Contagion Channels for Financial Systemic Risk

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Our Research Project

Main Aim

To create a computational framework that provides justifiable answers to a broad range of "what if?" questions about systemic risk in random financial networks.

Aspects of the Main Aim

1. random financial network (RFN): stochastic model for N banks, their balance sheets, behaviour and mutual exposures.
2. systemic risk (SR): the risk that default or stress of one or more banks will trigger default or stress of further banks, leading to large scale cascades of failures in the RFN.
3. computational framework:
   1. rigorous asymptotic analysis as $N \to \infty$;
   2. Monte Carlo simulations for finite N.
4. Typical what if? question: What if the RFN with parameter θ experiences a random shock? Is there a critical "knife-edge" value θ^* sharply separating cascading from non-cascading?
5. justifiable answers:
   - clear, reasonable assumptions;
   - rigorous analysis;
   - robust conclusions.

Why Study Systemic Risk?

1. The climax of the crisis in 2008 was predominantly a network crisis driven by two major explosions:
    - The buyers of CDS protection from AIG were unaware of the huge exposures AIG had taken on to its balance sheet.
    - Similarly, the true nature of Lehman Bros' highly levered balance sheet was massively obscured by their illegal use of the infamous "Repo 105" transactions.
2. Much of Basel III is macroprudential: Reporting and limits on large exposures to individual counterparties or groups of counterparties; the Liquidity Coverage Ratio (LCR) and the Net Stable Funding Ratio (NSFR); the capital surcharges on SIFIs.
3. New interbank exposure databases will need new theory.
4. It's fun.

Channels of Systemic Risk

There are at least four important channels of Systemic Risk:

1. **Correlation:** The system may be impaired by a large correlated asset shock.
2. **Default Contagion:** Default of one bank may trigger defaults of other banks.
3. **Liquidity Contagion:** Funding illiquidity of one bank may trigger illiquidity of other banks.
4. **Market Illiquidity:** Large scale asset sales by one or more distressed banks may trigger a "firestorm" or downward price spiral, further impairing the entire system.

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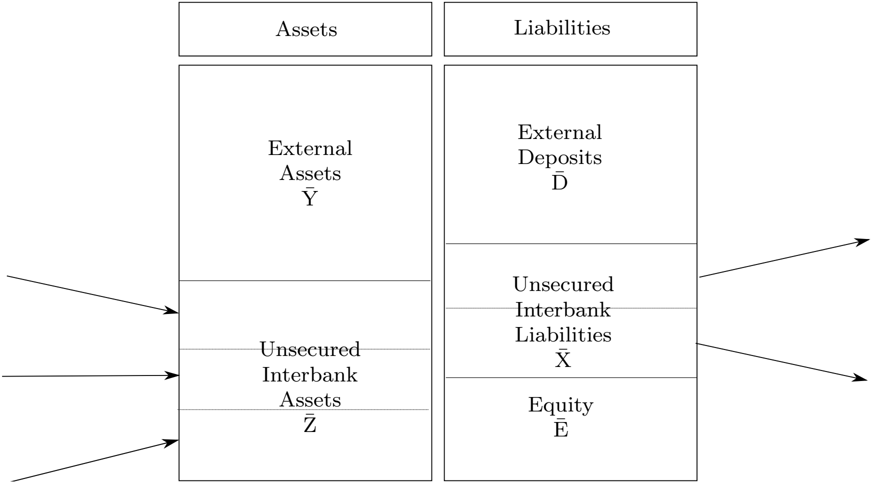
Static Cascade Models

- Contagion effects in financial networks are analogous to the spread of disease.
- A number of distinct mechanisms can be identified.
- We model such mechanisms first in static cascades.
- Static means during the cascade we ignore external shocks (in particular central bank actions) and focus only on internally generated shocks.

Eisenberg-Noe 2001 Model: Balance Sheets



EN2001 Insolvency Cascade

- Total nominal assets $= \bar{\mathrm{Y}}_v + \bar{\mathrm{Z}}_v, \ \bar{\mathrm{Z}}_v = \sum_w \Omega_{wv}$;
- Total nominal liabilities $= \bar{\mathrm{D}}_v + \bar{\mathrm{X}}_v + \bar{\mathrm{E}}_v, \ \bar{\mathrm{X}}_v = \sum_w \Omega_{vw}$.
- Ω_{vw} = amount bank v owes bank w.
- We assume a bank v defaults whenever its mark-to-market equity becomes zero (it can't go negative):

$$\mathrm{E} = \text{Assets} - \text{Liabilities} = 0$$

- Then any creditor bank w is forced to mark down its interbank assets, thus receiving a default shock.

EN2001 Cascade Mapping

1. At the onset of the cascade, some banks have $\Delta_v^{(0)} = \bar{\mathrm{E}}_v \leq 0$ and become primary defaults.
2. Let $p_v^{(n)}$ be amount of interbank debt v can pay after n steps of the cascade.
3. The mark-to-market value of interbank assets is then

$$\mathrm{Z}_v^{(n)} = \sum_w \Pi_{wv} p_w^{(n-1)}, \ \Pi_{wv} = \Omega_{wv}/\bar{\mathrm{X}}_w$$

4. and

$$p_v^{(n)} = F_v^{(EN)}(\mathrm{p}^{(n-1)}); F_v^{(EN)}(\mathrm{p}) := \max(0, \min(\bar{\mathrm{X}}_v, \bar{\mathrm{Y}}_v + \sum_w \Pi_{wv} p_w - \bar{\mathrm{D}}_v))$$

5. Clearing condition is fixed point of mapping, guaranteed to exist by Tarski Fixed Point Theorem:

$$\mathrm{p} = F^{(EN)}(\mathrm{p})$$

EN2001 Default Buffer Mapping

1. If $\Delta_w^{(n)}$ denotes the default buffer after n cascade steps, then

$$\begin{aligned} \Delta_w^{(n)} &= \Delta_w^{(0)} - \sum_v \Omega_{vw} \left(1 - h(\Delta_v^{(n-1)}/\bar{X}_v)\right) \\ p_w^{(n)} &= \bar{X}_v \, h(\Delta_v^{(n-1)}/\bar{X}_v) \end{aligned}$$

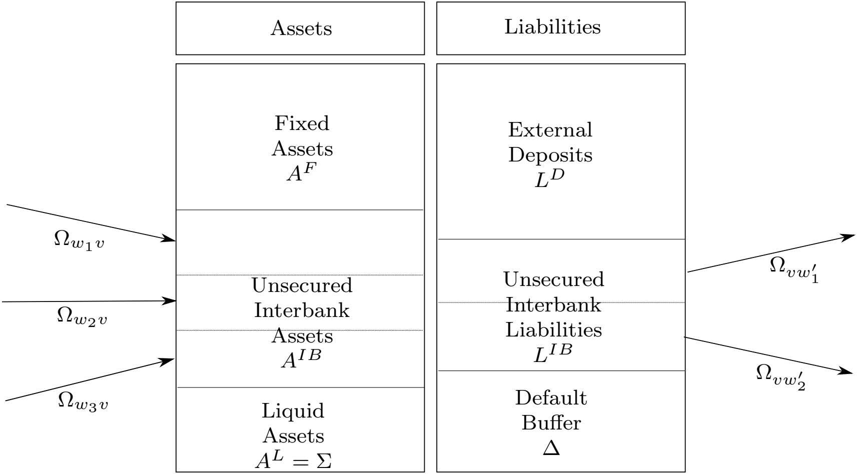
2. Threshold functions such as

$$\begin{aligned} h(x) &= \max(x+1,0) - \max(x,0) \\ \text{or} \quad \tilde{h}(x) &= \mathbf{1}_{x>0} \end{aligned}$$

   determine fractional recovered value of defaulted assets.

3. As $n \to \infty$, buffers $\Delta_w^{(n)}$ converge to unique fixed point $\Delta^+ = \{\Delta_v^+\}$ of solvency cascade mapping.
4. Gai-Kapadia 2010 Model is formally identical to EN2001, but with h replaced by $\tilde{h}$.

Illiquidity Cascade: Balance Sheets



Illiquidity Cascade: Seung Hwan Lee 2013 Model

1. At time 0, banks experience deposit withdrawals $\Delta d_v \geq 0$.
2. These are paid immediately in order of seniority by...
3. First liquid assets $\bar{Z} + \bar{Y}^L$, then fixed assets $\bar{Y}^F$.
4. Debtor banks receive liquidity shocks;
5. Let bank v have initial liquidity buffer $\Sigma_v^{(0)} = -\Delta d_v \leq 0$
6. After $n-1$ cascade steps, then

$$\Sigma_w^{(n)} = \Sigma_w^{(0)} - \sum_v \Omega_{wv} \left(1 - h(\Sigma_v^{(n-1)}/\bar{Z}_v)\right)$$

7. As $n \to \infty$, buffers $\Sigma_w^{(n)}$ converge to unique fixed point $\Sigma^\infty = \{\Sigma_w^\infty\}$ of liquidity cascade mapping.
8. Mathematically identical to EN 2001! The Gai-Haldane-Kapadia 2011 Liquidity Cascade is also formally identical to GK 2010.

Single Buffer Models

1. In these models, each bank's behaviour, and hence the cascade itself, is determined by a single buffer Δ_v or Σ_v.
2. Single buffer models can involve multiple thresholds.

A Double Buffer Model

1. In more complex models, banks' behaviour is determined by two or more buffers.
2. HCCMS 2013 introduces a double cascade model of illiquidity and insolvency, intertwining two buffers Δ_v, Σ_v, that combines the essence of both [GK, 2010a] default cascade and [GK, 2010b] liquidity cascade.
3. No non-contagion channels of SR: We assume them away.

Question

What effect does a bank's behavioural response to liquidity stress have on the probable level of eventual defaults in entire system?

Crisis Timing Assumptions

1. The crisis commences on day 0 after initial shocks trigger default or stress of one or more banks;
2. Balance sheets are recomputed daily;
3. Banks respond daily ;
4. External cash flows, interest payments, asset and liability price changes are ignored throughout crisis.

Bank Behaviour Assumptions

On each day of the crisis:

1. Insolvent banks, characterized by $\Delta = 0$, default 100% on their IB obligations. Its creditor banks write down their defaulted exposures to zero thereby experiencing a *solvency shock*.
2. A stressed bank, any non-defaulted bank with $\Sigma = 0$, reduces its IB assets A^{IB} to $(1-\lambda)A^{IB}$, transmitting a *stress shock* to the liabilities each of its debtor banks.
3. λ is a constant across all banks.
4. A newly defaulted bank also triggers maximal stress shocks.

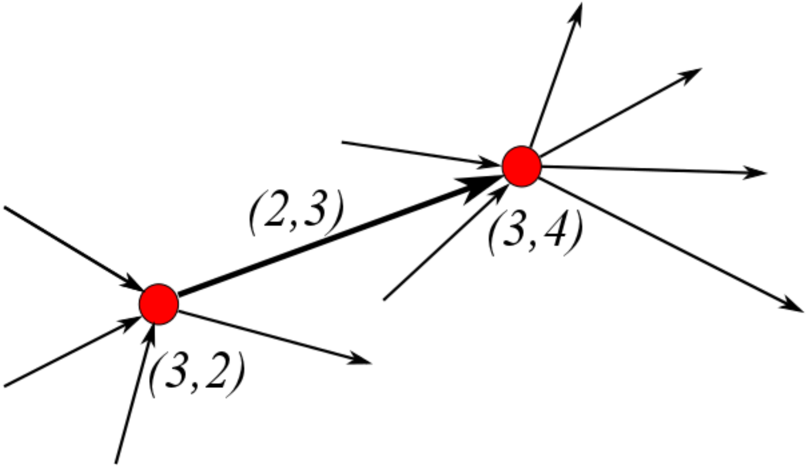
Critique of Static Cascade Models

1. Real world financial systems are far from these models.
2. Bank balance sheets are hugely complex.
3. Interbank exposure data are never publicly available.
4. Interbank exposures are known to change rapidly day to day.
5. Banking networks are often highly heterogeneous.

3 Reasons to Study Large Stochastic Networks

1. Even a completely known deterministic system, if it is large enough, can be well described by the average properties of the system.
2. Balance sheets of banks, between reporting dates, are not observed even in principle, and change quickly.
3. Even a fully known hypothetical financial system will be hit constantly by random shocks from the outside, stochastic world.

2 Nodes and 1 Edge



Random Financial Network (RFN)

...is a quintuple $(\mathcal{N}, \mathcal{E}, \Delta, \Sigma, \Omega)$ where

- $\mathcal{N}, \mathcal{E}$ is a directed random configuration graph (the "skeleton"):
  - nodes $v \in \mathcal{N}$ represent "banks";
  - directed links $\ell \in \mathcal{E}$ represent interbank exposures.
- $\Delta = (\Delta_v)_{v \in \mathcal{N}}$ is the set of random default buffers;
- $\Sigma = (\Sigma_v)_{v \in \mathcal{N}}$ is the set of random stress buffers;
- $\Omega = (\Omega_\ell)_{\ell \in \mathcal{E}}$ is the set of random interbank exposures.

1. Random configuration graphs are characterized by in/out degree distribution matrices $\{P_{jk}, Q_{kj}\}$.
2. Random variables have CDFs $\{D_{jk}(x), S_{jk}(x), W_{kj}(x)\}$.
3. Initially insolvent (or stressed) banks have $\Delta_v \leq 0$ $(\Sigma_v \leq 0)$.

The Cascade Problem

Define conditional stress and default probabilities after n cascade steps:

$$p_{jk}^{(n)} = \mathbb{P}\left[v \in \mathcal{D}_n | v \in \mathcal{N}_{jk}\right] \ , \qquad (1)$$
$$q_{jk}^{(n)} = \mathbb{P}\left[v \in \mathcal{S}_n | v \in \mathcal{N}_{jk}\right] \ .$$

Problem

Given the RFN $(\mathcal{N}, \mathcal{E}, \Delta, \Sigma, \Omega)$, compute p_{jk}^{∞} and q_{jk}^{∞}, the probabilities that a type (j, k) bank eventually defaults or becomes stressed.

LTI: Locally Tree-like Independence property

$N = \infty$ configuration graphs have a locally tree-like (LT) property. We extend this notion to RFNs by assuming a certain conditional independence on balance sheet random variables:

Assumption

LT **independence property**

The Role of LTI

It leads to conditions under which probabilities like this can be computed using independence:

$$\mathbb{P}[\ \Delta_v \leq \sum_{w\in\mathcal{N}_v^-} \Omega_{wv}\xi_{wv}^{(n)}, \Sigma_v \leq \sum_{w\in\mathcal{N}_v^+} \Omega_{vw}\zeta_{vw}^{(n)} | \text{conditions}]$$

fractional default on link (↑ $\xi_{wv}^{(n)}$) fractional stress on link (↑ $\zeta_{vw}^{(n)}$)

Cascade Mapping Theorem (Simplified)

Suppose quantities $p_{jk}^{(n-1)}, q_{jk}^{(n-1)}, t_{kj}^{(n-1)}$[1] are known. Then

$$p_{jk}^{(n)} = \left\langle D_{jk}, \left(g_j^{(n-1)}\right)^{\circledast j} \right\rangle$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product on $\mathbb{R}_+$ and $\circledast$ denotes convolution. Here

$$\begin{aligned} g_j^{(n-1)}(x) &= \sum_{k'} \Big[(1 - p_{k'}^{(n-1)})\delta_0(x) + t_{k'j}^{(n-1)} w_{k'j}(x) \\ &\quad + (p_{k'}^{(n-1)} - t_{k'j}^{(n-1)}) \cdot \frac{1}{1-\lambda} w_{k'j}(x/(1-\lambda)) \Big] \cdot Q_{k'|j} \end{aligned}$$

Similar formulas hold for $q_{jk}^{(n)}, t_{kj}^{(n)}$.

[1] $t_{kj}^{(n-1)}$ is probability link is 100% defaulted.

Poisson Experiment 1A: LTI vs MC

- Poisson random directed graphs $(\mathcal{N}, \mathcal{E})$ with mean connectivity $z = 10$;
- Buffer distributions $\Delta_v = 0.04$ and $\Sigma_v = 0.02$ where total assets are $A_v = 1$;
- Edge distribution Ω_ℓ: log normal with means $\mu_\ell = \frac{1}{5 j_\ell}$, standard deviation $\sigma_\ell = 0.4\mu_\ell$;
- Initial shock: random subset of nodes that default;
- $\lambda \in [0, 1]$ represents the "stress response" parameter.
- Analytic formulas using $N = \infty$ LTI approximation are compared with $N = 20000$ Monte Carlo estimators.

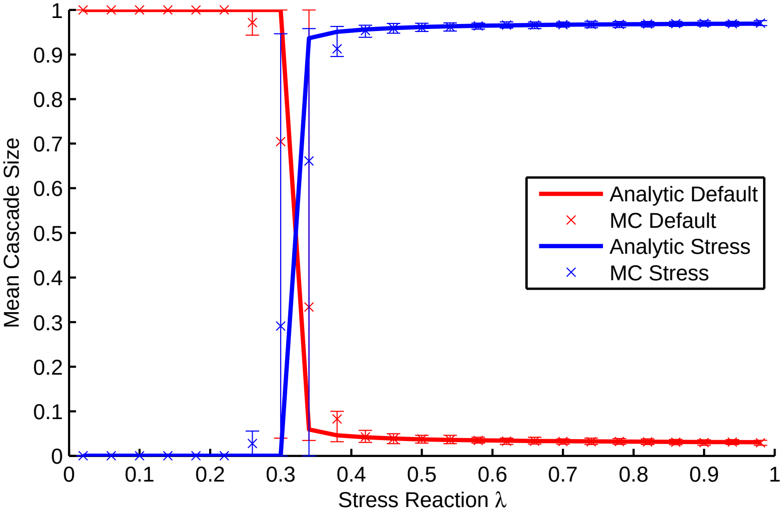


Figure : Experiment 1A: Comparison of MC vs LTI analytics on Poisson network, with errors bars for MC

Rules of Thumb: LTI Analytics vs Monte Carlo

Remark

- The discrepancies are concentrated around the knife-edge, that is, the cascade phase transition.
- Monte Carlo variance is also extremely high around the knife-edge.
- Stress and default are negatively correlated.

Poisson Experiment 1B: Default Size vs Δ and Σ

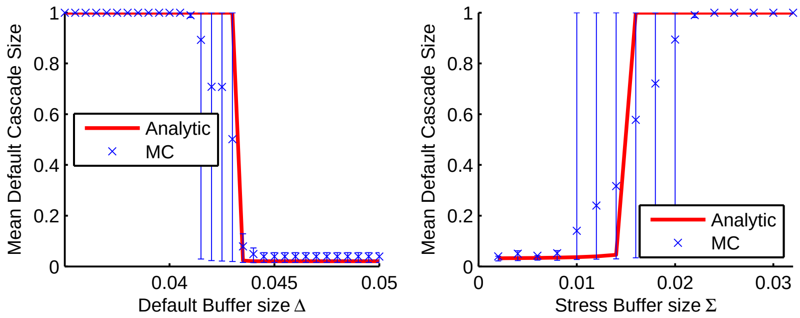


Figure : (l) The effect of default buffer. (r) The effect of stress buffer. MC error bars are shown.

Poisson Experiment 1C: Cascade Size vs z and λ

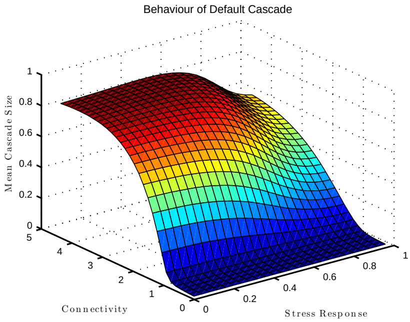


Figure : Stress and default cascade sizes on Poisson networks as functions of z and λ.

Experiment 2: Real-World Model of EU System

- Skeleton graph: $N = 90$ node, $L = 450$ edge subgraph of a single realization of a 1000 node scale-free graph.
- Default buffers $\Delta_v = (k_v j_v)^{\beta_1} \exp[a_1 + b_1 X_v]$;
- Stress buffers $\Sigma_v = \frac{2}{3}(k_v j_v)^{\beta_1} \exp[a_1 + b_1 \tilde{X}_v]$;
- Exposures $\Omega_\ell = (k_\ell j_\ell)^{\beta_2} \exp[a_2 + b_2 X_\ell]$;
- $\{X_v, \tilde{X}_v, X_\ell\}$ are I.I.D. standard normals;
- Parameters match moments of interbank exposure data

$$\beta_1 = 0.3, a_1 = 8.03, b_1 = 0.9, \beta_2 = -0.2, a_2 = 8.75, b_2 = 1.16$$

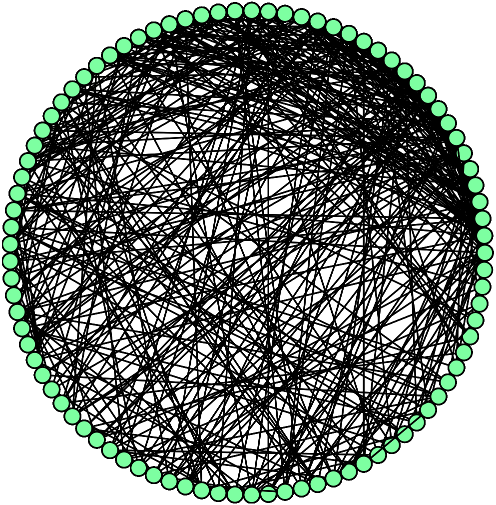


Figure : Undirected skeleton graph of stylized 90 bank EU network.

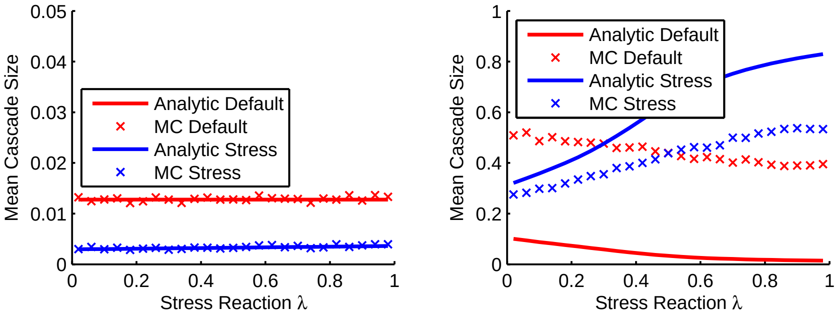


Figure : (l) EU resilience in normal times; (r) EU cascade after an extreme crisis.

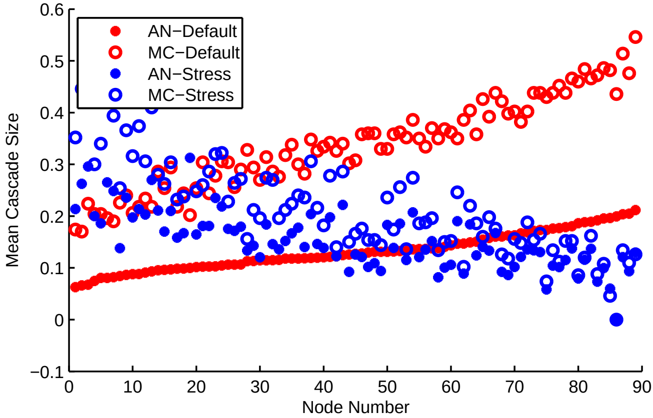


Figure : Default and stress probabilities of individual EU banks after extreme crisis.

Overall Summary

- We have developed a static framework for understanding general cascade mechanisms in financial networks.
- We have defined systemic risk (SR) in random financial networks (RFNs);
- We have a flexible computational framework, analytical for $N = \infty$ and Monte Carlo, even in complex model specifications;
- We have justifiable answers to a host of what if? questions.

Rules of Thumb: Double Cascade Model

- The stress response parameter λ and stress buffer Σ strongly control network resilience to default;
- These complex cascade models exhibit critical regions just as predicted by simple cascade models.
- LTI analytics and Monte Carlo work best, and agree best, when the system is far from critical;

Some General Observations

- Our RFNs are a powerful laboratory for studying such complex problems;
- Our experiments reveal systemic responses that are difficult to predict, but explicable in hindsight;
- In "realistic" networks, cascades are not triggered unless conditions become "extreme" for other reasons.
- Many model parameters that have strong effects on the stability of such systems still remain to be studied.
- There are many stories to tell about the network effects that can happen.

Some References

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