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The Macro-Financial Risk Assessment Framework (MFRAF): Model Features and Policy Use

Fields Quantitative Finance Seminar
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Agenda

1. Macro Stress Tests (MSTs) at the Bank of Canada (BoC)
2. MFRAF Overview
   a. Solvency risk
   b. Funding liquidity risk
   c. Information contagion / Contagious runs
   d. Network effects
3. Calibrating MFRAF
4. Some *hypothetical* results
5. Conclusion

1. Overview of MSTs

- Annual exercise conducted jointly by the BoC and OSFI
- **Objective:** To assess the resilience of the financial system to extreme but plausible shocks
- Involves the "big six" Canadian banks

1. MST scenario
2. Bottom-up stress test exercise:
    a. Bank's apply MST scenario to their balance sheet
    b. Focuses on solvency risk only
3. Top-down stress test exercise:
    a. MFRAF

1. Example of a MST scenario

- Materialization of key risks identified in the FSR, e.g.,
  - Euro area crisis
  - Canadian household finance and housing price shock

- **Trigger:** Disorderly default of a peripheral eurozone country

- **Transmission mechanisms:** Disruption in funding markets; financial contagion; adverse confidence and wealth effects

- **Outcome:** Severe and persistent economic recession and slow recovery over a 3-year horizon

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1. Bottom-up stress testing

- Banks apply the MST scenario to their balance sheets using internal models and report the results back to OSFI
- The exercises yield detailed information on the resilience of banks
- Drawbacks:
  - Does not offer a systemic perspective, as it ignores liquidity risks and network effects

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1. Top-down stress testing: MFRAF

- MST scenario is consistently applied to a suite of internally developed and calibrated models (MFRAF) that accounts for different risks:
  - Solvency risk
  - Funding liquidity risk and contagious runs
  - Interbank network spillovers

- MFRAF provides a systemic perspective on risks to the banking sector, and also serves as a consistency check for the bottom-up stress test exercise

- Disadvantage: “A model is only as good as its assumptions”

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1. Objectives of MFRAF

- Provides a consistency check for the bottom-up exercise
- Quantitative tool for assessing the systemic impact of key risks to the financial system
- Framework to look at policy options, e.g.
  - Capital vs. liquidity requirements
  - Measure of systemic risk contribution of an individual bank

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2. Structure of MFRAF

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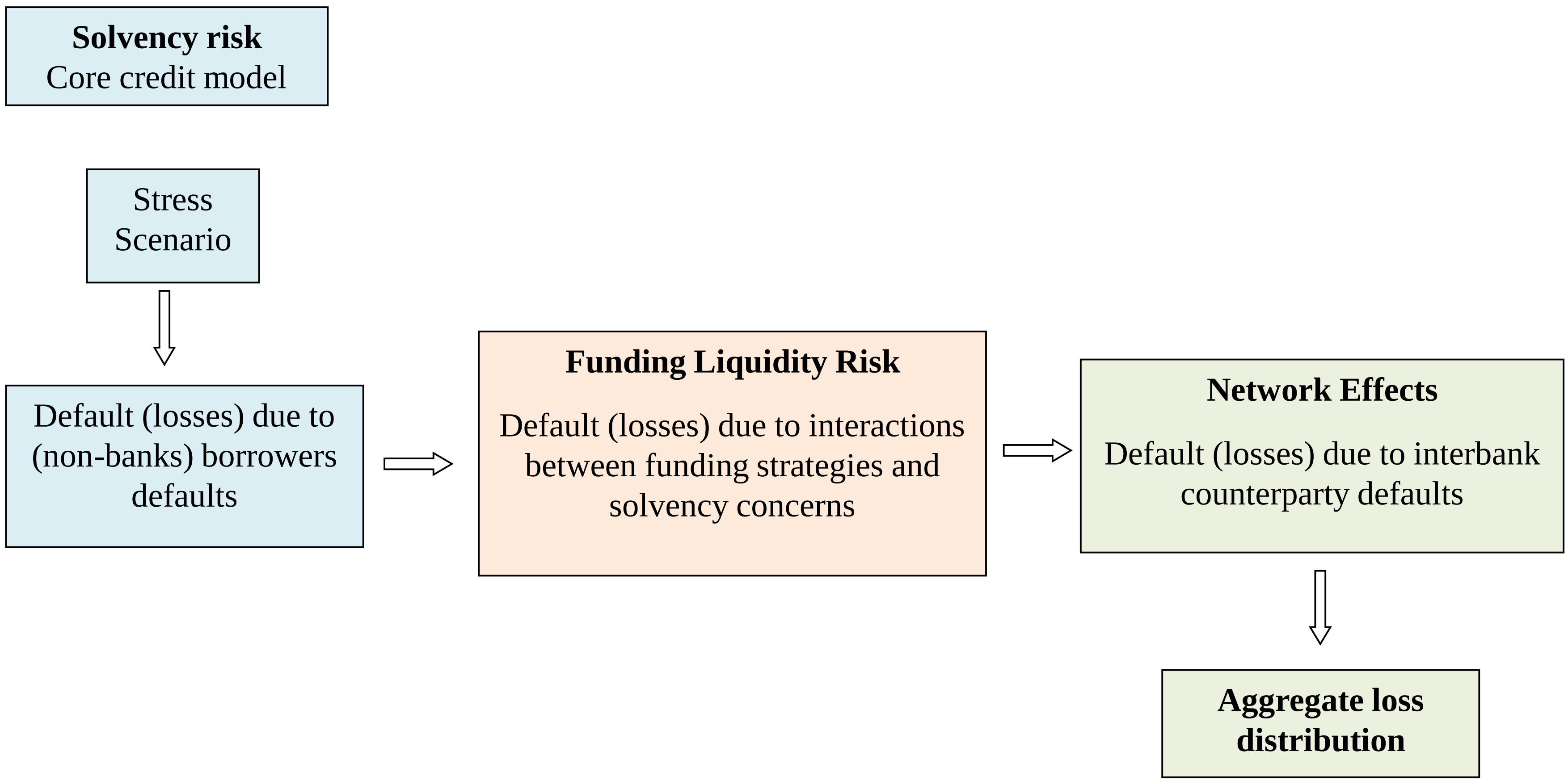
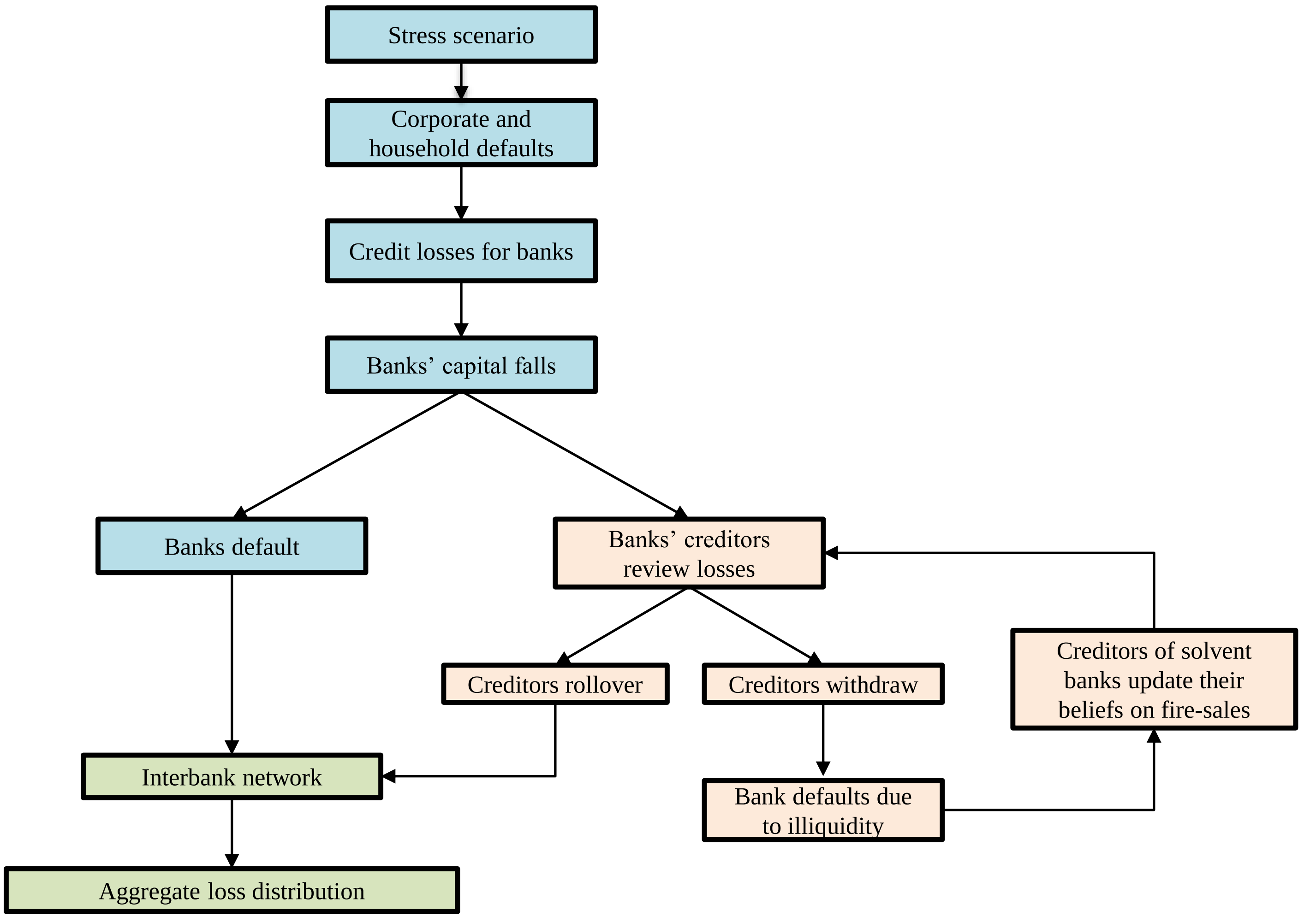
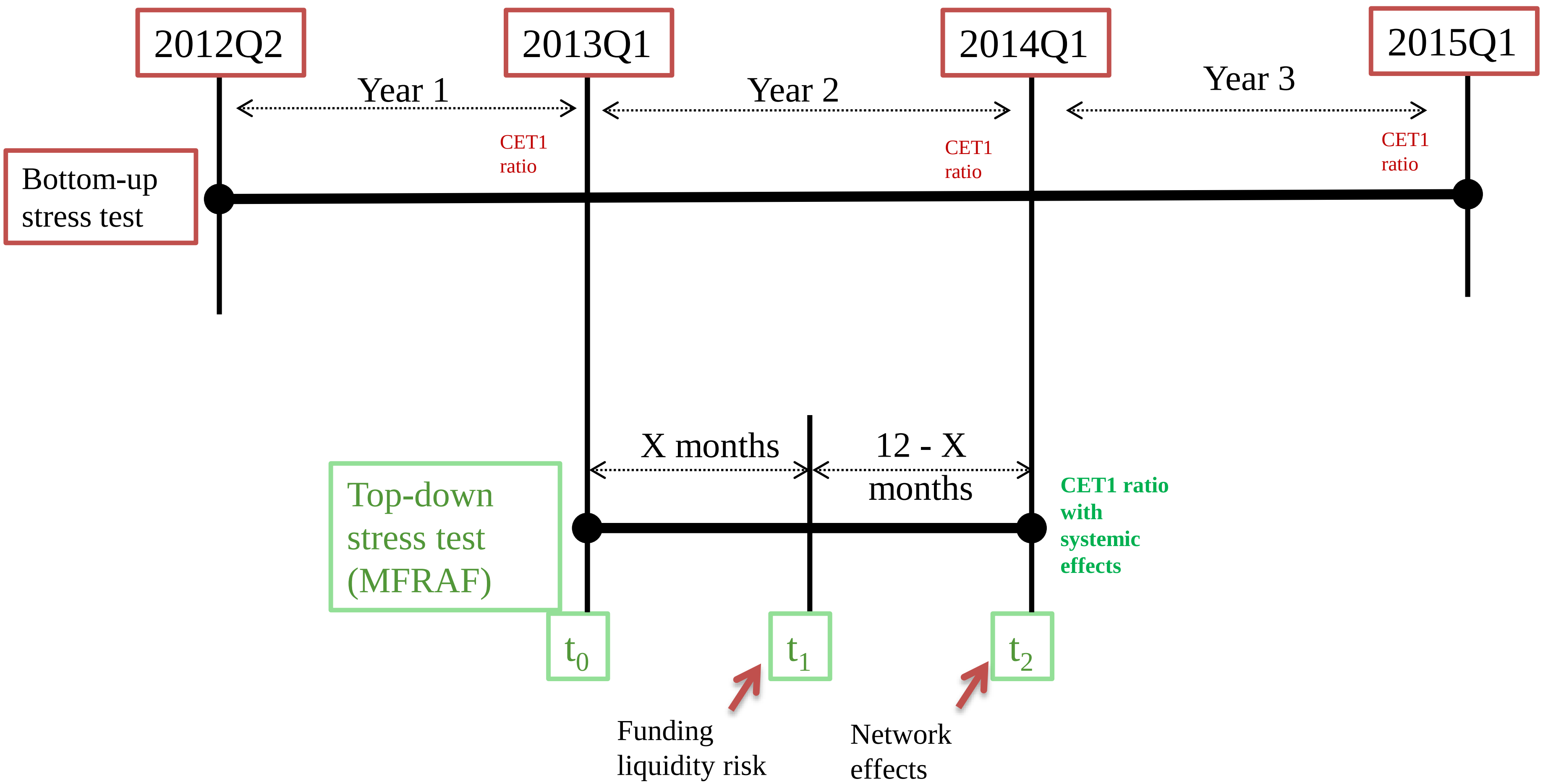
2. MFRAF: sequential framework



2. MFRAF: sequential framework (continued)



2. Timelines


2012Q2
2013Q1
2014Q1
2015Q1
Year 1
Year 2
Year 3
Bottom-up stress test
CET1 ratio
CET1 ratio
CET1 ratio
X months
12 - X months
Top-down stress test (MFRAF)
CET1 ratio with systemic effects
t0
t1
t2
Funding liquidity risk
Network effects

2. Bank's t_0 (initial) balance sheet

Illiquid assets I_0	Long term liabilities
Liquid assets M_0	Short term liabilities (coming due in X months) S_0
	Capital E_0

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2.a Solvency risk

- Banks' loan portfolios subject to credit risk across different sector, e.g., business, government, consumer

$$\text{Expected Losses} = \text{Probability of Default} \times \text{Loss Given Default} \times \text{Exposure at Default}$$

  - PDs (distribution) – function of macro-variables.
  - LGDs – judgement based, e.g., from bottom-up exercises
  - EADs – banks' regulatory reported values

- Derive **annual loss distributions** for each sector and for each bank

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2.a Solvency risk

- Each realization of the expected annual losses, $P^{(E)}$, must be translated into the time structure of MFRAF

- Losses $P^{(1)}$ realized at date t_1 (interim period)

- Losses $P^{(2)}$ realized at date t_2 (final period)

$$P^{(1)} = \frac{P^{(E)}}{12/X}, \text{and} \;\; P^{(2)} = P^{(E)} \times \left(1 - \frac{1}{12/X}\right)$$

2.a Bank's ex-post (t_2) balance sheet

Illiquid assets $I_0 - P^{(1)} - P^{(2)}$	Long term liabilities
Liquid assets M_0	Short term liabilities (coming due in X months) S_0
	Capital $E_0 - P^{(1)} - P^{(2)}$

Ex-post solvency condition

$$E_0 - P^{(1)} - P^{(2)} > 0.$$

2.b Liquidity risk

- At the interim date, t_1, following the realization of the $P^{(1)}$ losses, a bank's creditors may decide to run
- Runs may occur due to:
  - Concerns over the bank's **future solvency**;
  - **Low liquidity**, relative to it's wholesale funding

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2.b Liquidity risk

- **Illiquidity condition:** a bank fails if the fraction of creditors who foreclose (ℓ) is greater than the banks' recourse to liquidity, i.e.,

$$\ell \times S_0 > M_0 + \bar{\psi} \times \left(I_0 - P^{(1)}\right),$$

where $\bar{\psi}$ is the expected fire-sale price for the bank's illiquid assets

Balance Sheet Liquidity: $$\lambda \equiv \frac{M_0 + \bar{\psi} \times \left(I_0 - P^{(1)}\right)}{S_0}$$

2.b Liquidity risk – the rollover game

- Decisions of creditors modeled as a simultaneous move coordination game

- Binary choice model – each creditor must decide whether to
  (2) withdraw deposits, or
  (1) rollover deposits

- Payoffs for an individual creditor:
  - Withdraw – r^F, irrespective of whether the bank survives, or not
  - Rollover – $r^S > r^F$, if the bank survives, and zero otherwise

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2.b Liquidity risk – the rollover game

	$\ell \leq \lambda$	$\ell > \lambda$
Rollover	r^S	0
Withdraw	r^{F}	r^{F}

2.b Liquidity risk – the rollover game

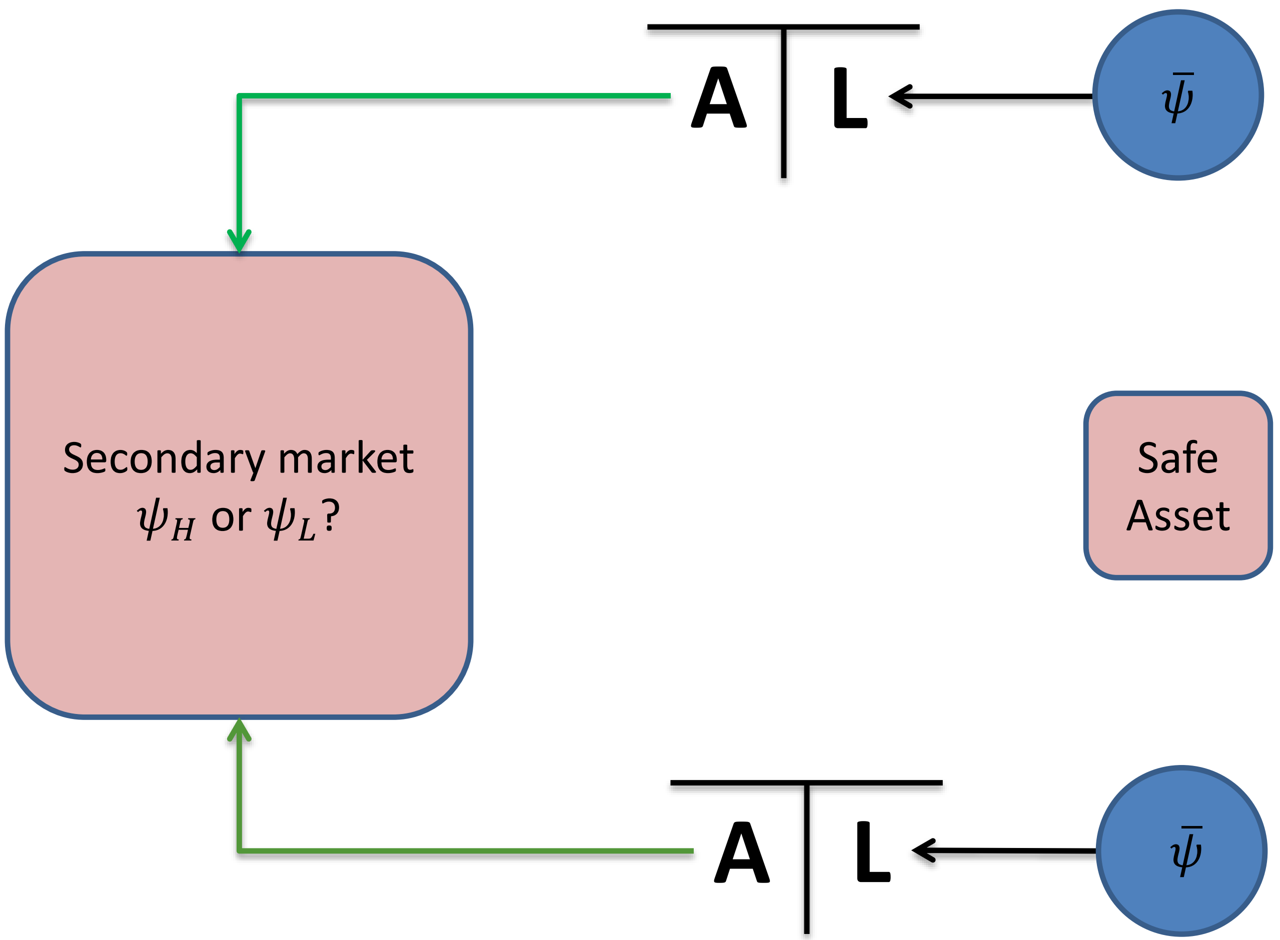
- Solve using the global games paradigm

- Creditors use threshold strategies
  - rollover if $P^{(1)} < P^{\star}$
  - foreclose otherwise

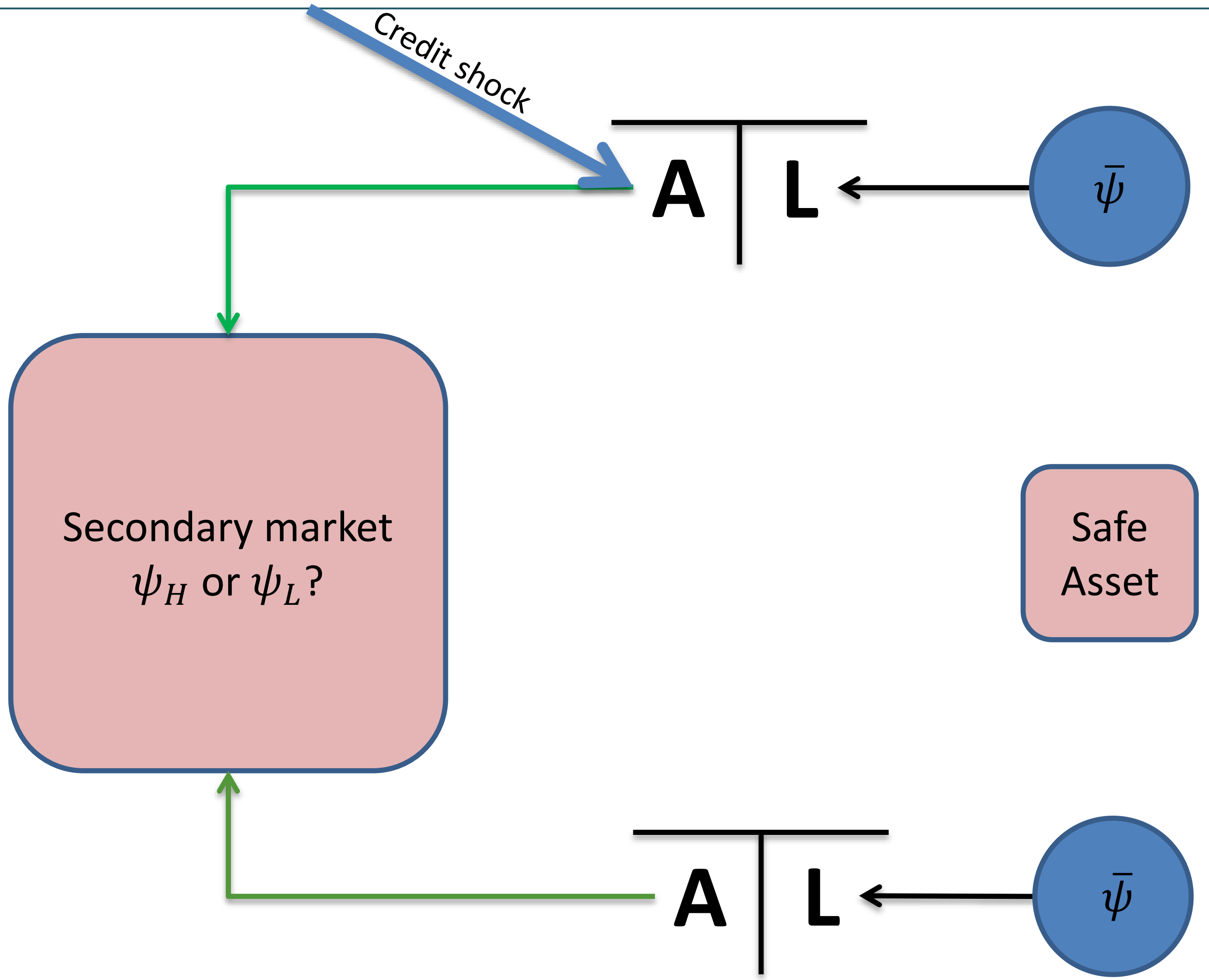
- Bayes-Nash Equilibrium – $P^{\star}$ solved from FPE

$$\lambda(P^{\star}) \times \text{Prob}\big(E_0 - P^{\star} - P^{(2)} > 0\big) \times r^S = r^F$$

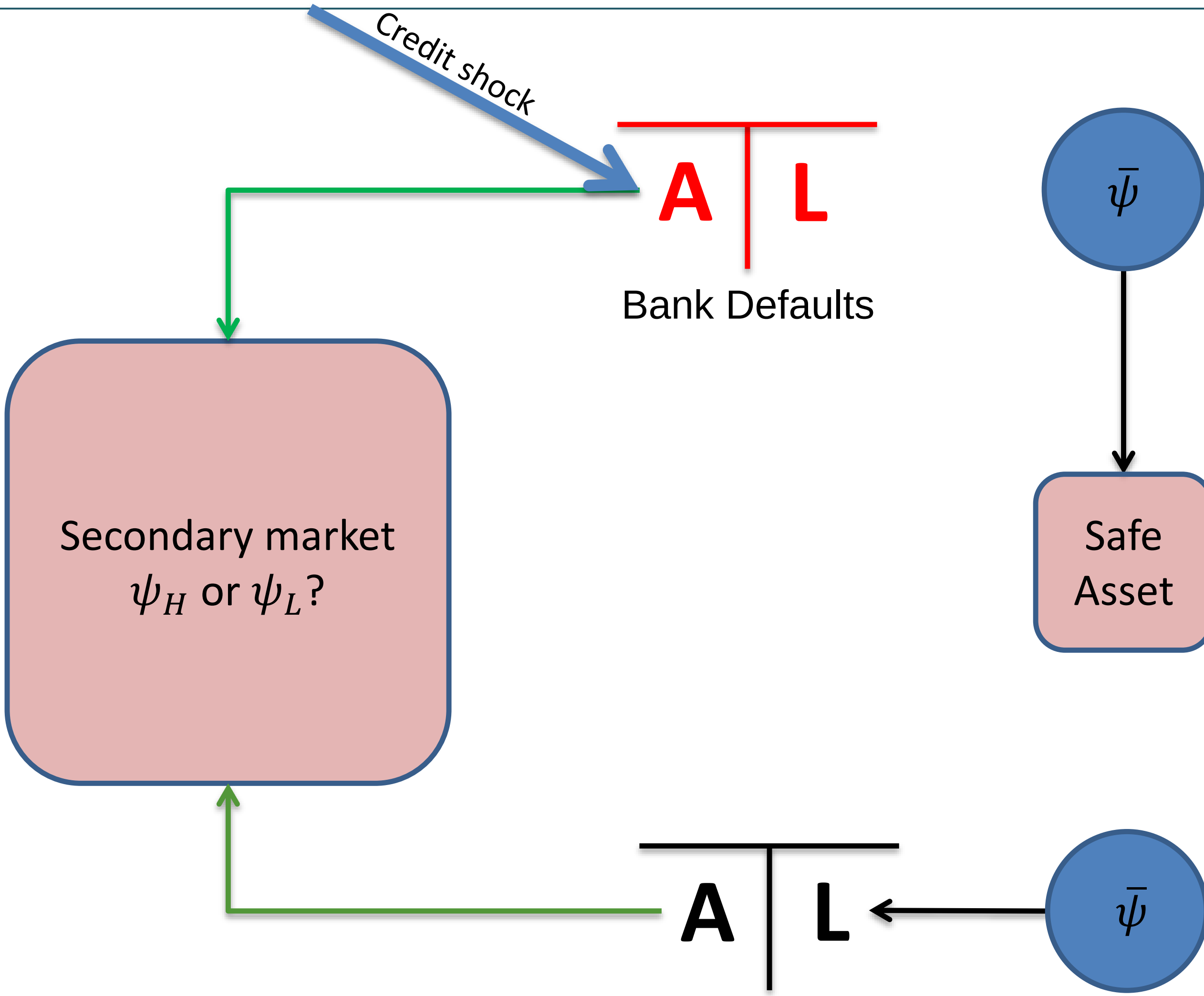
2.c Contagious runs



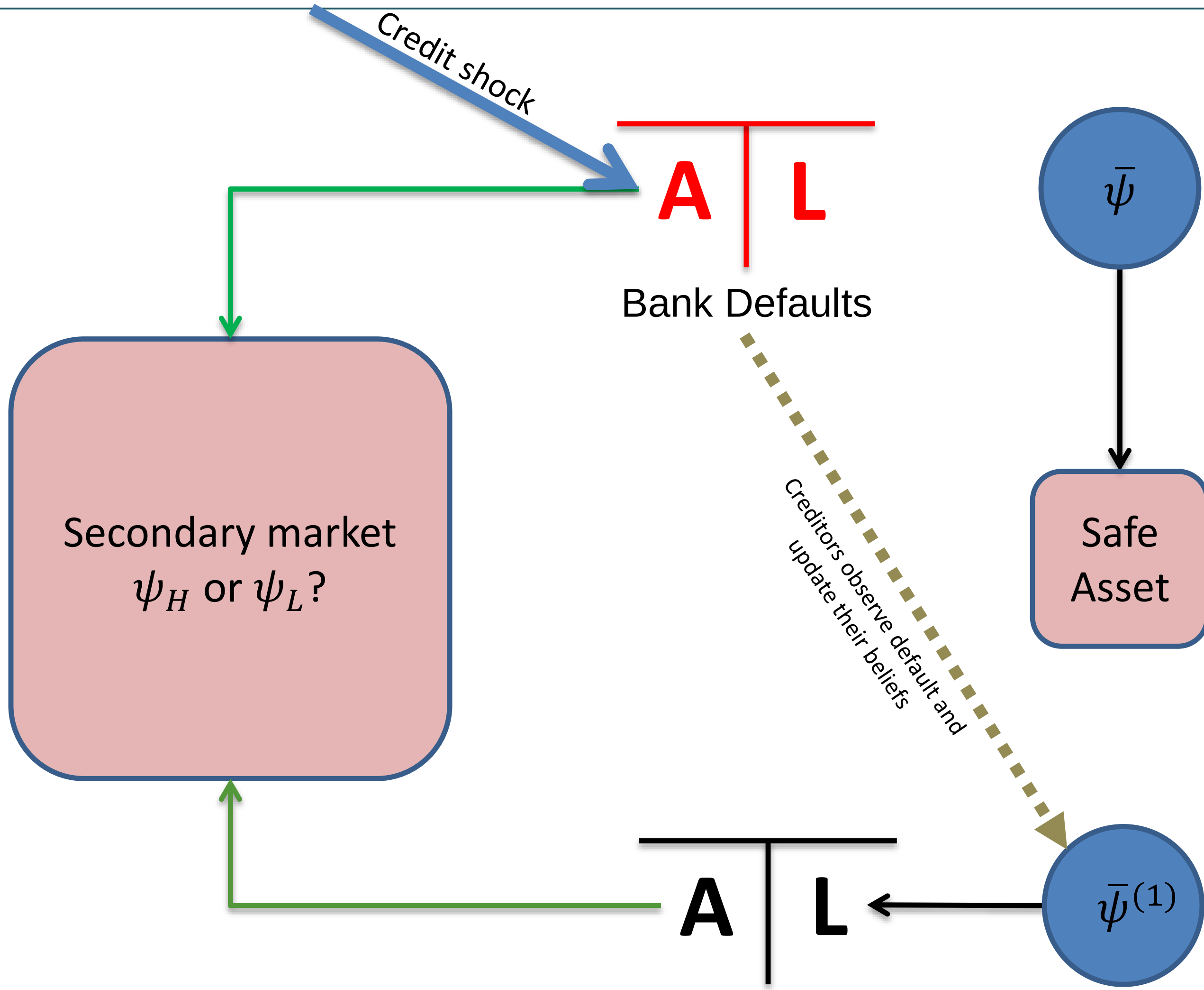
2.c Contagious runs



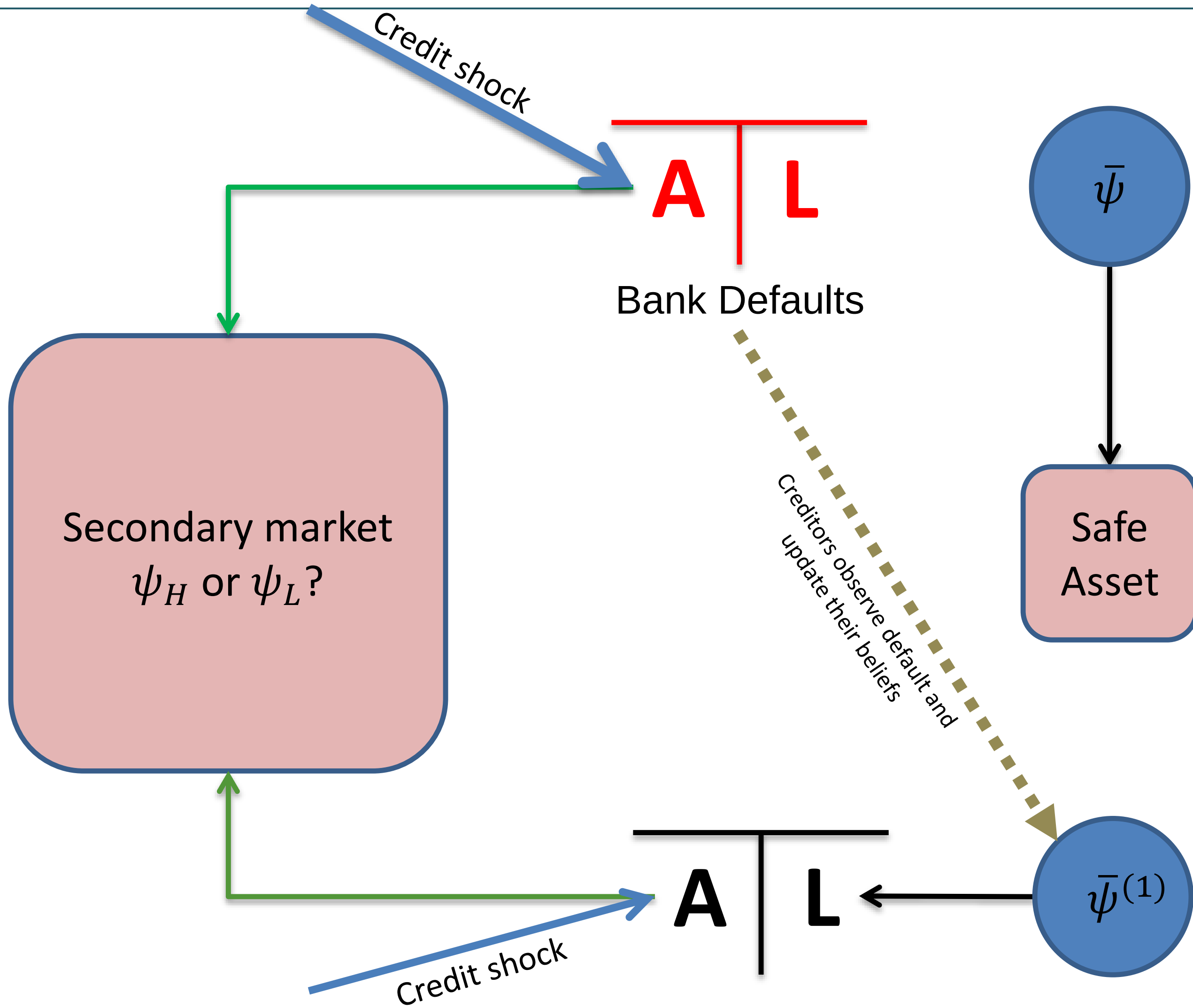
2.c Contagious runs



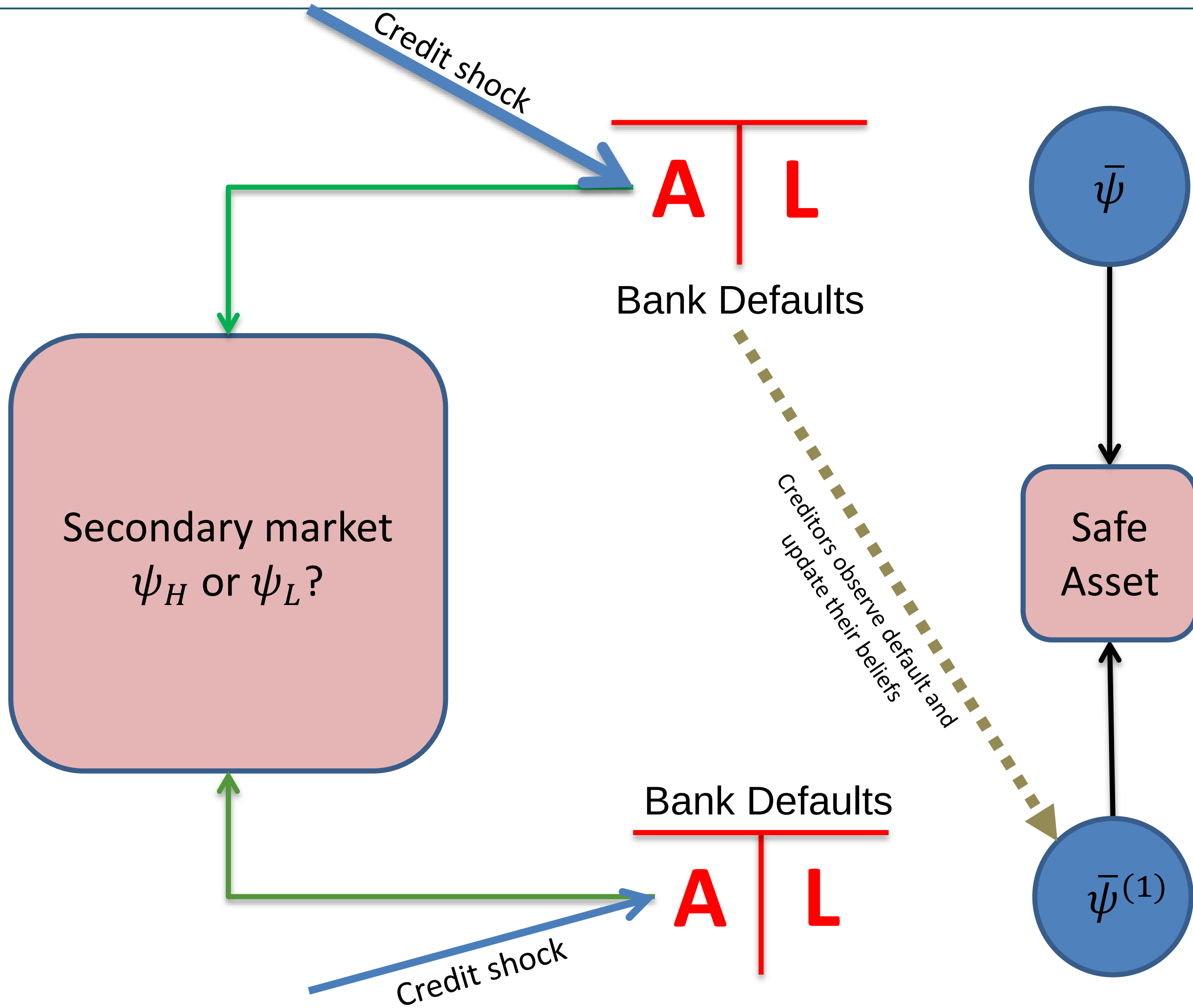
2.c Contagious runs



2.c Contagious runs



2.c Contagious runs



2.c Contagious runs– Bayesian updating

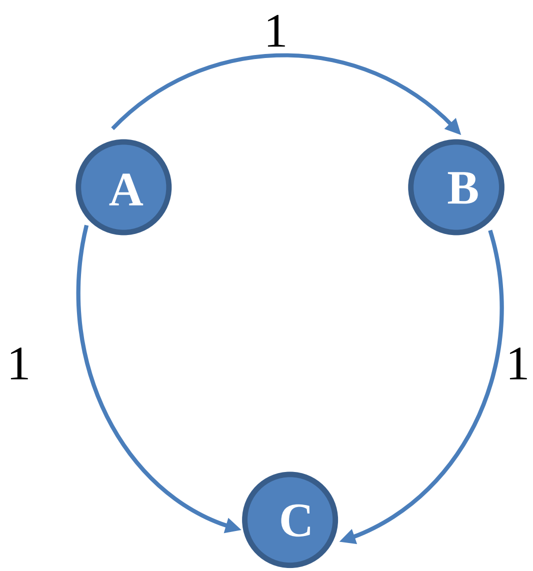
- Define w_j to be the subjective belief held by the creditors of bank j that $\psi = \psi_H$, and $\eta_k \in \{0,1\}$ as an indictor for whether bank k has defaulted (1), or not (0), and i as the iteration-step

$$w_j^{(i+1)} = Prob\left(\psi = \psi_H \mid \left\{\eta_k^{(i)}\right\}_{k\neq j}\right)$$

$$= \frac{Prob\left(\eta_1^{(i)} \mid \left\{\eta_k^{(i)}\right\}_{k\neq j,1}, \psi_H\right) \times \cdots \times Prob\left(\eta_{N-1}^{(i)} \mid \eta_N^{(i)}, \psi_H\right) \times Prob\left(\eta_N^{(i)} \mid \psi_H\right) \times Prob(\psi = \psi_H)}{Prob\left(\left\{\eta_k^{(i)}\right\}_{k\neq j}\right)}$$

$$= Prob(\psi = \psi_H) \times \prod_{k\neq j} \frac{Prob\left(\eta_k^{(i)} \mid \psi_H\right)}{Prob\left(\eta_k^{(i)}\right)}$$

2.d Network effects

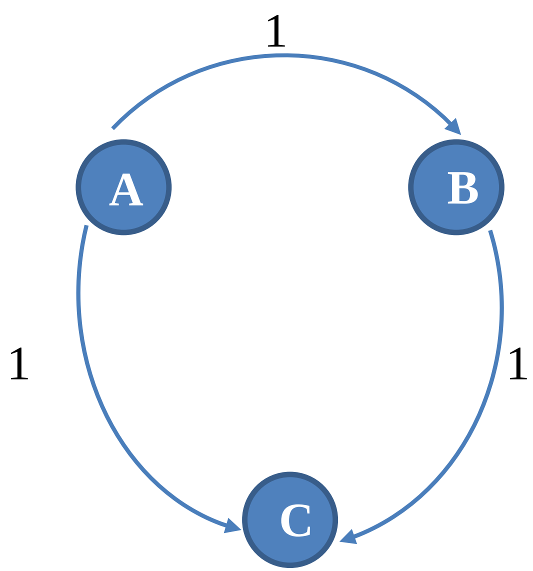


A owes 1 to B and 1 to C
B owes 1 to C
C owes to nothing to A and B

Bank	IA	IL	Net IA	Net non-IA	Net worth
A	0	2	-2	1	(-2)+1 = -1
B	1	1	0	0	0 + 0 = 0
C	2	0	2	0	2 + 0 = 2

IA: interbank assets
IL: interbank liabilities
Net non-IA: net non-interbank assets after credit losses

2.d Network effects



Bank	IA	IL	Net IA	Net non-IA	Net worth
A	0	2	-2	1	(-2)+1 = -1
B	1	1	0	0	0 + 0 = 0
C	2	0	2	0	2 + 0 = 2

A in solvency default

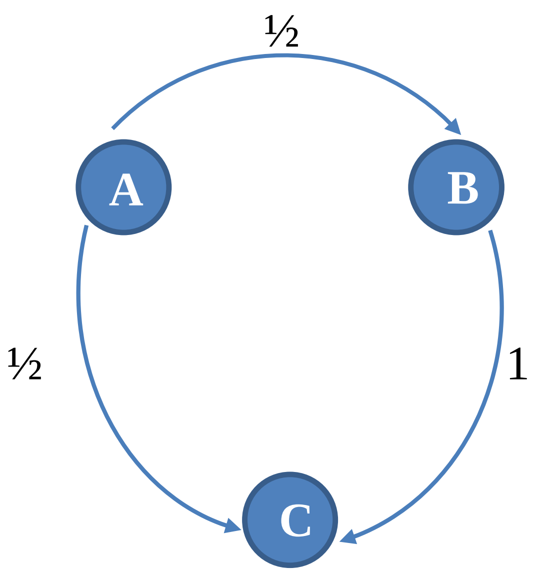
A promised to pay 1 to B and 1 to C but is only willing to pay 1

How to allocate 1 between B and C?

B holds 50% of A's interbank liabilities → ½ to B

C holds 50% of A's interbank liabilities → ½ to C

2.d Network effects



Bank	IA	IL	Net IL	Net non-IA	Net worth
A	0	2	-2	1	(-2)+1 = -1
B	~~1~~ ½	1	½ -1 = - ½	0	- ½ + 0 = - ½
C	~~2~~ 1 ½	0	1 ½	0	1 ½ + 0 = 1 ½

B gets ½ instead of 1 => B defaults because A has not made full payment: spillover default

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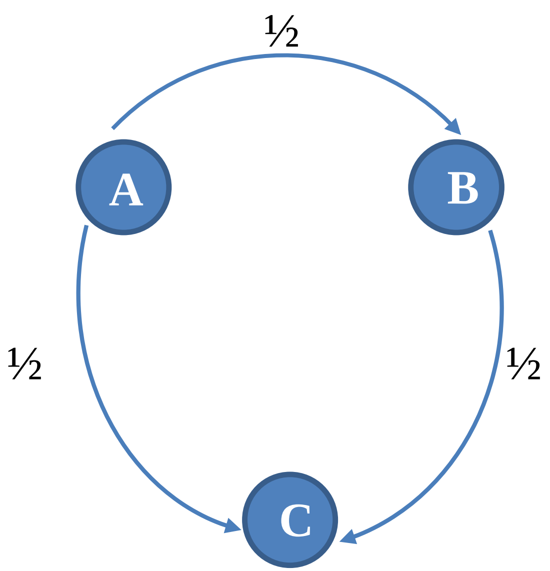
2.d Network effects



Bank	IA	IL	Net IL	Net non-IA	Net worth
A	0	1	-2	1	-1
B	½	1	0	0	- ½
C	~~2~~ 1	0	1 – 0 = 1	0	1

B promised to pay 1 to C but is willing to pay ½ only
C remains solvent

3. Calibrating MFRAF

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Variable	Description	Source for calibration
I_0	Dollar value of illiquid assets	NCCF report
ψ_H	Liquidation value of assets in the "high" state	Judgement on haircuts
ψ_L	Liquidation value of assets in the "low" state	Judgement on haircuts
M_0	Dollar value of liquid assets	NCCF report
S_0	Cumulative short term liabilities that come to maturity in t_1	NCCF report
RWA	Risk weighted assets (CET1 Basel III)	Provided by the banks
$INCOME$	Operating income (internally generated capital)	Satellite models
κ	Bank's starting capital levels (CET1 Basel III)	Provided by the banks
τ	Minimum threshold level for bank's capital ratio (7% or 4.5%).	
X	Interbank network	Regulatory filings

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3 Calibrating MFRAF

- Banks reported their holdings of liquid and illiquid assets using the Net Cumulative Cash Flow (NCCF) definitions
- Liquid assets have to be unencumbered and eligible for central bank open market operations:
  - Cash and deposit accounts at the BoC
  - Government securities (Canada, U.S., and Euro Area)
  - Other eligible securities (e.g. BAs and NHA-MBS)

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3. Assumptions on recovery rates (1 – haircuts)

Instrument	State H	State L
Deposits with banks		
Other Securities		
Other government		
Mortgage Backed Securities		
Asset Backed Securities		
Corporate CP		
Corporate bonds		
Equities		
Precious Metals		
Other commodities		

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3. Assumptions on recovery rates

Instrument	State H	State L
Loans		
Residential mortgages - insured		
Residential mortgages - uninsured		
Personal loans		
Credit cards		
Business and government loans		
Customers' liabilities under BAs		
Swapped Intra-bank Loans		
Call Loans		
Reverse Repurchase Agreements		
Securities borrowed		
Derivatives related amounts		
Other Assets		

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3. Starting capital level (CET1 Basel III)

- “Front-load” income generated over the 1-year MFRAF horizon onto the starting capital level, i.e.,

$$E_0 = \kappa \times RWA_0 + Income$$

- To determine the default threshold, we look at the level of capital in excess of the regulatory minimum,

$$E_0 = \kappa \times RWA_0 + Income - \tau \times RWA_0$$

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3. Accounting for losses

- Credit risk losses

$$P^{(1)} + P^{(2)}$$

- Losses following a bank run

z percent of $\tau \times RWA_0$

- Losses after default due to network contagion

endogenous clearing

4. Some *hypothetical* results

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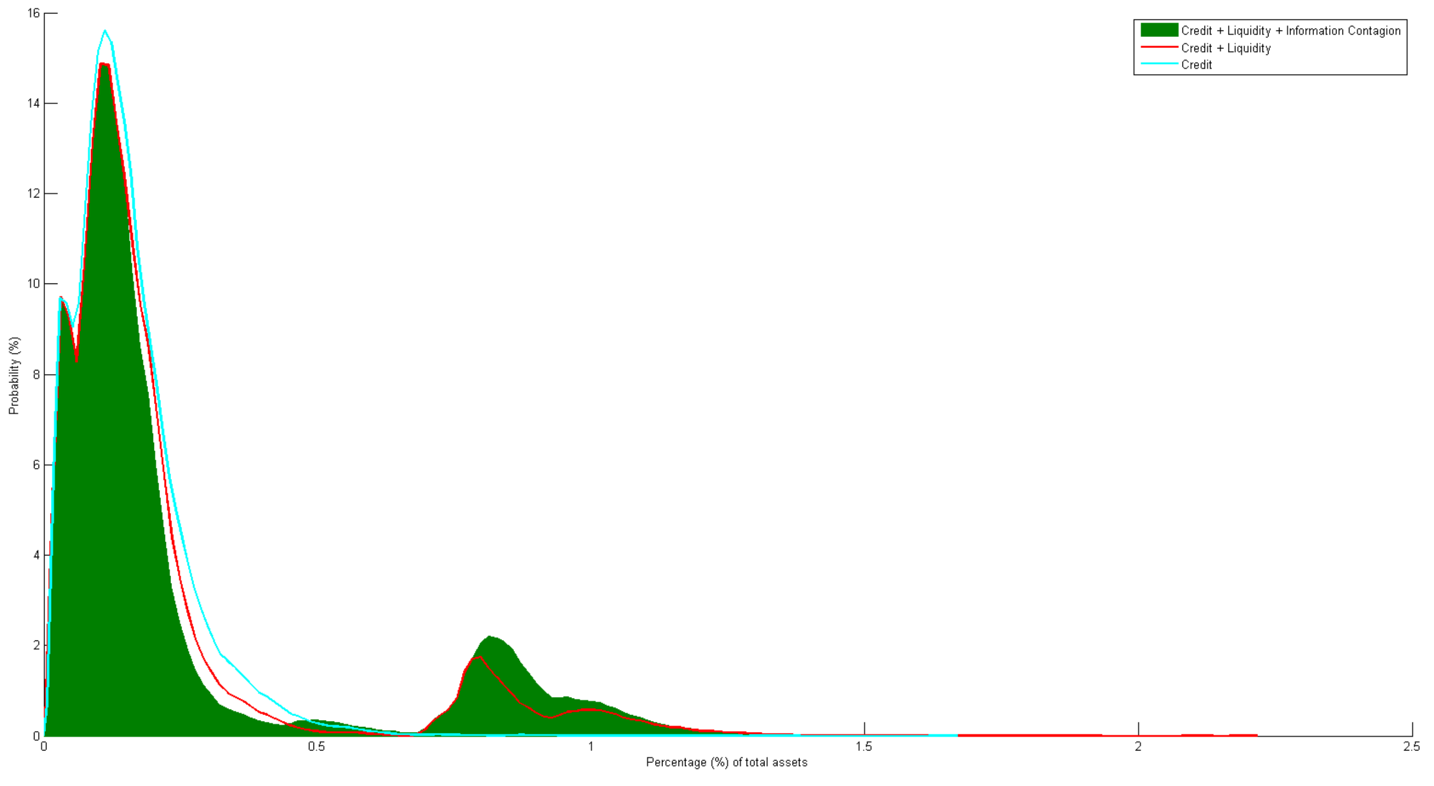
4. Results – loss distribution (solvency & liquidity)



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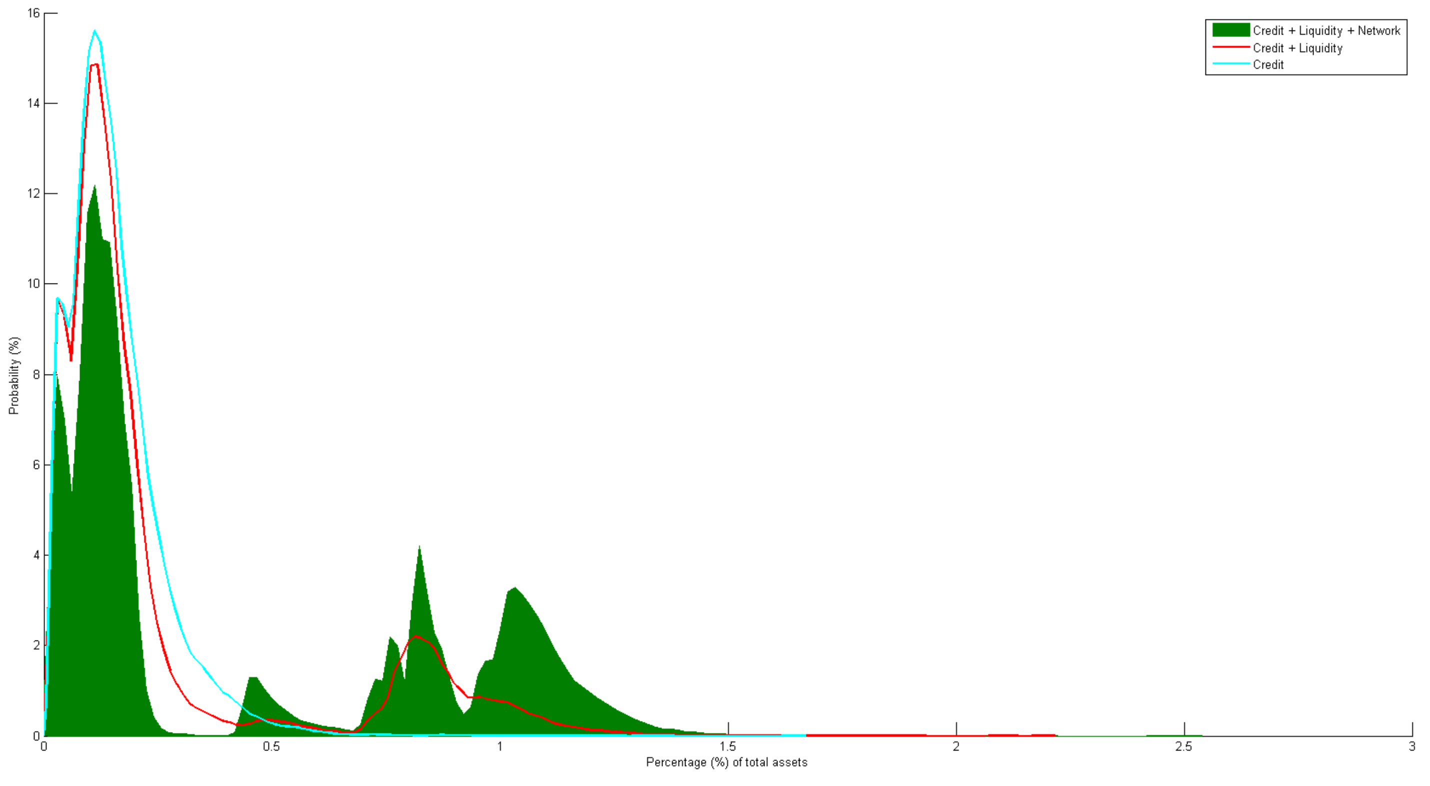
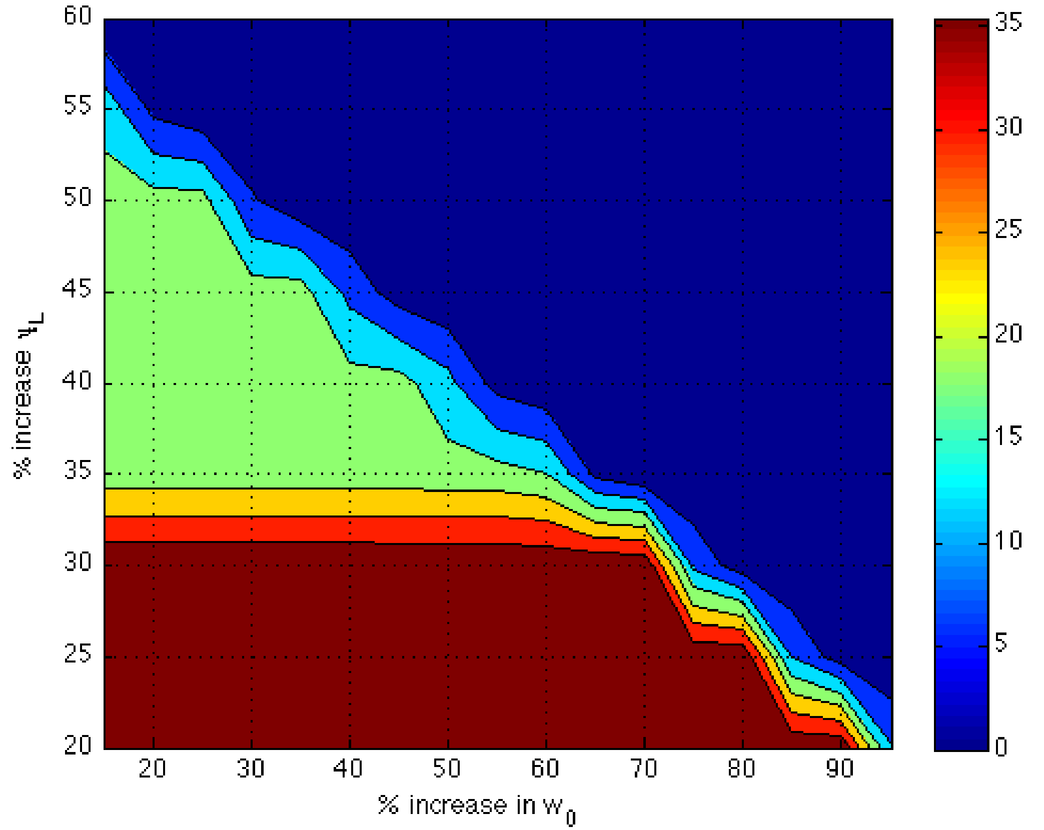
4. Results – loss distribution (all effects)



4. Results – sensitivity to beliefs and prices



5. Conclusions

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5. Conclusions

- MFRAF is a top-down stress testing tool that investigates the interactions between solvency and liquidity risk.
- Results depend starting capital ratios and balance sheet liquidities.
- Calibrating prices is very much an art form, and ideas for a more robust modeling would be very welcome.

5. Conclusions: Model Improvements – Key priorities

- Feedback effects to the real economy
  - TVAR with endogenous Financial Stress Index (FSI) to generate stress scenarios
  - Link FSI to outputs from MFRAF (e.g., via losses).

- RWA model to account for impact of liquidity risk and network effects.

- Link market liquidity (ψ parameters) with funding liquidity risk, i.e., endogenous relationship.

Thank you!