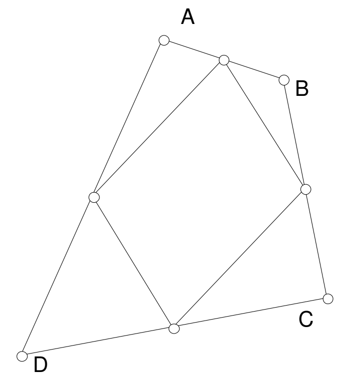
## <u>Thinking, Inquiry and Problem-Solving</u> Name:____________________

Complete the following with the use of your notes and textbook.

1. Given the scalar equation of a plane, $5x - 7y + z - 20 = 0$, and a point not on the plane, A(-5,16,7) here are two methods to find the shortest distance from A to the plane. Demonstrate each method, showing the steps clearly.

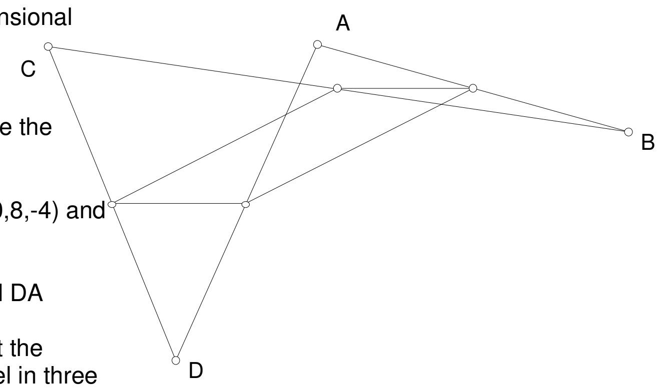
Method 1: Find any point on the plane. Call it P. Find the vector projection of the vector $\overline{AP}$ on the normal of the plane. The magnitude of the projection is the distance from the point A to the plane.

Method 2: Find a vector equation of the line through A that is perpendicular to the plane. Find the point of intersection of the line found and the plane. Call the point F (*the foot of the perpendicular from A to the plane*). Find the magnitude of the vector $\overline{AF}$. This is the shortest distance from the point A to the plane.




2. It is possible to prove that the lines joining consecutive midpoints of the sides of a convex quadrilateral form a parallelogram.
(see example 3 on page 18 of your text)
In three dimensions, this property of quadrilaterals can be extended. The original quadrilateral can be replaced with a set of 4 points in any position, even a set points that are not in the same plane. It is possible to show that the lines joining consecutive midpoints of the sides still form a parallelogram (as illustrated below at right).

Your job is to demonstrate, **or** prove this three dimensional property is true.
**<u>You have a choice</u>** to do **either** a) or b) below:




a) **<u>For level 3</u>**, prove a **particular set of points** have the property.

Here are the points to use: A(-2,0,-2), B(6,2,10), C(0,8,-4) and D(-6,-2,4).

Find M,N,P and Q, the midpoints of AB, BC, CD and DA respectively.
Show that MNPQ is a parallelogram by showing that the opposite sides of the quadrilateral MNPQ are parallel in three space (slope won't work, you need another way to show they are parallel).

b) **<u>For level 4</u>**, prove a **general set of points** have the property.

Here are the points to use: O(0,0,0), A(2a,0,0), B(2a,2b,0) and C(2d,2e,2f) . Notice that 0,A and B are coplanar (on the xy plane), but C is not on their plane, and so these four points can be considered to represent any four locations in space.

Find M,N,P and Q, the midpoints of OA, AB, BC and CO respectively.
Show that MNPQ is a parallelogram by showing that the opposite sides of the quadrilateral MNPQ are parallel in three space for all possible values of the constants a, b, d, e and f.