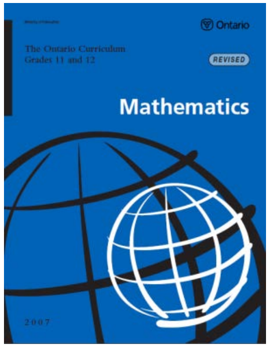
. . . But Growing Success does not refer to our

MATHEMATICAL PROCESS EXPECTATIONS

The mathematical processes are to be integrated into student learning in all areas of this course.

Problem Solving
Reasoning and Proving
Reflecting
Selecting Tools and Computational Strategies
Connecting
Representing
Communicating

The processing skills and critical/creative thinking processes in the Thinking category include some but not all aspects of the mathematical processes described on pages 17–22 of this document. Some aspects of the mathematical processes relate to the other categories of the achievement chart.



How has Growing Success Supported/Changed My Practice? My personal philosophy statements

Collaboration is important in my class.

- Students can learn as much from each other as from me.
- Assessment is part of the lesson. Students can self assess and peer assess while working on rich problems and thinking questions together. I can assess by wandering through the class, listening to what they are saying, and watching what they are doing. I can intervene to provide support only when my intervention is needed.
- They expect the teacher to be able to do it but that does not motivate them in the same way as seeing that their peers are able to do a question.
- Students are learning through teaching each other.

How has Growing Success Changed My Practice?
My personal philosophy statements

- If students still need me by the end of the course, then I haven't been a very successful teacher.

- I want students to be excited about doing math. Rich questions/tasks excite them.

- If students argue about math, HUZZAH!

- Students find their own questions more interesting than they do *my* questions. When learning/assessment is personalized by allowing students to insert themselves into their work, they care more and do better.

Wherever possible, don't test. When testing, test when students are ready, not when the teacher is ready.

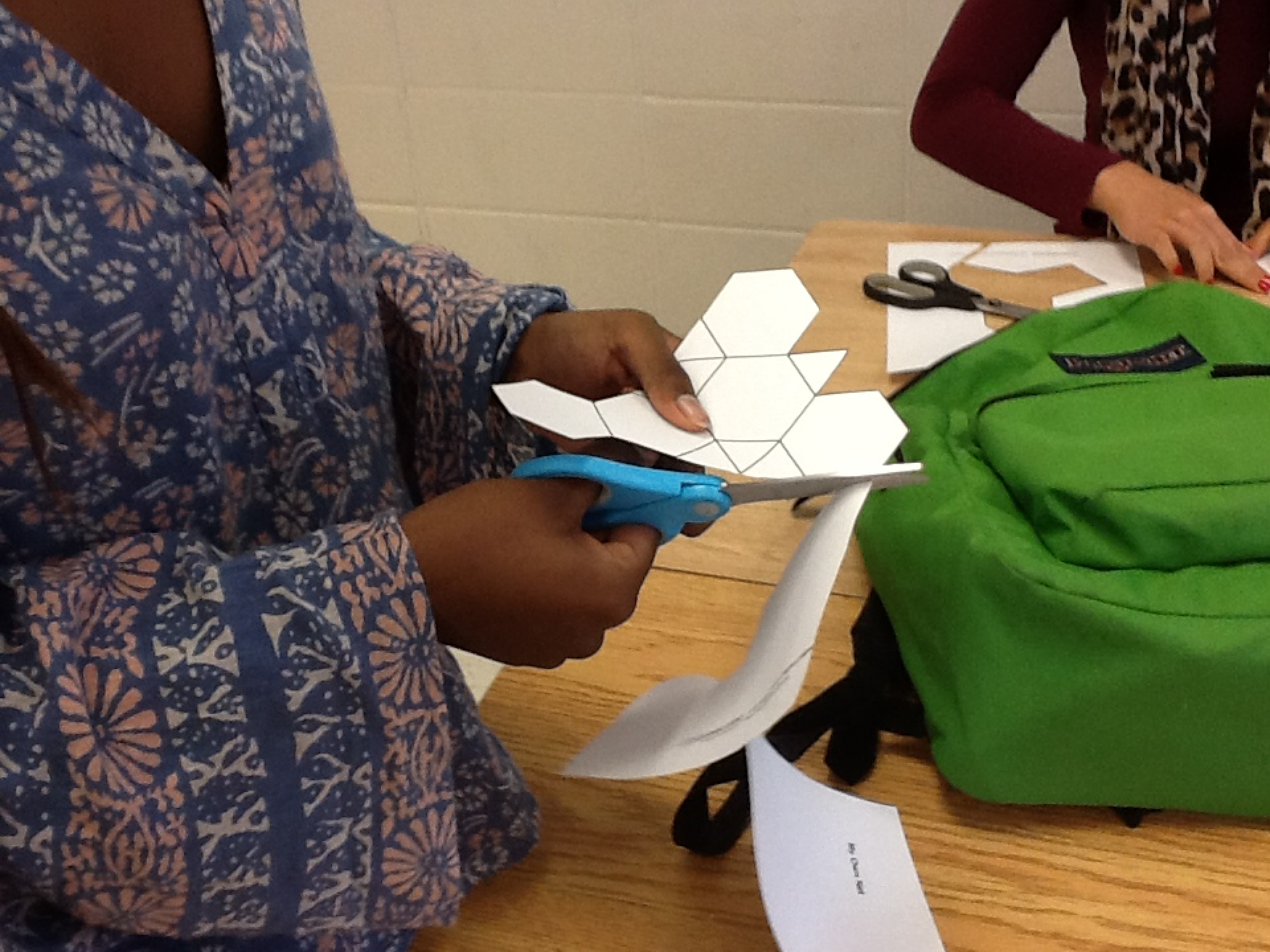
How has Growing Success Changed My Practice?
My personal philosophy statements

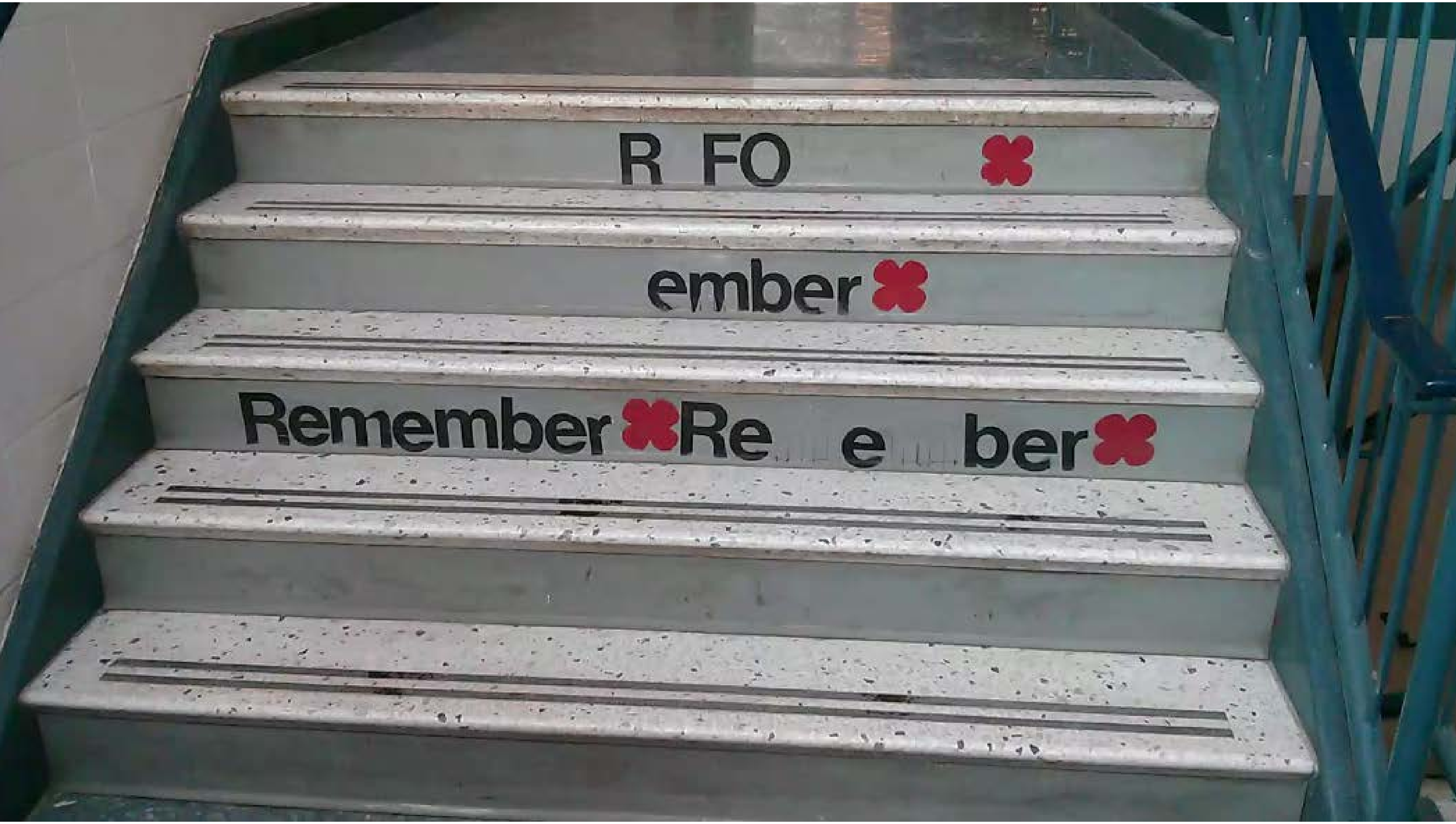
- Wherever possible, don’t test. When testing, test when students are ready, not when the teacher is ready.

Rich explorations, tasks, and questions, as well as projects and assignments, can easily become the foundation for learning as well as assessing and evaluating mathematics.

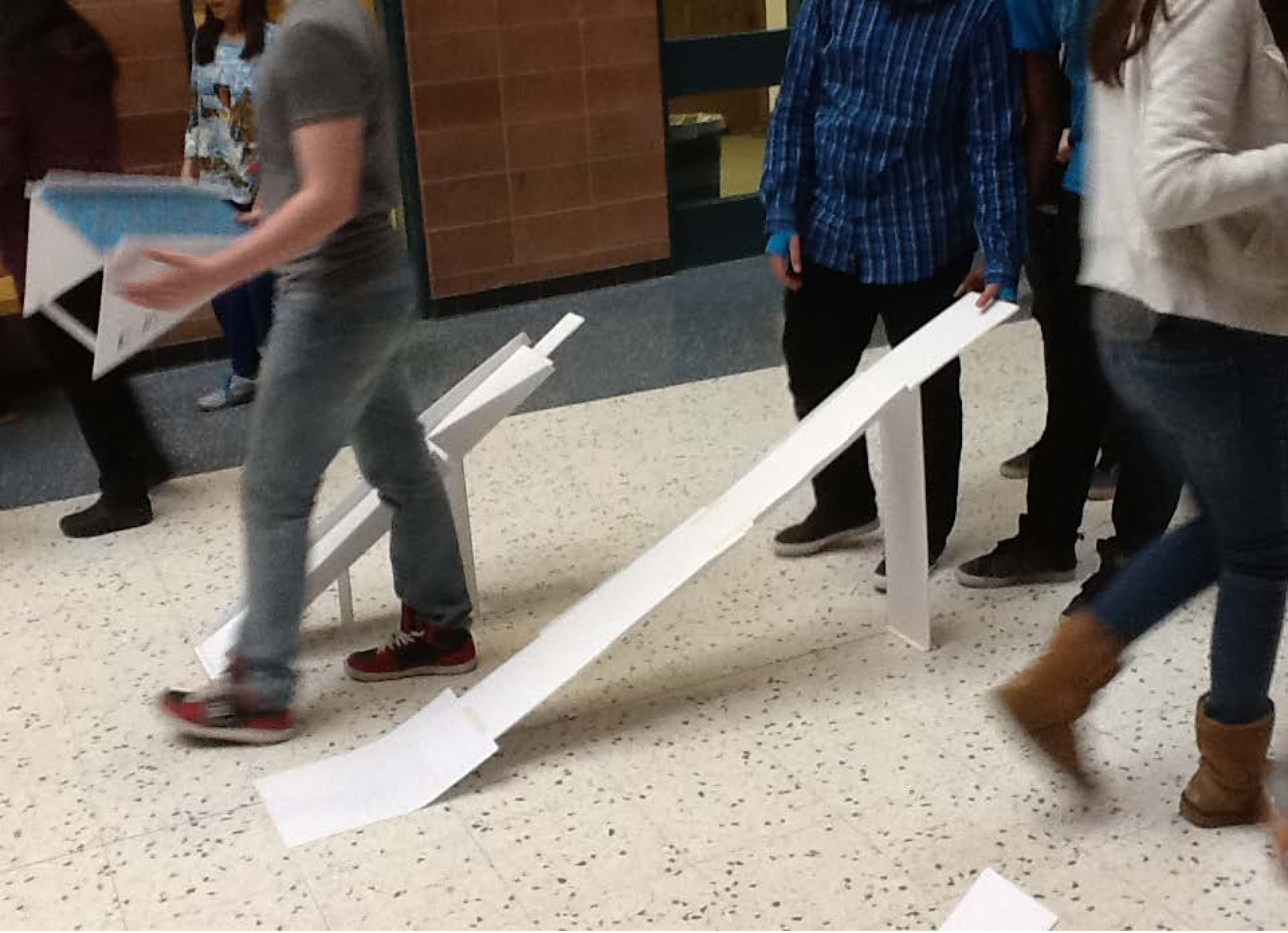
Pictures followed of students working together as well as of student work. Pictures that allow students to be identified have been removed.

Exploring patterns using algebra tiles working through University of Nottingham representation matching challenges; whole class trying to find a complete set of matching cards (linear equation in two forms/slope/y-intercept/graph); using netbooks, GSP, and photos of actual structures to determine scale factors; bending pipe cleaners to capture the flow of water out of a drinking fountain; modelling linear and non-linear data using penny circles; creating 3D structures from nets; checking is staircases are built to Ontario Building Code standards; ramp-building challenge; bicycle gear ratio project and bicycle frame design analysis; function art projects from grades 9 and 11 (handed in multipletimes for feedback); graphic organizers are used successfully as pre-assessment and post-assessment tools; Functioneer journals from grade 12U (handed in multipletimes for feedback); students working on problems identified from reading through Functioneer journals (working collaboratively before individual assignment is to completed and handed in)





Do our staircases meet the Ontario Building Code?



Research:

On your own

Working with a partner:

- View the following GSP files and answer the questions in the sketches.
  - Circle Basics
  - GEARS

When you are done with these,

- View the following internet files:
  - http://auto.howstuffworks.com/gears.htm

*(R) = Reduced

Table #1

	Front Gears			Rear Gears		
Gear #	¹1	¹2	¹3	²1	²2	²3
# of Teeth	60	70	80	20	30	39

Table #2

Combination	Front Gear		Rear Gear		Gear Ratio	Gear Ratio (R)
	Gear #	# of teeth	Gear #	# of teeth		
¹1, ²1	1	60	1	20	60:20	3:1
¹1, ²2	1	60	2	30	60:30	2:1
¹1, ²3	1	60	3	39	60:39	1.5:1
¹2, ²1	2	70	1	20	70:20	3.5:1
¹2, ²2	2	70	2	30	70:30	2.3:1
¹2, ²3	2	70	3	39	70:39	1.7:1
¹3, ²1	3	80	1	20	80:20	4:1
¹3, ²2	3	80	2	30	80:30	2.6:1
¹3, ²3	3	80	3	39	80:39	2.1:1

Smallest ... Biggest

1.5:1, 1.7:1, 2:1, 2.1:1, 2.3:1, 2.6:1, 3:1, 3.5:1, 4:1

0.2 0.3 0.1 0.2 0.3 0.4 0.5 0.5

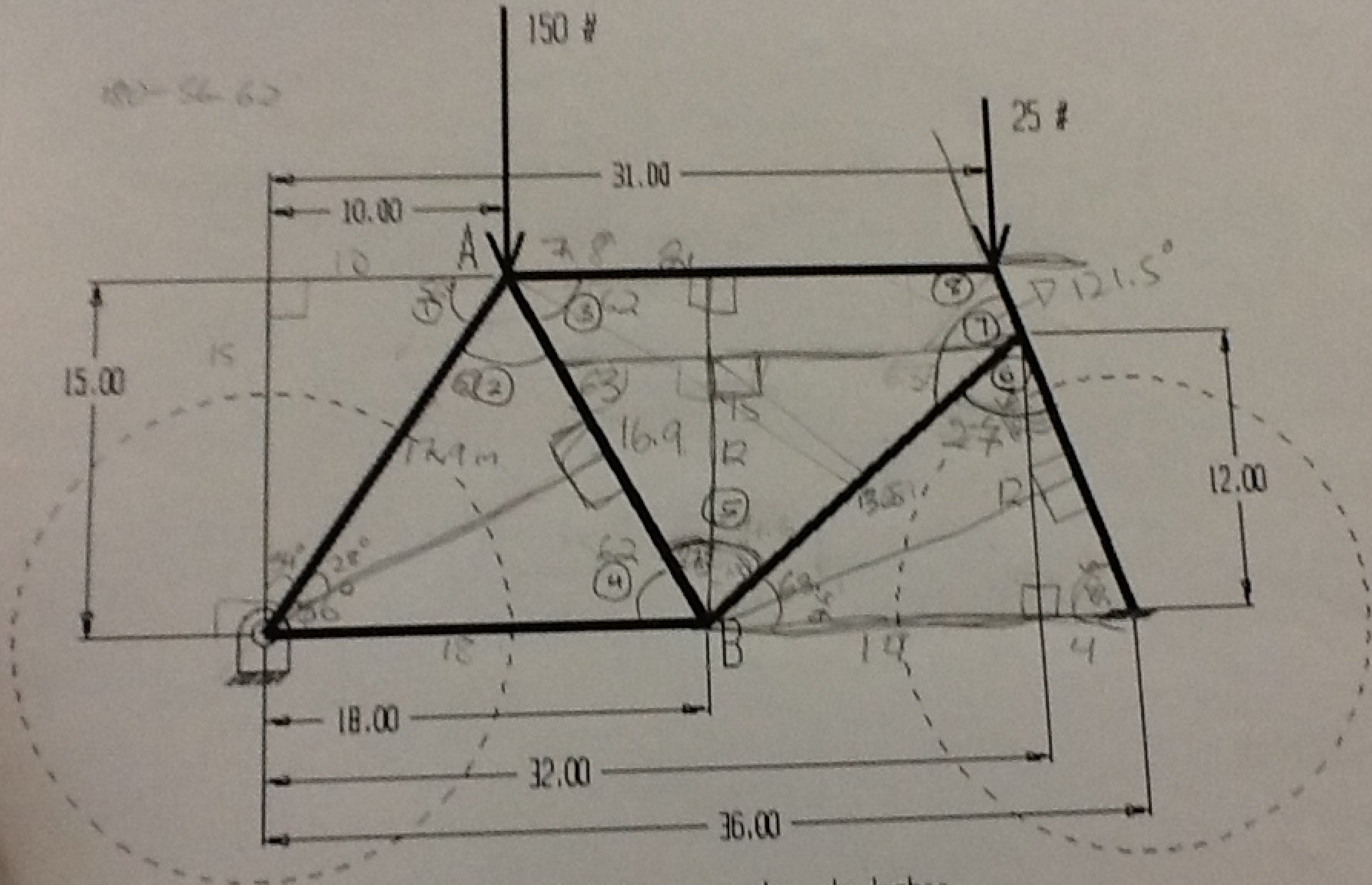
Thinking

What number did you start with?
-we started with low numbers like 5, 10, 15, 20, 30

What did you have to change? How did you know you needed to change this/these? What did you change to?
-We needed to change all the numbers because we found out that they were all too low. We knew that we needed to change them because when we lowered them into the ratio it was below 1.5. We changed all the numbers.

Which of the criteria were the most difficult to meet?
-We think it was not to make them the same.

What surprised you?
-That so many were the same. Me too!

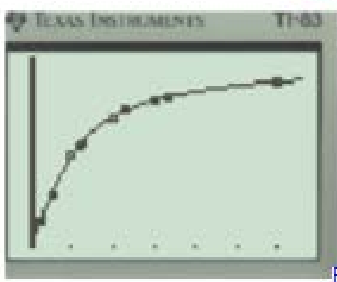


All dimensions are given in inches.

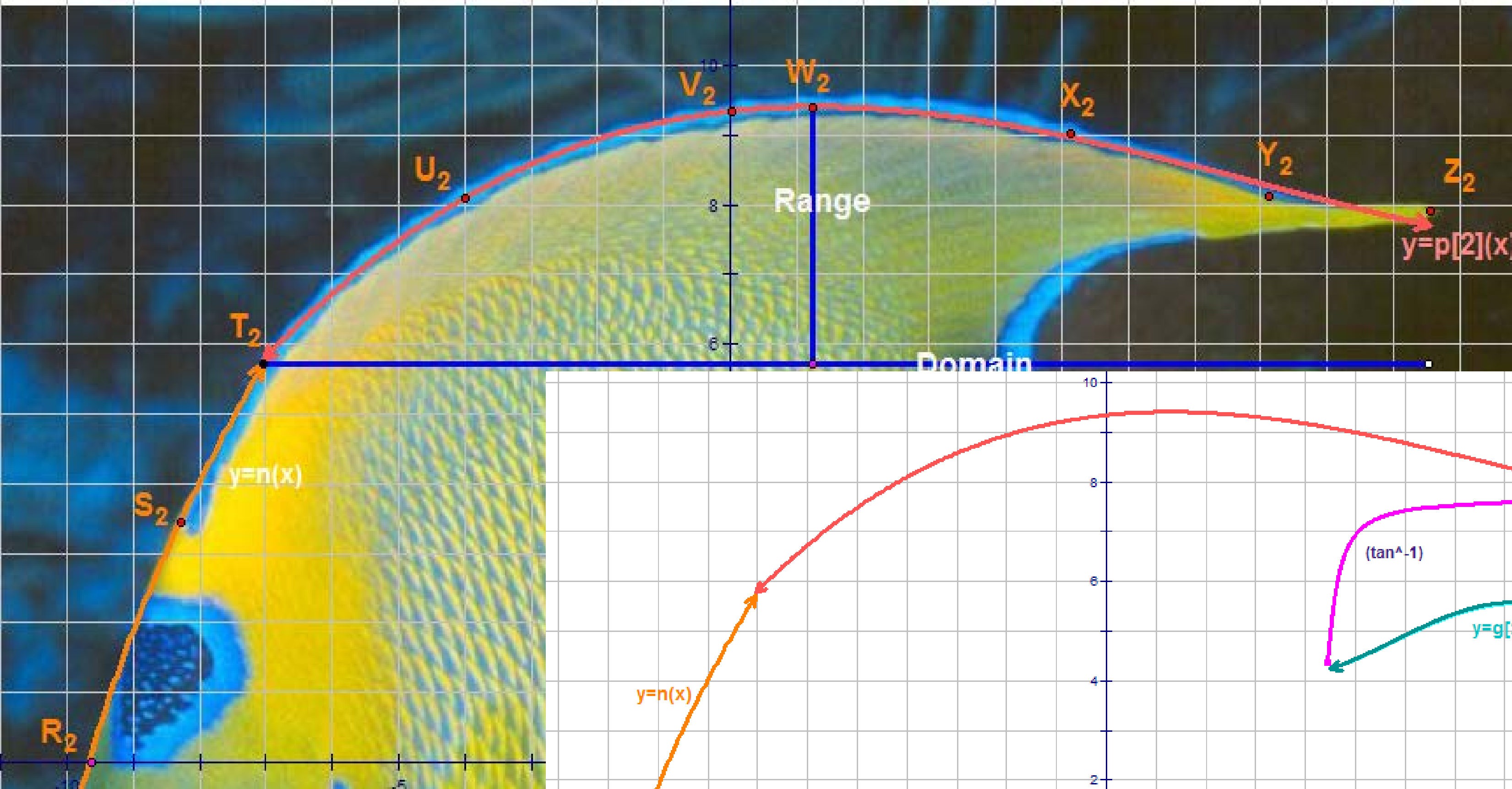
Data

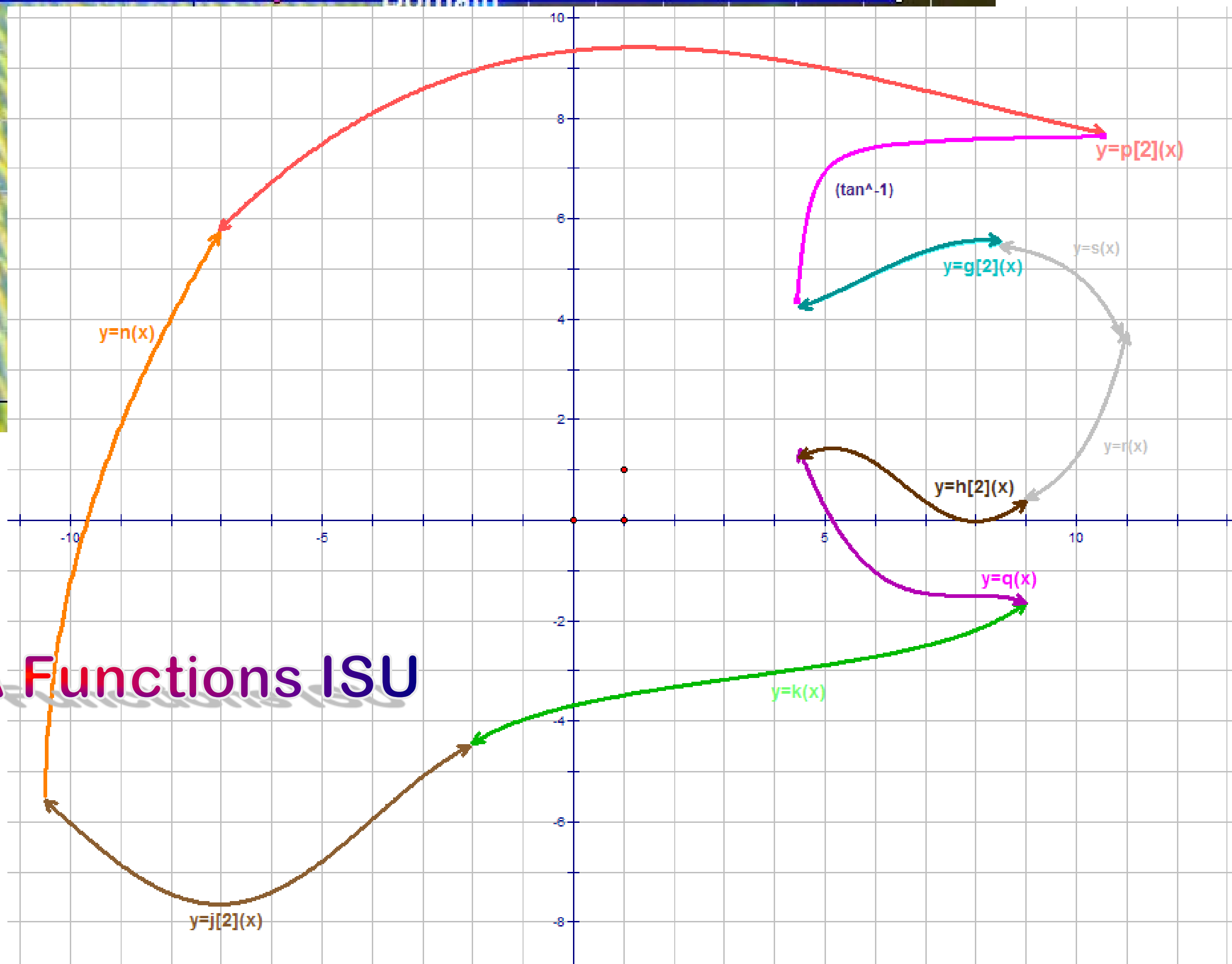
Grade 9/10 Applied split class

Steepness	Slope	Angle
1	1	45°
2	2	63.43°
3	5/4	51.34°
4	7/3	66.80°
5	3	71.57°
1	10/3	73.30°
2	2/9	12.53°
3	2/4	26.57°
4	3/6	26.57°
5	6/1	80.54°
6	6/5	50.19°
	Grade 9	grade 10



Extend Page





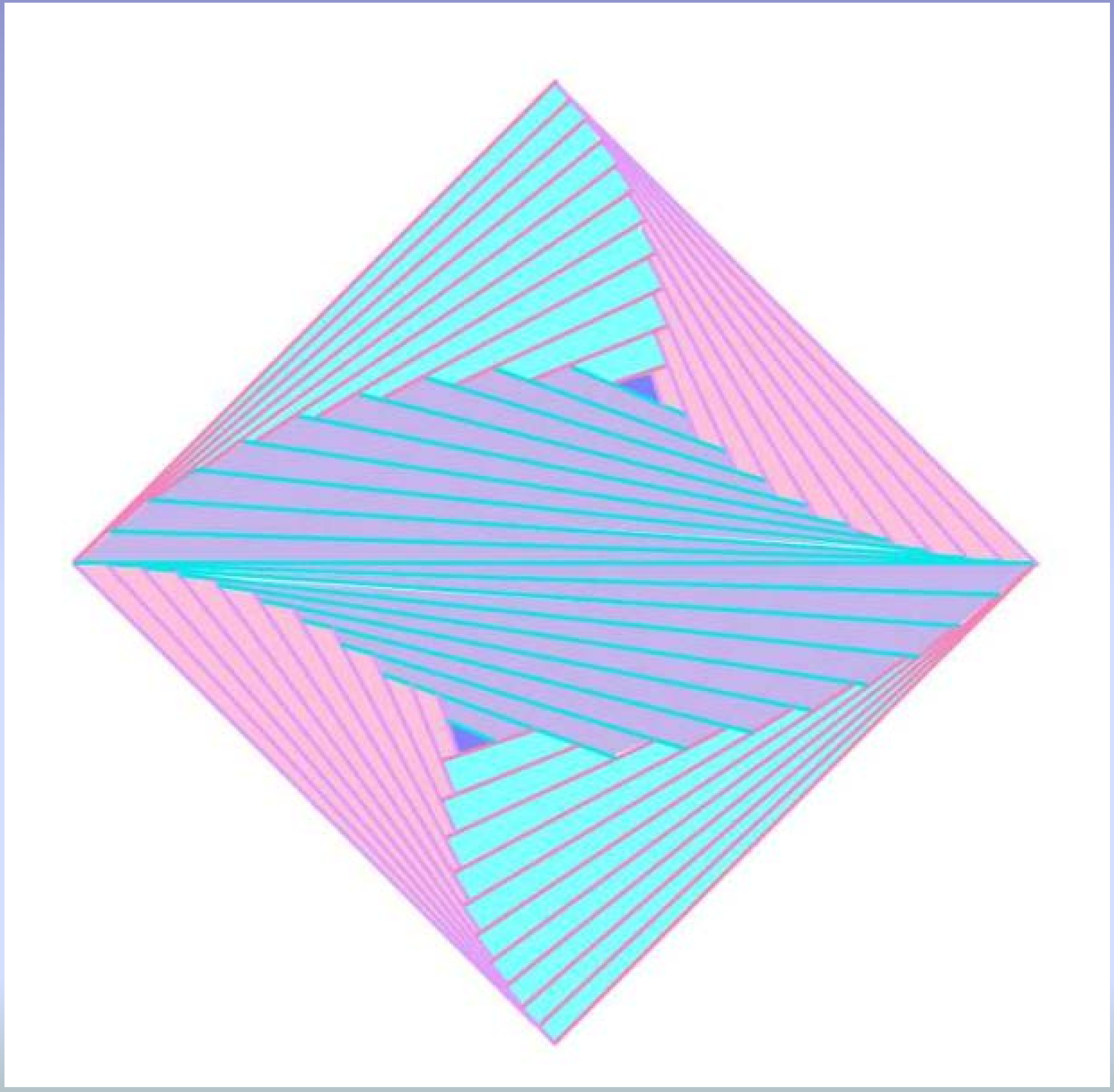
Picture This - A Functions ISU

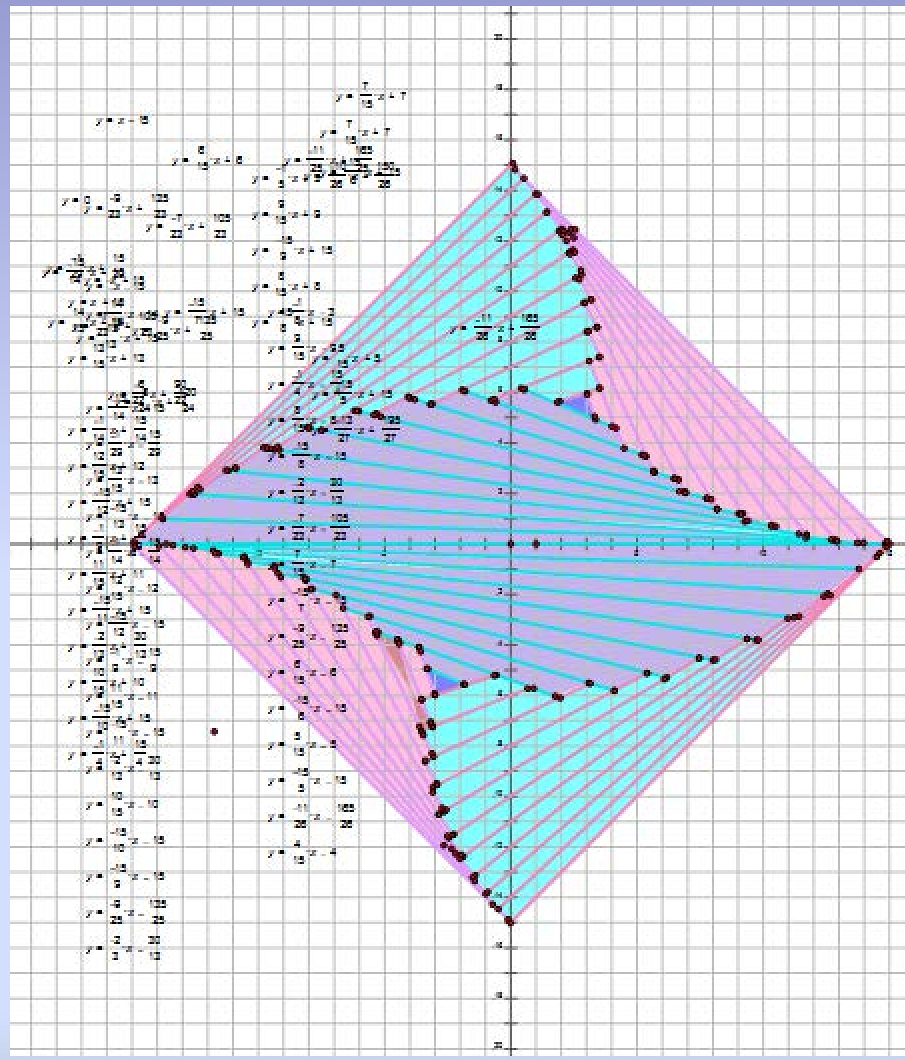
LINEAR ASSIGNMENT ART

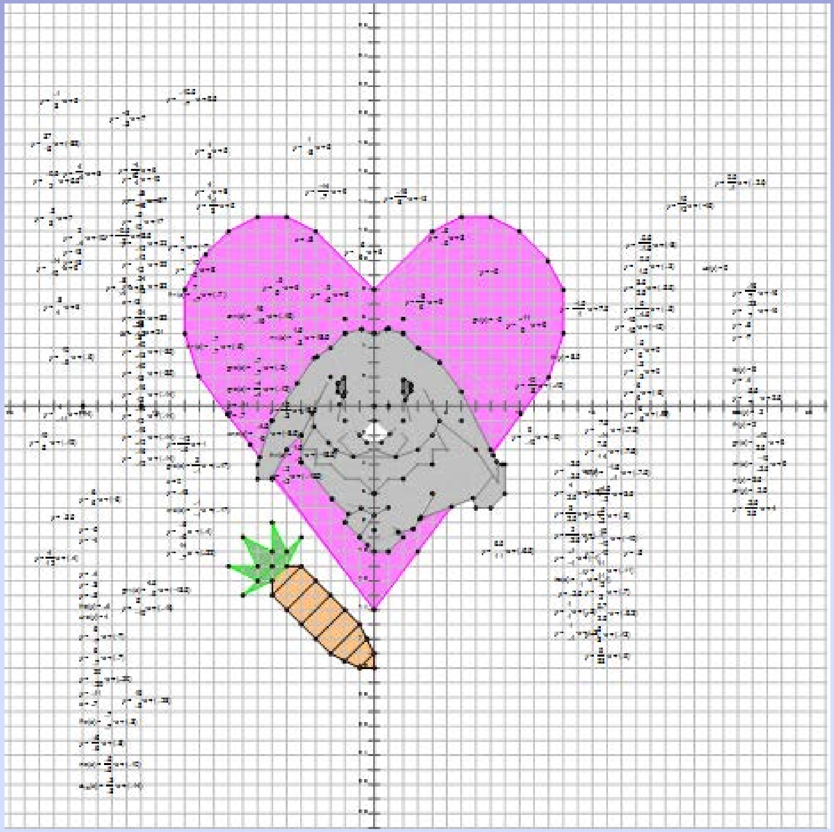
For this assignment, you will create an interesting piece of artwork using your knowledge of graphing linear relations.

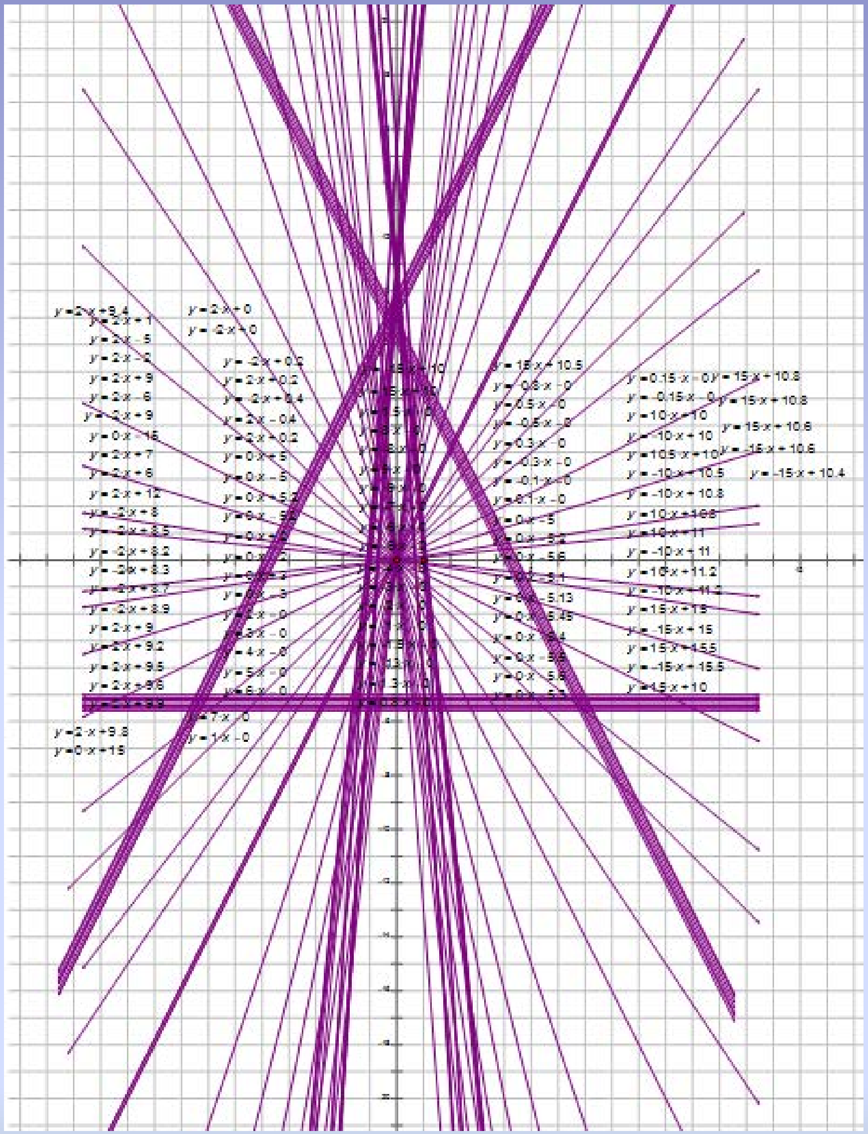
Here are your criteria:

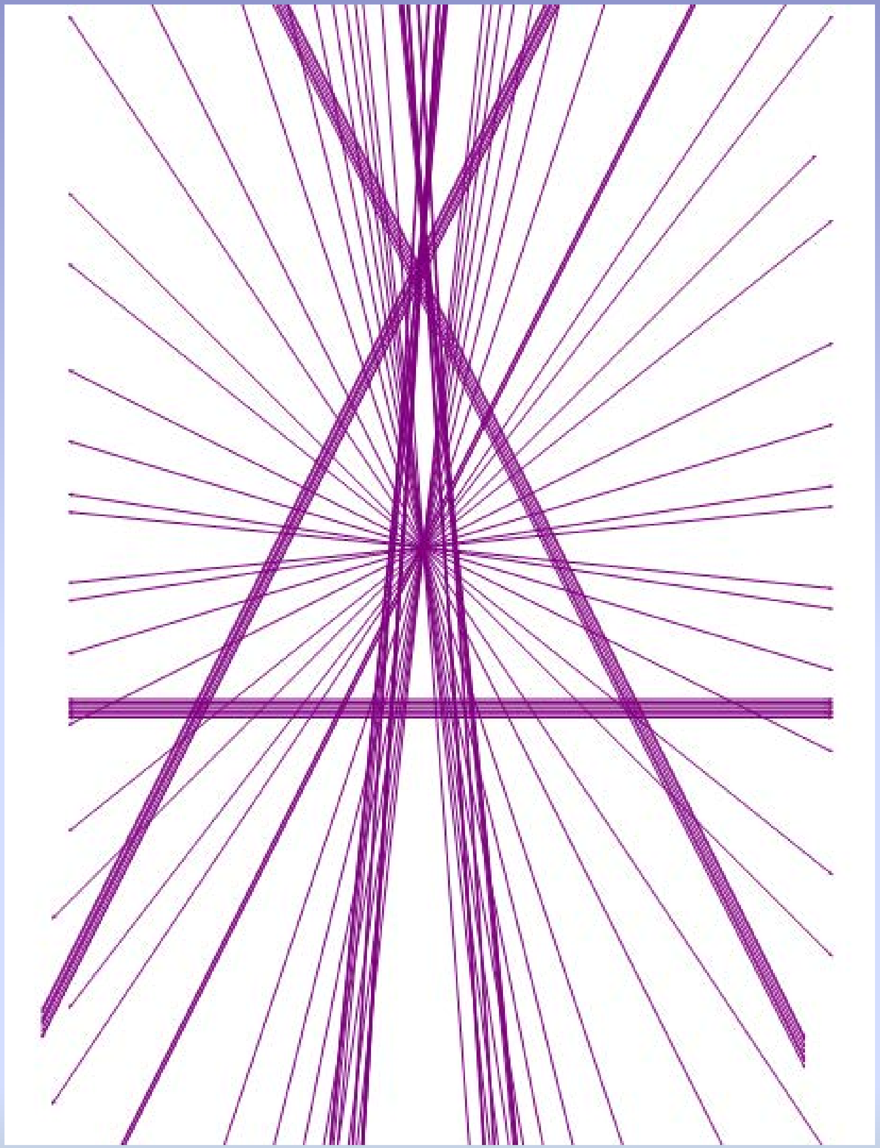
1. Your art piece is to be created using the Graph tool (along the top), (not the drawing tool along the left side) on Geometer's Sketchpad.
2. Your artwork may be in colour or black and white.
3. Your artwork must be submitted in <u>both electronic form and in print form</u>. In print form, it should fit on a standard letter-sized page (8.5" x 11").
    a. The print copy:
        i. You may choose to Hide the axes (under Display menu) and/or Hide the grid (under the Graph menu) or you may choose to incorporate these into your work. You will make this decision based on the artistic impression you are trying to make. However, as you continue to refine your piece, you will probably find that you need to bring this information back into your work to help you create the equations you are looking for.
        ii. Your final work should show no equations, points, or numerical values (such as you use to define your scale on the coordinate axes). However, you will need to record the relation equations and some of the characteristics of your relations in a separate document that must be submitted electronically along with your artwork.

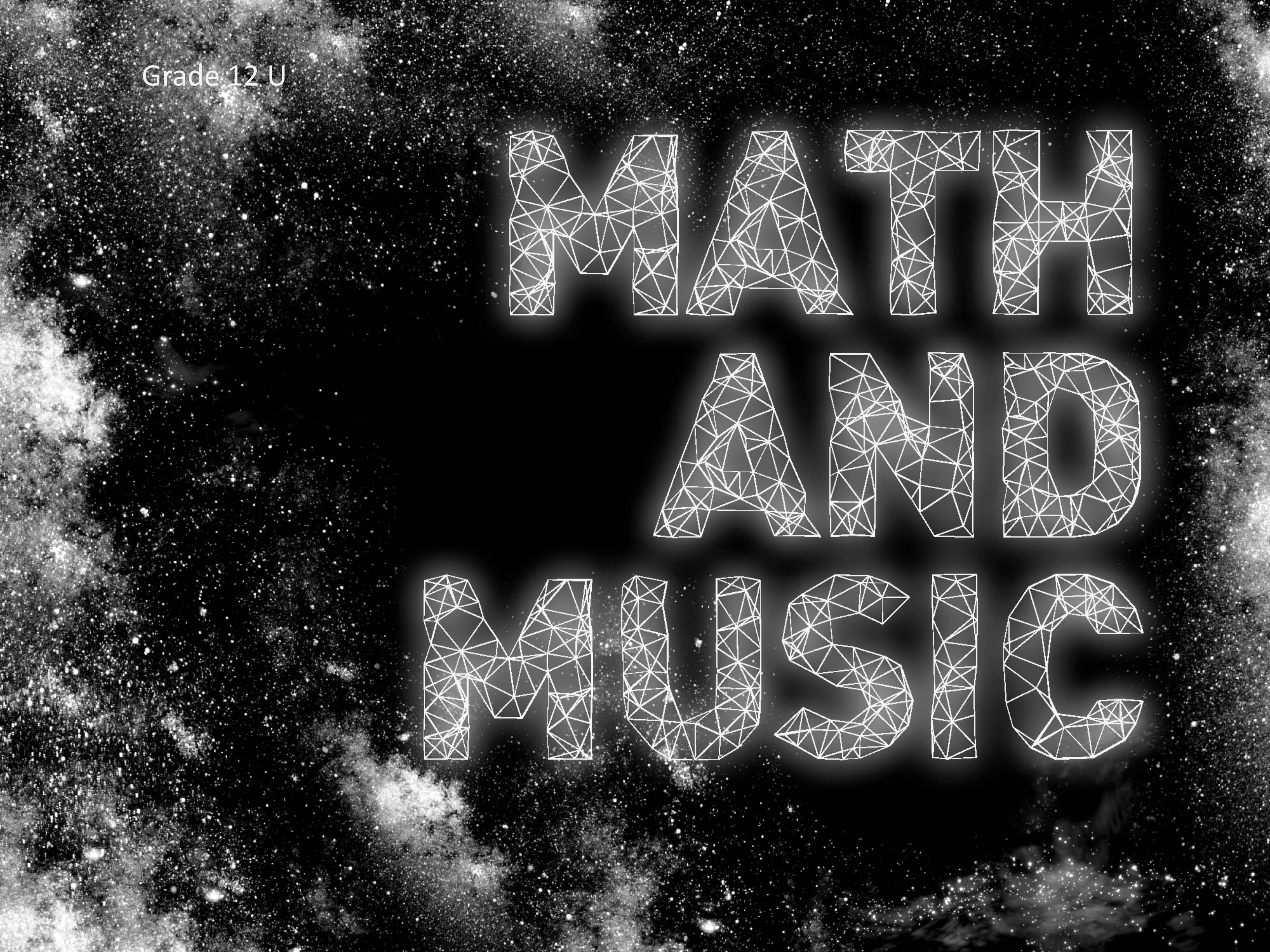


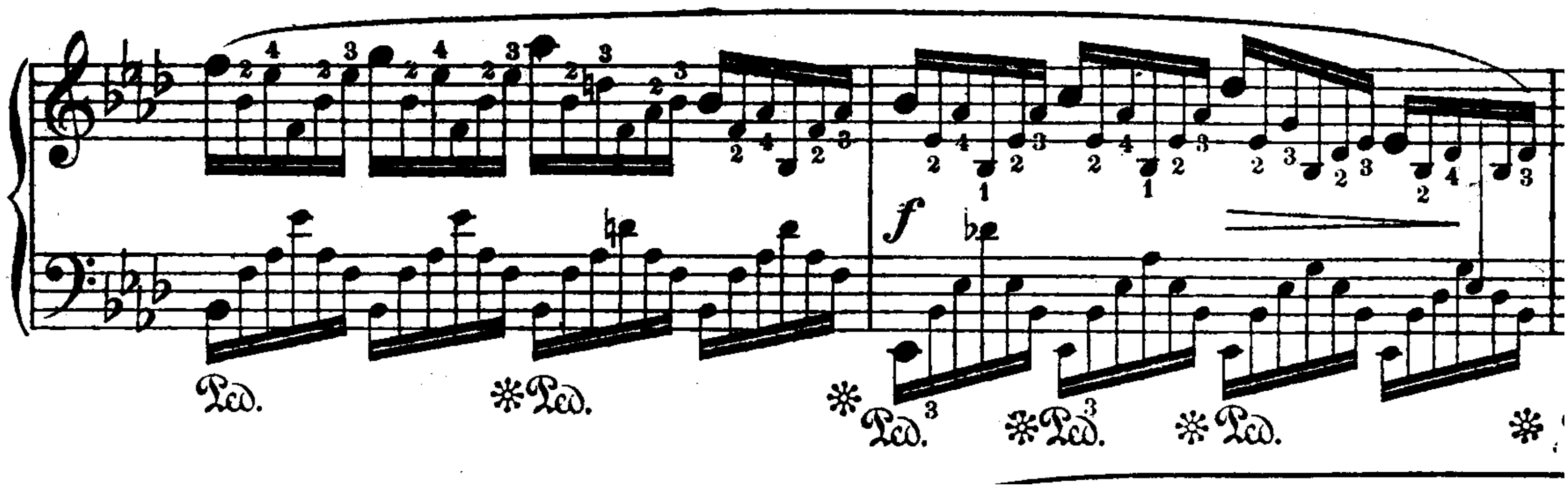




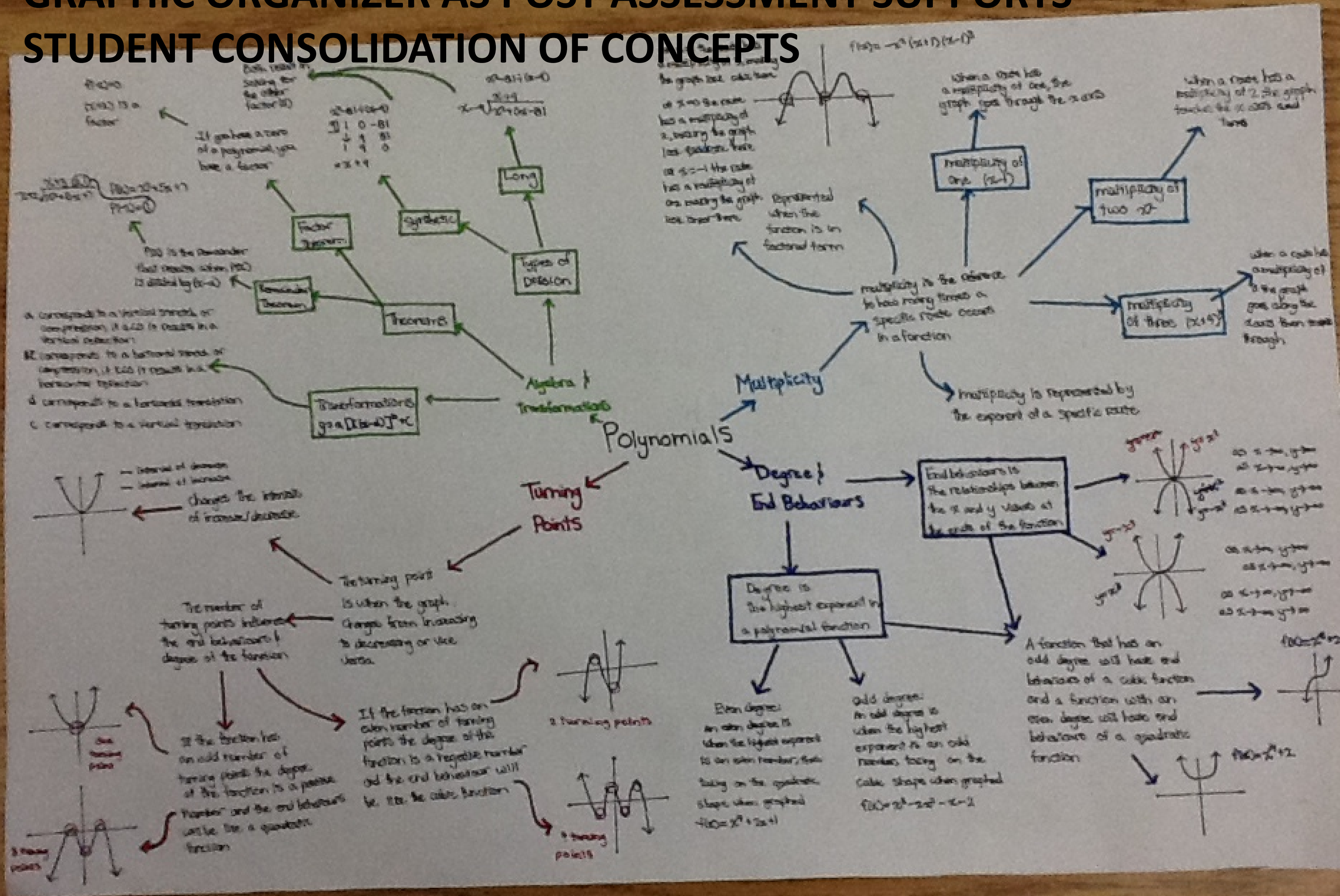





Grade 12 U
MATH
AND
MUSIC


f
Ped.
Ped.
Ped.
Ped.
Ped.
Ped.

GRAPHIC ORGANIZER AS POST ASSESSMENT SUPPORTS STUDENT CONSOLIDATION OF CONCEPTS



GRAPHIC ORGANIZER AS PRE-ASSESSMENT WHEN STARTING A NEW UNIT ALLOWS FOR INSTRUCTION TO EXPOSE STUDENT MISCONCEPTIONS

Discontinuities,
equivalences,
asymptotes,
and
holes

MYTHBUSTERS

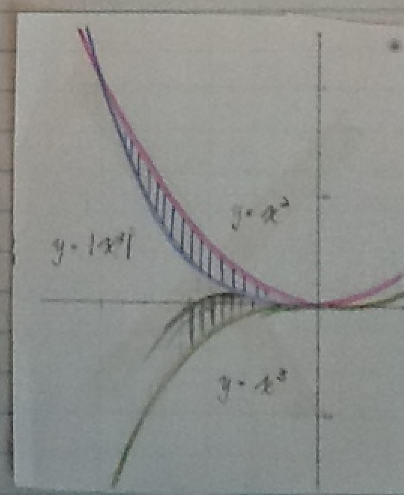
STUDYING
RATIONAL
FUNCTIONS

FUNCTIONEER JOURNALS

- In quadrant I, the graph of $f(x) = x^2 + x^3$ constantly ↑ ∴ when we substitute any positive # for x, we are going to get positive y-values ∴ the graph ↑ Note that when x is between 0 & 1, $x^2 > x^3$.
- The left portion of the graph ↑ and then ↓ unlike a normal cubic function which constantly ↑. The left side of the graph rises into quadrant II ∴ the square of a # between -1 & 0 is greater than the the cube of that function. So in $-1 < x < 0$, $x^2 > x^3$ & x^2 is going to generate a greater positive # than negative, so when adding $x^2 + x^3$ (+ & -), the corresponding y-values will be positive ∴ ↑
- The reason why the graph is in quadrant 3 is because when $x < -1$ ~~[illegible]~~ x^3 is going to generate a greater negative # than the positive # x^2 generates ∴ the resulting y-values are negative when $x < -1$
- Continue to explain the bump that's created in quadrant II. The bump then ↓ back to 0 ∴ x^2 ↓ at a faster rate than x^3 ↑ when x is between 0 & -1. As x values ↑ within this domain, the difference between the rates ↓ ∴ the bump ↓ back to the origin.

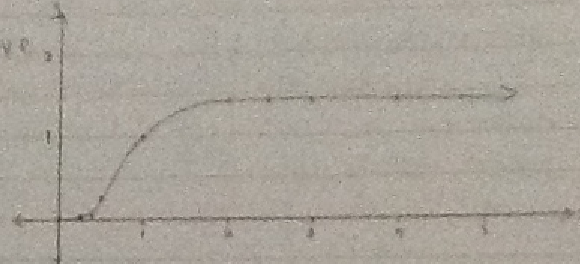
Ex. If $x = -2$, then $x^2 = (-2)^2 = 4$; $x^3 = (-2)^3 = -8$
$x^2 + x^3 = 4 + (-8) = -4$

- The difference in the rates of decreasing between a parabola & an absolute valued cubic function when $-1 < x < 0$ creates the bump in quadrant II that wouldn't normally be there in a regular cubic function ($f(x) = x^3$)



$f(x) = x^{\frac{1}{x}} \rightarrow \sqrt[x]{x}$

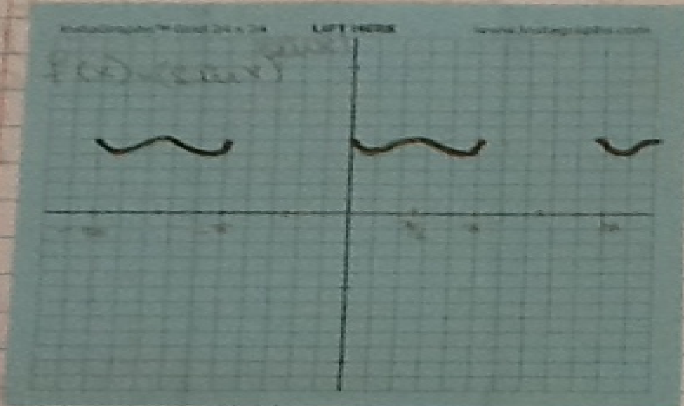
Rule: the radicand ≠ -ve,
↳ ∴ $x \geq 0$
D: $x \in (0, \infty)$



x	y
0	0
1	1
1/4	1/256
1/3	1/27
1/2	1/4
2	1.414
2.5	1.443
3	1.442
4	1.414
6	1.348
10 000	1.001

Initially, the x-values of this function are very small but as it approaches the value of $\frac{1}{2}$, the corresponding y-value is 1/4, and since the square root of 1 is always 1, a point on this function is (1, 1). This graph peaks at a value between 2 and 3, where it then begins to slowly decrease. This is because as the numbers get larger, the n^{th} root of that number also decreases in value. For example, the square root of 1 is 1 because the index and radicand values are both 1. However, if the x-value was 4, the index and radicand values would then both be 4, resulting in the quad root of 4, which is 1.414. This graph has a horizontal asymptote at y = 0 as the y-value will never be negative because of the vertical asymptote at x = 0. The vertical asymptote ensures that a negative x-value will always produce an undefined y-value as it is impossible to take the n^{th} root of a negative number. Just a domain restriction.

Having fun with sin x :)



Since this course is a majorly graphing course, I thought of experimenting and finding graphs of weird equations!

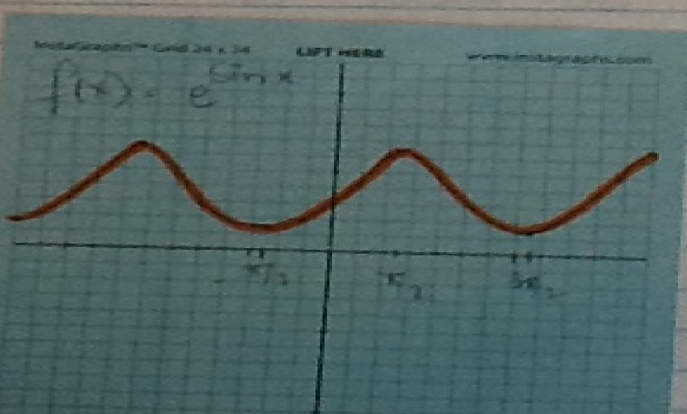
$f(x) = (\sin x)^{\sin x}$

This is just the beginning :)

I named this graph "Moustache". Don't you feel it looks like a moustache? :)

...you ...ht ...why ...get those ...s ...you can explain it ...using Grade 11 math!

Domain = {x ∈ R}



So guess what I found: an equation that could be an equation for BEAUTIFUL MOUNTAINS

$f(x) = e^{\sin x}$

4. What is the definition of an asymptote?

A ~~value~~ line or curve which the function can never touch. I agree.

A ~~value~~ line or curve of x for which f(x) is undefined.

Not a value. An asymptote is a 1-dimensional geometric object. It is a straight or curved line.

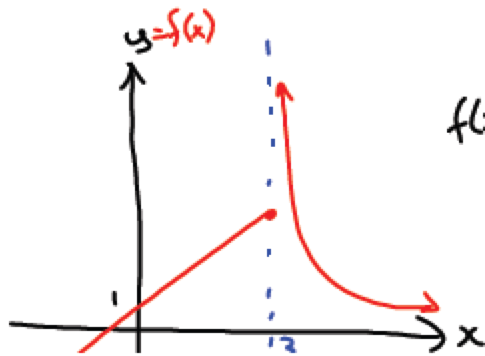
Asymptotes.gsp

HA Crossing.gsp

Dispelling another myth Rational Functions.gsp

Parabolic asymptote.gsp

From Wolfram: An asymptote is a line or curve that approaches a given curve arbitrarily close.



$$f(x)=\begin{cases} x+1, & x\leq 3 \\ \dfrac{1}{x-3}, & x>3 \end{cases}$$

FROM FUNCTIONEERS

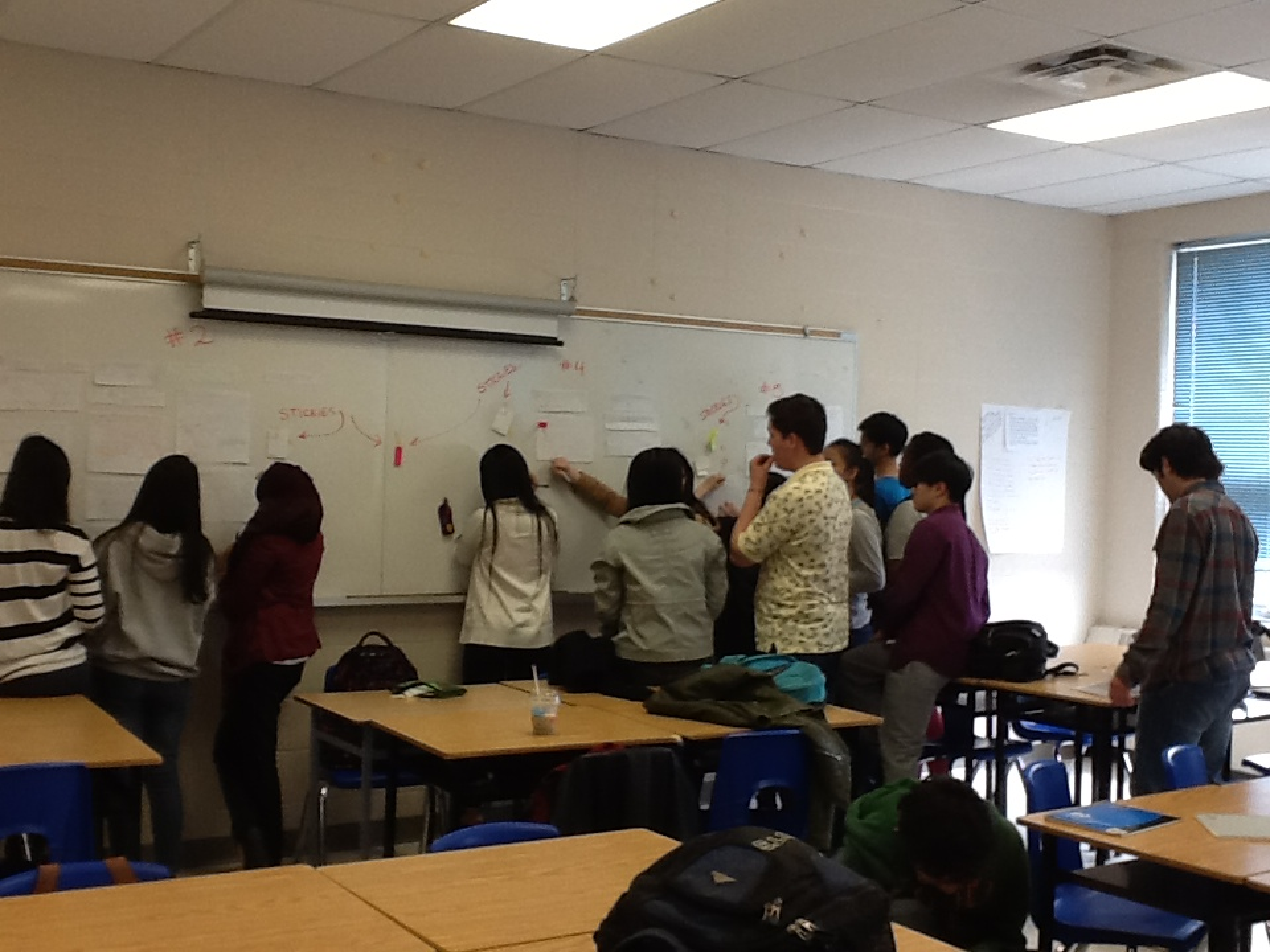
1. Is it possible a reciprocal transformation of a function to have a hole rather than a vertical asymptote?

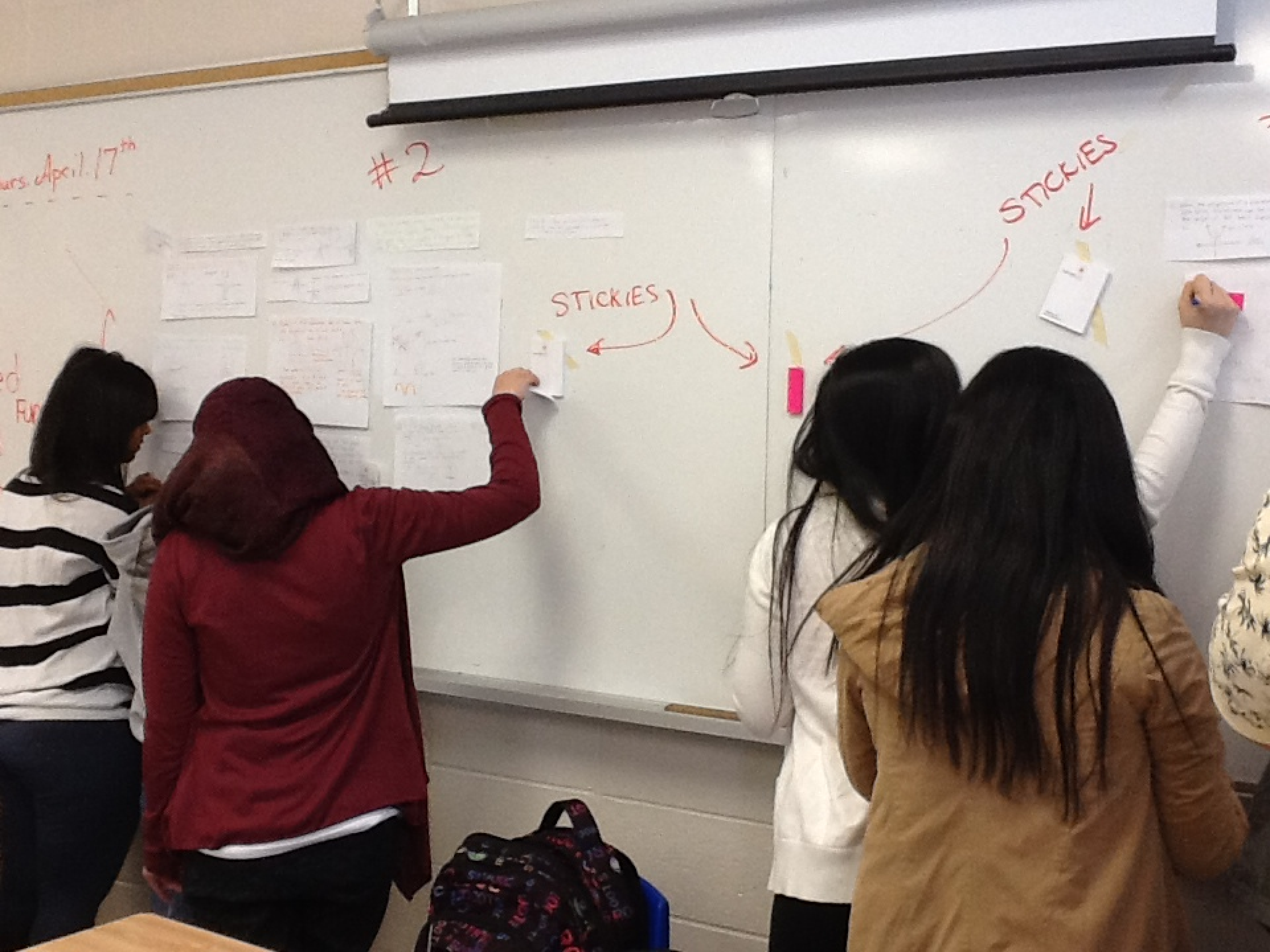
2. True or false? If a function has a maximum value, then its reciprocal has a minimum value. Explain/argue/justify/support with examples.

3. What accounts for the differences between the reciprocals of $f(x) = \sqrt{x}+3$ and $g(x) = \sqrt{x+3}$?

4. True or False? The graph of a reciprocal of a function always intersects the graph of the base function. Explain/argue/justify/ support with examples.

5. True or false? The reciprocals of functions are special types of rational functions. Explain/argue/justify/support with examples.


#2
STICKIES


April 17th
#2
STICKIES
STICKIES