A Report on the Fall, 2003 Calculus Experience at The University of Guelph

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We have three streams of one semester calculus courses at U. of G.

Math 1080–Calculus for the Biological Sciences
Math 1000–Calculus for the Social Sciences
Math 1200–Calculus for the Physical Sciences.

My report is exclusively concerned with Math 1200 and its sequel, Math 1210.

Here are topics that are either new to the first semester curriculum, marked by *, or whose coverage had been allocated more time.

-compound angle formulas for trigonometric functions*
-reciprocal trigonometric functions*
- $\lim_{h \to 0}\left(\frac{\sin(h)}{h}\right) = 1$*
-derivation of the trigonometric derivatives (I always shiver with discomfort when I use the expression, "derivation of a derivative"!)*
-Riemann sums
-Definition of the definite integral
-Applications to area problems
-The Fundamental Theorem of Calculus
-Anti-derivatives including a brief introduction to the substitution rule

Here is what we removed from the first semester sequence to accommodate these additions.
-Reduction of time on related rates
-Reduction of time on applications of the differential
-Applications of the exponential and logarithm functions to growth and decay problems
-Extensive practice on introduction by substitution

In the Calculus II course, we have added work on the substitution rule. We have omitted intersection and area problems in polar coordinates. We have cut back the introduction to calculus of more than one variable to brief discussions of domain, range, and partial derivatives. There are other topics listed as optional. They will be included if time permits.

Now, to end, I present for your consideration our proposed sequence for Calculus, semesters I and II. This is meant to be a work in progress. We will certainly have improvements to implement for September, 2004. In the meantime, I invite your constructive criticisms. I will also, the editor and reviewers of this journal being willing, write an update for the June, 2004 OMG, with a progress report.

MATH 1200 OUTLINE , FALL 2003

TOPIC	DETAILS	LECTURES
Trigonometry	• Graphs of the six trigonometric functions • Compound angle formulas	2
Exponents and Logarithms	• Definition, domain, range, and graphs of $y = a^x$, $y = \log_a(x)$ • $y = e^x$, $y = \ln(x)$ • Change of base formula • Properties of Exponents and Logarithms	2
Intuitive Limits	• Straightforward evaluation • "0/0" • Limits at infinity • Limits as x approaches infinity • Discussion of vertical and horizontal asymptotes	2
Formal Definition of a Limit	Formal definition of a limit • ε / δ proofs, linear case (eg., $\lim_{x\to 3}(3x-7) = 2$) • Limits with constraints ($\lim_{x\to 3}(x^2) = 9$) • The Squeeze Theorem • Special examples such as $\lim_{x\to 0}\left(x\sin(1/x)\right) = 0$ and $\lim_{\theta\to 0}\left(\frac{\sin(\theta)}{\theta}\right) = 1$ • Limit theorems (This section will incorporate inequality and absolute value questions formerly covered in the first lectures.) • When does $\lim_{x\to a} f(g(x)) = f(\lim_{x\to a} g(x))$? • Formal definition and examples of limits involving infinity	4
Continuity	• Definition of continuity at a point, on open and closed intervals • Continuity theorems including careful discussion of composition, ie., when does $\lim_{x\to a} f(g(x)) = f(g(a))$? • The Extreme Value Theorem • The Intermediate Value Theorem	2
Differentiation	• Average rate of change • The difference quotient; instantaneous rate of change; • Formal definition of the derivative • The definition of the tangent line to a curve as the limit of secants • Finding tangent and normal lines	2
Derivative Formulas	• The derivative of $y = x^n$ (Prove the formula for natural numbers, and extend by example to real numbers = - 1) • Derivatives of simple polynomials • Sum, Difference, Product, Quotient, and Chain Rules	3
More Differentiation	• Implicit Differentiation • Higher order derivatives	1

Applications of the Derivative Formulas	• Distance/velocity/acceleration • Related rates **(Optional)**	1
The Differential	• Definition and geometric interpretation of the differential • Approximating functions using the differential	1
More Applications	• Fermat's Theorem • Rolle's Theorem • The Mean Value Theorem • Applications to optimization problems • The second derivative test	3
Graphing	• Graphing WITHOUT calculus • Detailed graphing with calculus	2
Calculus and Trigonometry	• Derivatives of trigonometric functions	1
Exponential and Logarithm functions	• Derivatives of logs and exponential functions, emphasizing bases e and 10 • Logarithmic differentiation	2
Indefinite integral	• Straightforward examples of the indefinite integral (eg., $\int \sin(3x+1)dx,\ \int \frac{x}{x+5}dx,\ \int e^{5x}dx$)	1
Definite Integral	• The Riemann Integral • Riemann sums applied to simple area problems, eg., the area bounded by $y = x^2$, $y = 0$, $x = 0$, $x = 2$	2
The Fundamental Theorem Of Calculus	• Proof of the Fundamental Theorem of Calculus • Application to definite integrals, area problems (both using dy and dx) • Derivative of an integral	1
The Substitution Rule for Integration	• The substitution rule ($\int f'(u)\frac{du}{dx}dx = f(u) + C$)	2
		Sum = 34

OUTLINE FOR MATH 1210, WINTER 2004

TOPIC	DETAILS	LECTURES
Inverse Functions	•One to one functions •Finding inverses algebraically •Finding inverses by reflecting in $y = x$ •Finding inverses by "flipping and rotating" •The reciprocal relationship between the derivative of a function and its inverse	1
The Inverse Trig Functions	•Restricted domains •Defining the six inverse trigonometric functions •Derivatives •Integrals yielding inverse trigonometric functions	4
Hyperbolic Functions	•Definition in terms of exponential functions •Relationship to the hyperbola $x^2 - y^2 = 1$ •The "circular" functions versus the "hyperbolic functions" •Formulas involving hyperbolic functions (including Left Side/Right Side proofs) •Derivatives and integrals of hyperbolic functions	2
Inverse Hyperbolic Funtions	•Definitions •Deriving "ln" formulas for inverse hyperbolic functions •Derivatives •Using the derivatives as an alternate method of deriving the ln formulas for the inverse hyperbolic functions •integrals yielding inverse hyperbolic functions	2
L'Hopital's Rule	•Indeterminate forms $\left(\text{"}\frac{0}{0}\text{"}, \text{"}\frac{\infty}{\infty}\text{"}, \text{"}0^0\text{"}, \text{"}1^\infty\text{"}, \text{etc.} \right)$ •Proofs of Cauchy's Mean Value Theorem and L'Hopital's Rule	3
Techniques of Integration I	•basic formulas, substitution (in examples such as $\int \frac{x\,dx}{\sqrt{5x+2}}$) •"Chain Rule in Reverse": $\int f'(u)\frac{du}{dx}dx = f(u) + C$	1
Techniques of Integration II	•Integration by parts (including application to examples such as $\int e^x \sin(x)dx$, $\int x^n e^x dx$, $\int \sec^3(x)dx$)	2
Techniques of Integration III	•Products of trigonometric functions	1
Techniques of Integration IV	•Integration by trigonometric substitution	2
Techniques of Integration V	Integration of Rational Functions by Partial Fractions •Linear factors •Irreducible quadratics	2

Techniques of Integration VI	Improper Integrals	1
Applications of Integration	•Arc length •Volume of a solid with known cross-sections •Volumes of solid of revolution (disc, washers and shells) •Area of a surface of revolution **(Optional)**	4
Parametric Equations	•Definition, domain and range • $\frac{dy}{dx}, \frac{d^n y}{dx^n}$ • the cycloid: definition, graph, area and length **(Optional)** •Graphing parametric equations	3
Polar Coordinates	•definition as a special case of parametric equations $(x = r(\theta)\cos(\theta), y = r(\theta)\sin(\theta))$ •relation to rectangular coordinates •dy/dx and interpretation of dy/dx at the pole •basic polar graphs (circles, three and four leaf roses, cardioids, lemniscates)	2
Taylor Series Polynomials	•Taylor and MacLaurin Series for functions such as $y = \sin(x)$, $y = e^x$, $y = \tan(x)$ •Taylor polynomials with remainder **(Optional)** •Taylor's Inequality ($\lvert R_n(x) \rvert \le \frac{M}{(n+1)!} \lvert x-a \rvert^{n+1}$ **(Optional)**	3
Functions of Several Variables	•Examples, domain, range •Partial derivatives •Double integrals (optional)	3
		$\sum = 36$

What's new and what's gone in Math 1210?

Gone:

-plotting points in R^3, distance formula, equations of planes (deferred to Advanced Calculus) (1 class)
-Quadric surfaces (deferred to Advanced Calculus) (1 class)
-symmetry conditions in polar coordinates (½ class)
-alternate equations for a given polar graph (eg., $r = 1 + \cos(\theta)$ and $r = -1 + \cos(\theta)$) (½ class)
-intersection of polar graphs (1 class)
-area in polar coordinates (1 class)

New:

-extra class on basic integration and substitution (1 class)

-Taylor and MacLaurin polynomials and series (3 classes)

Math 1200 Test 2A Tuesday, October 14, 2003

Name: ____________________ ____________________ ___
(First name) (Last name) 25

I.D. Number: ______________________________________

1(a). Prove using the ε / δ approach: $\lim_{x \to -6} \left(\frac{x-9}{x+3} \right) = 5$

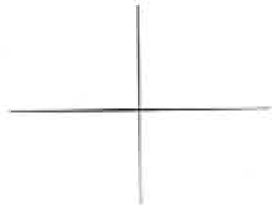
/6

(b) Your answer to part (a) should look like,"$0 < \delta \leq$ YOUR ANSWER IN TERMS OF ε".
Explain why we write this instead of
"$\delta =$ YOUR ANSWER IN TERMS OF ε".
(Explain both why $\delta \neq 0$ and why $\delta \leq$ YOUR ANSWER IN TERMS OF ε.)
/2

2. (a) Complete and illustrate the definition:

$\lim_{x \to \infty} f(x) = L$ if

/2



(b) Prove, using the definition in (a), that $\lim_{x \to \infty} \left(\frac{3x+1}{x+1} \right) = 3$

/3

3. Evaluate $\lim_{x \to -\infty} \left(\sqrt{x^2 - 5x} + x \right)$. Note: This is of the form " $\infty + (-\infty)$".

/5

4. Given that $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} g(x) = M$,

prove using the ε / δ approach that $\lim_{x \to a} \left(f(x) - g(x) \right) = L - M$.

/4

5. Let $f(x) = \begin{cases} 0, \text{ if } x \in Q \\ 1, \text{if } x \notin Q \end{cases}$, where Q is the set of rational numbers.

Prove that $\lim_{x \to 0} f(x)$ does not exist.

/4

UNIVERSITY OF GUELPH
MATH 1200 FINAL EXAM
FALL SEMESTER, 2003
Instructor: J. Weiner

First Name: ______________________

Last Name: ______________________

I.D. Number: ______________________

Please **check** that you have **19** pages, **INCLUDING** this cover page, **plus** a separate formula sheet and a computer answer sheet. Use the space beside each question for rough work. There are 40 multiple choice questions.

On the multiple choice answer sheet, enter your **seven digit ID in the first seven spaces** and leave the last two blank. In the e-mail address, enter your webct login Id. (This is the same User ID you use to log in to webct. For example, mine is jweiner.) Don't forget to enter your name! Ignore the "section number".

GOOD LUCK!

FILL IN YOUR NAME AND ID NUMBER (USE THE FIRST SEVEN DIGITS AND LEAVE THE LAST TWO BLANK–IGNORE THE SECTION NUMBER) ON THE COMPUTER BUBBLE SHEET. IN THE E-MAIL ADDRESS, ENTER YOUR WEBCT LOGIN ID. (THIS IS THE SAME USER ID YOU USE TO LOG IN TO WEBCT.)

FOR EACH QUESTION, PLEASE CIRCLE YOUR CHOICE FOR THE CORRECT ANSWER AND FILL IN THE CORRESPONDING BUBBLE ON THE COMPUTER ANSWER SHEET. (USE THE SPACE TO THE RIGHT OF THE QUESTIONS OR THE BACK OF THE EXAM PAGES FOR YOUR ROUGH WORK.)

1. The secant function $y = \sec(x)$ is undefined

A) when x equals $k\pi$, where k is any integer

B) when x equals $\pi/2 + k\pi$, where k is any integer

C) when x equals $k\pi/2$, where k is any integer

D) when x equals $-k\pi/2$, where k is any integer

E) never. $y = \sec(x)$ is defined for all real numbers x.

2. The solution to the inequality $\dfrac{-2(x-3)(x^2+1)}{(x+2)(x+1)} \geq 0$ is

A) $(-\infty, -2] \cup [-1, 3]$

B) $(-\infty, -2] \cup [-1, 3)$

C) $(-\infty, -2] \cup (-1, 3)$

D) $(-\infty, -2) \cup (-1, 3)$

E) $(-\infty, -2) \cup (-1, 3]$

3. Which of the following is false?

A) $-b \le |b| \le b$

B) $|a| - |b| \le |a - b|$

C) $|a| \le |b| \Leftrightarrow a^2 \le b^2$

D) $|a| \le b \Rightarrow a^2 \le b^2$

E) $-|b| \le -b \le |b|$

4. For which one of the following is it a **really bad idea** to solve by squaring both sides?

A) $|g(x)| < f(x)$

B) $g(x) = |f(x)|$

C) $|g(x)| < |f(x)|$

D) $g(x) < |f(x)|$

E) $1 \le |g(x)|$

5. The value of $\lim_{x \to -\infty} \left(\dfrac{\sqrt{3x + 16x^2}}{-7x+1} \right)$ is

A) $-4/7$

B) $-16/7$

C) $4/7$

D) $-\infty$

E) ∞

6. Find the value of $\lim_{x \to -64} \left(\dfrac{x+64}{x^{1/3}+4} \right)$.

A) -48

B) $1/48$

C) $-1/48$

D) 48

E) The limit does not exist since the expression approaches ∞ from one side and $-\infty$ from the other as x approaches -64.

7. Find the value of $\lim_{x \to 0} \left(\dfrac{1-\cos x}{5x} \right)$.

A) 1

B) 1/5

C) 0

D) $-\infty$

E) 5

8. The solution of $[[x^2]] = 143$ is

A) $\sqrt{143} \le x < 12$ or $-\sqrt{143} \le x \le -12$

B) $\sqrt{143} \le x < 12$ or $-12 \le x < -\sqrt{143}$

C) $\sqrt{143} \le x < 12$ or $-12 < x \le -\sqrt{143}$

D) $\sqrt{143} < x \le 12$ or $-\sqrt{143} \le x < -12$

E) $\sqrt{143} \le x \le 12$ or $-12 \le x \le -\sqrt{143}$

9. In proving an ϵ/δ limit, we correctly find only two conditions on $|x - a|$:
$0 < |x - a| < 1$ and $0 < |x - a| < \epsilon/6$. The best answer for δ is

A) $0 < \delta \le \min\{1, \varepsilon/6\}$

B) $0 \le \delta \le \min\{1, \varepsilon/6\}$

C) $0 < \delta < \min\{1, \varepsilon/6\}$

D) $0 \le \delta < \min\{1, \varepsilon/6\}$

E) There is not enough information. We need the original limit question.

10. Which of the following is the definition of $\lim_{x\to a} f(x) = -\infty$?

A) For every $N > 0$ there is an $\epsilon > 0$ such that if $|x - a| > N$ then $|f(x) - \infty| < \epsilon$

B) For every $N > 0$ there is an $\epsilon > 0$ such that if $0 < |x - a| < N$ then $|f(x) - \infty| < \epsilon$.

C) For every $N > 0$ there is an $\delta > 0$ such that if $0 < |x - a| < \delta$ then $f(x) > N$.

D) For every $N < 0$ there is an $\delta > 0$ such that if $0 < |x - a| < \delta$ then $f(x) < N$.

E) For every $N > 0$ there is an $\delta > 0$ such that if $0 < |x - a| < \delta$ then $f(x) < N$.

11. In proving $\lim_{x\to -1} \frac{1}{(x+1)^2} = \infty$, we want δ such that

when $0 < |x+1| < \delta$, then $\frac{1}{(x+1)^2} > N$. The best choice for δ is

A) $0 < \delta \le \frac{1}{\sqrt{N}}$

B) $0 < \delta \le \sqrt{N}$

C) $0 < \delta \le \frac{1}{N^2}$

D) $0 < \delta \le N^2$

E) $\delta \ge \frac{1}{\sqrt{N}}$

12. If $f(x) = \dfrac{x^2 - 9}{x + 3}$ and $g(x) = \dfrac{1}{x + 3}$, then

A) f and g have essential discontinuities at $x = -3$ and f has a removable discontinuity at $x = 3$.

B) f has an essential discontinuity at $x = 3$ and g has a removable discontinuity at $x = 3$.

C) f has a essential discontinuity $x = -3$ and g has a removable discontinuity at $x = -3$.

D) f has a removable discontinuity at $x = 3$ and g has an essential discontinuity at $x = 3$.

E) f has a removable discontinuity $x = -3$ and g has an essential discontinuity at $x = -3$.

13. For the function $f(x) = \begin{cases} 2x + A, & \text{if } x \le -1 \\ x^2 + 2x, & \text{if } -1 < x < 1 \\ 3Bx - 1, & \text{if } x \ge 1 \end{cases}$

to be **continuous at** $x = -1$, the value of A must be

A) You need the value of B in order to determine the value of A.

B) -1

C) 0

D) 1

E) 4

14. At what value(s) of x is $f(x) = \begin{cases} 2x + 1, & \text{if } x < -1 \\ x^2 + 4x + 2, & \text{if } -1 \le x < 1 \\ 6x + 1, & \text{if } x \ge 1 \end{cases}$ **not differentiable**?

A) -1

B) 1

C) -1 and 1

D) -1, 0, and 1

E) The function is differentiable for every $x \in \mathbb{R}$.

15. If $f(x) = \sec(h(x))$, then $f'(x)$ equals

A) $\sec(h'(x))\tan(h'(x))$

B) $\tan^2(h(x))\,h'(x)$

C) $\sec(x)\tan(x)\,h(x) + h'(x)\sec(x)$

D) $\sec(h(x))\tan(h(x))\,h'(x)$

E) $(\sec)(\tan)\,h(x) + (\sec)\,h'(x)$

16. If $x^2 + xy = 8 + y^3$, then $\dfrac{dy}{dx}$ equals

A) $\dfrac{3y^2 - x}{y + 2x}$

B) $\dfrac{y + 2x}{x - 3y^2}$

C) $\dfrac{-y - 2x}{3y^2 + x}$

D) $\dfrac{y + 2x}{3y^2 - x}$

E) $\dfrac{3y^2 - x}{-y - 2x}$

17. Given $x^2 + y^2 = 25$ and $\frac{dy}{dt} = 5$, find $\frac{dx}{dt}$ when $x = 4$.

A) $dx/dt = 15/4$ at the point $(4, -3)$ and $dx/dt = -15/4$ at the point $(4, 3)$

B) $dx/dt = -15/4$ at the point $(4, -3)$ and $dx/dt = 15/4$ at the point $(4, 3)$

C) $dx/dt = -20/3$ at the point $(4, -3)$ and $dx/dt = 20/3$ at the point $(4, 3)$

D) $dx/dt = 20/3$ at the point $(4, -3)$ and $dx/dt = -20/3$ at the point $(4, 3)$

E) None of the above.

18. To estimate $\frac{1}{63.8^{1/3}}$ using differentials, choose

A) $f(x) = x^{1/3}, \ x = \frac{1}{64}, \ dx = -0.2$

B) $f(x) = \left(\frac{1}{x}\right)^{1/3}, \ x = 64, \ dx = 0.2$

C) $f(x) = \left(\frac{1}{x}\right)^{1/3}, \ x = \frac{1}{64}, \ dx = -0.2$

D) $f(x) = \left(\frac{1}{x}\right)^{1/3}, \ x = 64, \ dx = -\frac{1}{0.2}$

E) $f(x) = \frac{1}{x^{1/3}}, \ x = 64, \ dx = -0.2$

19. The first derivative with respect to x of $x^3 + y^3 = 5$ is $dy/dx = -x^2/y^2$. The second derivative with respect to x, d^2y/dx^2 is equal to

A) $\dfrac{5x}{y^5}(x^3 - y^3)$

B) $\dfrac{-10x}{y^5}$

C) $\dfrac{10x}{y^5}$

D) $\dfrac{2x}{y^5}$

E) $\dfrac{-2x}{y^5}$

20. Which of the statements (1), (2), and (3) are true?

(1) Let $y = f(x)$ be a continuous function with a maximum value at $x = c$.
Then $f'(c) = 0$ or $f'(c)$ does not exist or $(c, f(c))$ is an end point.

(2) If $f(x)$ is differentiable on (a, b) then $f(x)$ is continuous on (a, b).

(3) If $f(x)$ is differentiable on the open interval (a, b), and $(c, f(c))$ is a maximum point for some $c \in (a, b)$, then $f'(c) = 0$.

A) Only (1) and (2) are true.

B) Only (2) and (3) are true.

C) Only (1) is true.

D) Only (2) is true.

E) All three are true.

21. Consider the function $f(x) = x^2$ on the interval [1, 2]. According to the Mean Value Theorem, there must be a number c in the interval (1, 2) such that $f'(c)$ is equal to a particular number M. What are c and M?

A) $c = 3/2$ and $M = 3$

B) $c = 2/3$ and $M = 3$

C) $c = 3$ and $M = 3/2$

D) $c = \sqrt{3}$ and $M = 3$

E) $c = \pm\sqrt{3}$ and $M = 3$

22. According to Rolle's Theorem, what is the number of real roots that the equation $x^5 + 3x + c = 0$ must have, where c is a constant?

A) 1

B) 2

C) 3

D) 4

E) 5

23. Let $f(x) = \dfrac{2x - 9}{3x + 4}$. Then $f(x)$ has

A) vertical asymptote $x = 2/3$ and horizontal asymptote $y = -9/4$

B) vertical asymptote $y = -4/3$ and horizontal asymptote $x = 2/3$

C) vertical asymptote $x = -4/3$ and horizontal asymptote $y = 2/3$

D) vertical asymptote $y = 2/3$ and horizontal asymptote $x = 2/9$

E) vertical asymptote $x = 2/9$ and horizontal asymptote $y = -3/4$

24. The function $f(x) = \left[(x+1)^2(x-1)^2\right]^{1/3}$ has first and second derivatives

$f'(x) = \dfrac{4x}{3(x+1)^{1/3}(x-1)^{1/3}}$ and $f''(x) = \dfrac{4(x-\sqrt{3})(x+\sqrt{3})}{9(x+1)^{4/3}(x-1)^{4/3}}$. The function is

decreasing on the intervals

A) $(-\infty, 0]$

B) $[-1, 0], [1, \infty)$

C) $(-\infty, -1], [0, 1]$

D) $[0, \infty)$

E) $[0, 1], [1, \infty)$

25. The function $f(x) = \left[(x+1)^2(x-1)^2\right]^{1/3}$ has first and second derivatives

$f'(x) = \dfrac{4x}{3(x+1)^{1/3}(x-1)^{1/3}}$ and $f''(x) = \dfrac{4(x-\sqrt{3})(x+\sqrt{3})}{9(x+1)^{4/3}(x-1)^{4/3}}$. The function has

A) a vertical tangent minimum at $x = -1$, a vertical tangent minimum at $x = 1$, a point of inflection at $x = 0$.

B) a vertical tangent maximum at $x = -1$, a horizontal tangent minimum at $x = 0$, a vertical tangent maximum at $x = 1$.

C) an end point minimum at $x = 1$ only since the function is not defined for $x < 1$.

D) a horizontal tangent minimum at $x = -1$, a vertical tangent maximum at $x = 0$, a horizontal tangent minimum at $x = 1$.

E) a vertical tangent minimum at $x = -1$, a horizontal tangent maximum at $x = 0$, a vertical tangent minimum at $x = 1$.

26. The function $f(x) = \left[(x+1)^2 (x-1)^2 \right]^{1/3}$ has first and second derivatives

$$f'(x) = \frac{4x}{3(x+1)^{1/3}(x-1)^{1/3}} \text{ and } f''(x) = \frac{4(x-\sqrt{3})(x+\sqrt{3})}{9(x+1)^{4/3}(x-1)^{4/3}}.$$ The function

A) is concave up on the interval $[-1, 1]$.

B) has points of inflection at $x = -\sqrt{3}$, $x = -1$, $x = 1$, and $x = \sqrt{3}$.

C) is concave down on the intervals $(-\infty, -\sqrt{3}]$, $[\sqrt{3}, \infty)$.

D) is concave down on the interval $[-\sqrt{3}, \sqrt{3}]$.

E) has points of inflection only at $x = -1$ and $x = 1$.

27. Geometrically, if $y = f(x)$ is continuous, then

$\int_a^b f(x)\,dx$ measures from $x = a$ to $x = b$ and bounded by $y = f(x)$

A) the area above the x axis

B) the area above the x axis together with the area below the x axis

C) the area above the x axis minus the area below the x axis

D) the average value of the function

E) none of the above is necessarily true. It depends on the function.

28. $\int \frac{x + x^2 + 2}{\sqrt{x}}\,dx$ equals

A) $\frac{\left(\frac{x^2}{2} + \frac{x^3}{3} + 2x\right)}{\left(\frac{2x^{3/2}}{3}\right)} + C$

B) $\frac{2}{3}x^{3/2} + \frac{2}{5}x^{5/2} + 4x^{1/2} + C$

C) $\frac{\left(\frac{x^2}{2} + \frac{x^3}{3} + 2x + C\right)}{\left(\frac{2x^{3/2}}{3} + D\right)}$

D) $\frac{3}{2}x^{3/2} + \frac{5}{2}x^{5/2} + x^{1/2} + C$

E) $\frac{2}{3}x^{3/2} + \frac{2}{5}x^{5/2} + x^{1/2} + C$

29. $\int -\csc x \cot x\,(1 + \csc x)\,dx$ equals

A) $\frac{(\csc x \cot x)^2}{2} + C$

B) $\frac{(1 + \csc x)^2}{2} + C$

C) $\frac{1 + (\csc x)^2}{2} + C$

D) $-\frac{(1 + \csc x)^2}{2} + C$

E) $\frac{(\csc x \cot x)^2\,(1 + (\cot x)^2)}{4} + C$

30. $\int \frac{x^3}{\left(x^4+1\right)^2}\, dx$ equals

A) $\frac{-1}{4(x^4+1)}+C$

B) $\frac{4}{3}\ln(x^4 + 1)+C$

C) $\frac{1}{4(x^4+1)}+C$

D) $-\frac{4}{(x^4+1)}+C$

E) $\frac{1}{4}\ln(x^4 + 1)+C$

31. $\int(\cos^3 x - 3\cos^{1/2} x + 5)\sin x\, dx$ equals

A) $-\cos^4 x/4 + 9\cos^{3/2} x/2 - 5\cos x + C$

B) $-\cos^4 x/4 + 9\cos^{3/2} x/2 - 5x + C$

C) $\cos^4(x)/4 - 2\cos^{3/2} x + 5\cos x + C$

D) $-\cos^4 x/4 + 2\cos^{3/2} x - 5\cos x + C$

E) $-\cos^4 x/4 + 2\cos^{3/2} x - 5x + C$

32. Which one of the following indefinite integrals **CANNOT be evaluated using the "chain rule in reverse".**

A) $\int \frac{\sin(x^{2/3})}{x^{1/3}}\,dx$

B) $\int x\,e^{x^2+x}\,dx$

C) $\int e^x e^{e^x}\,dx$

D) $\int (7x + 5)^3 dx$

E) $\int \sec^2 x \sin(\tan x)\,dx$

33. If we approximate $\int_a^b x^2 + x + 5\,dx$ using n equal subintervals and z_i to be the left endpoint of each subinterval, then

A) $x_i = \frac{(b-a)i}{n}$, $\Delta x_i = \frac{b-a}{n}$, $z_i = \frac{(b-a)i}{n}$

B) $x_i = a + \frac{(b-a)i}{n}$, $\Delta x_i = \frac{(b-a)(i-1)}{n}$, $z_i = a + \frac{(b-a)(i-1)}{n}$

C) $x_i = a + \frac{(b-a)i}{n}$, $\Delta x_i = \frac{b-a}{n}$, $z_i = a + \frac{(b-a)(i-1)}{n}$

D) $x_i = a + \frac{(b-a)i}{n}$, $\Delta x_i = \frac{(b-a)i}{n}$, $z_i = a + \frac{(b-a)(i-1)}{n}$

E) $x_i = a + \frac{(b-a)i}{n}$, $\Delta x_i = \frac{b-a}{n}$, $z_i = \frac{(b-a)i}{n}$

34. Let f be a function continuous on the interval $[a, b]$ and define $A(x) = \int_a^x f(t)\,dt.$

Which of the following is **NOT** true?

A) $A(a) = 0$

B) $A'(x) = f(x)$

C) if $F'(x) = f(x)$, then $\int_a^b f(t)\,dt = F(b) - F(a) = A(b)$

D) if $F'(x) = f(x)$, then $F(x) = A(x)$

E) $A'(b) = f(b)$

35. $\int_{\sqrt{e}}^{e^4} \frac{1}{x}\,dx$ equals

A) 3.5

B) $\ln(e^4 - \sqrt{e})$

C) $\frac{1}{e^4} - \frac{1}{\sqrt{e}}$

D) $\ln(3.5)$

E) $e^4 - \sqrt{e}$

36. $\frac{d}{dx}\left(\int_{\ln(x)}^{\sin^2 x} f(x)\,dx\right)$ equals

A) $f'(2\sin x \cos x) - f'(1/x)$

B) $2\sin x \cos x\, f(\sin^2 x) - (1/x)\, f(\ln x)$

C) $f(2\sin x \cos x) - f(1/x)$

D) $f(\sin^2 x) - f(\ln x)$

E) $2\sin x \cos x\, f'(\sin^2 x) - (1/x)\, f'(\ln x)$

37. Let $f(x) = a^x$, where a is a constant, $a > 0$. Which of the following is false?

A) $f'(0) = \lim_{h \to 0}\left(\frac{a^h - 1}{h}\right)$

B) $\lim_{h \to 0}\left(\frac{a^h - 1}{h}\right) = \ln a$

C) $\frac{d(a^x)}{dx} = a^x \Leftrightarrow a = e$

D) $f'''(x) = a^x(3\ln a)$

E) $a^x > 0$ for all $x \in \mathbb{R}$

38. The area of the region bounded by

$y = \sin x$ and $y = \cos x$ from $x = 0$ to $x = \pi$ is given by

A) $\int_0^{\pi} \sin x - \cos x \, dx$

B) $\int_0^{\pi} \cos x - \sin x \, dx$

C) $\int_0^{\pi/4} \cos x - \sin x \, dx + \int_{\pi/4}^{\pi} \sin x - \cos x \, dx$

D) $\int_0^{\pi/2} \cos x - \sin x \, dx + \int_{\pi/2}^{\pi} \sin x - \cos x \, dx$

E) $\int_0^{\pi/4} \sin x - \cos x \, dx + \int_{\pi/4}^{\pi} \cos x - \sin x \, dx$

39. Let $y = (\sin x)^{\tan x}$. Then $\frac{dy}{dx}$ equals

A) $(\sin x)^{\tan x} (\cos x)^{\sec^2 x}$

B) $(\sin x)^{\tan x} \left(\sec^2 x - \cos x\right)$

C) $(\sin x)^{\tan x} \left(1 + \sec^2 x \ln(\sin x)\right)$

D) $(\sin x)^{\tan x} \left(-1 + \sec^2 x \ln(\sin x)\right)$

E) $(\cos x)^{\sec^2 x}$

40. Consider the function $f(x) = x^3$ on the interval [1, 3]. According to **The Mean Value Theorem for Integrals,** there is a number c in [1, 3] such that $f(c)$ (which measures the average value of the function on the interval [1, 3]) is equal to a particular number M. The value of c is

A) 10

B) $10^{1/3}$

C) $\left(\frac{13}{3}\right)^{1/2}$

D) 20

E) $20^{1/2}$

I truly hope you did well. Have a great holiday and come back in January happy and healthy. I look forward to seeing you in Math 1210.

Best wishes,

Jack Weiner