Central bank liquidity provision, risk-taking and economic efficiency

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Abstract

After the Lehman default, but also during the euro area sovereign debt crisis, central banks have tended to extend the ability of banks to take recourse to central bank credit operations through changes of the collateral framework (e.g. CGFS, 2008 – in consistence with previous narratives, such as Bagehot, 1873). We provide a simple four sector model of the economy in which we illustrate the relevant trade-offs, derive optimal central bank collateral policies, and show why in a financial crisis, in which liquidity shocks become more erratic and the total costs of defaults increase, central banks may want to allow for greater potential recourse of banks to central bank credit. The model also illustrates that the credit riskiness of counterparties and issuers is endogenous to the central bank's credit policies and related risk control framework. Finally, the model allows identifying the circumstances under which the counterintuitive case arises in which a relaxation of the central bank collateral policy may reduce its expected losses.

Keywords: central bank, risk-taking, collateral policy, economic efficiency

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Non-technical summary

It is well known at least since the 19th century experience of the Bank of England (as documented in Bagehot (1873) or King (1936)) that in a financial crisis central banks play a crucial role as lenders of last resort. These conclusions were substantiated during the current crisis when many central banks loosened their respective collateral frameworks to increase the potential of banks to fund their balance sheets with central bank credit operations, thus ensuring that no disorderly deleveraging takes place. But this also raises an important question: to what extent should central banks extend credit to banks under funding stress, given that such elastic credit provision might increase their risk-taking and promote moral hazard? More specifically, what are the key trade-offs the central bank needs to consider in limiting the elasticity of credit provision through the restrictiveness of its collateral and associated risk control framework?

We provide a stylized model representing the key trade-offs and allowing to derive optimal central bank policies from a risk-management and economic efficiency perspective. The model is cast in a comprehensive system of financial accounts, featuring four key sectors of the economy – households/investors, corporates, banks and the central bank – which ensures that all key financial flows are properly reflected. The model contains both asset value (solvency) shocks which exhibit persistence over time, and liquidity shocks, that are the actual trigger of default. The model reflects the empirical observations that default may occur despite an economic entity being solvent, and the insolvency of banks or corporates may remain unnoticed for an extensive period of time as they continue to be able to access funding of one or the other kind.

The model is driven by households/investors who receive noisy signals on the quality of banks' assets and may decide to withdraw funding on that basis. In contrast, the central bank is assumed to have no particular information on corporates' economic performance and the quality of loan portfolios, but it must provide liquidity to banks in a way that achieves the optimum with regard to minimizing the expected costs across two possible errors: (i) letting a bank default for liquidity reasons although it was viable in the sense that there was no reason to expect that it would produce sizable losses in the future; and (ii) preventing, through extensive liquidity provision, the default of a bank which is not sound and expected to generate future substantial losses if it is not wound down. The one model parameter of the central bank to achieve the optimum is the haircut it imposes on collateral.

The model shows that economic efficiency and central bank risk-taking are in many cases non-monotonous functions of haircuts, and even if the functions are monotonous, they can be either upward- or downward sloping. This means that depending on the haircut level and on economic circumstances, increasing haircuts can either increase or decrease central bank risk-taking, and either increase or decrease economic efficiency, with the two not necessarily aligned. One counter-intuitive insight is that in stressed market conditions, characterized e.g. by high costs of default and low correlation of liquidity shocks with fundamentals, central bank risk-taking can increase with the level of haircuts. Hence, paradoxically, loosening the collateral framework may under some circumstances be consistent with protecting the balance sheet of the central bank, as already implied by Bagehot's dictum that only the "brave" plan of the central bank is the "safe" plan. This is a specific consequence of a more general insight that financial sector risk tends to be endogenous with respect to central bank's emergency liquidity support.

Going beyond model specification, this phenomenon can be illustrated by the following mechanism: if the funding stress of banks, together with other macroeconomic factors, lead to a continued credit crunch and economic downwards spiral affecting collateral values, counterparties' solvency will deteriorate over time and PDs will increase, eventually increasing also central bank's risk parameters. To the extent that the central bank's emergency liquidity operations manage to overcome the negative feedback loops characteristic of a systemic financial turmoil, these actions should then also potentially reduce the central bank's long-term risk exposure. We believe this reasoning, illustrated formally by our model, goes a long way towards explaining why the major central banks have, over the course of the recent crisis, aimed at increasing the total post-haircut amount of collateral relative to the total balance sheet length of the banking system.

1 Introduction

That in financial crises, central banks should become lenders of last resort to the economy, while taking into account financial risk management and moral hazard concerns, is well known ever since the 19th century experience of the Bank of England as documented in Bagehot (1873) or King (1936). In this paper we propose a simple model that integrates the issue of central bank lender of last resort policies and financial risk management. The model is driven by both liquidity and solvency shocks hitting financial institutions, and by how the two are correlated. The model allows to derive optimal collateral eligibility and haircut levels for central banks. We thereby integrate also two normally remote strands of central banking literature, namely on financial risk management and on the management of liquidity crises. Consider briefly each of those strands and related recent debates.

Financial risk management of financial institutions is in principle as old as banking business itself, as any financial exposure is subject to credit, and often also to market risk. Modern financial risk management, as summarized for instance in Hull (2012), has evolved dramatically reflecting (i) the development of modern financial markets, (ii) the financial crisis that started in 2007, and (iii) financial regulation such as the Basle accords. Central banks face in principle similar risk management issues as private financial institutions. The length of central bank balance sheets, and hence financial exposures, has increased dramatically over the last 15 years for two reasons. First, according to IMF data, emerging and developing economies have increased steeply their foreign exchange reserves, namely from USD 660 billion in 2000 to USD 6,797 billion at end 2011 (i.e. more than a tenfold increase). Second, since 2007, central banks in industrialized countries have increased their balance sheet length in the context of measures taken to combat the financial crisis. From end 2006 to end 2011, the length of the balance sheets of the Fed, Bank of England, and the Eurosystem increased by 233%, 240%, and 138%, respectively (in absolute terms: USD 2,049 billion, GBP 205 billion, EUR 1,585 billion).

The modern literature on central bank risk management has developed in parallel to the rise of these exposures. Bernadell, Cardon, Coche, Diebold, and Manganelli (2004) is the first volume dedicated entirely to a central bank financial risk management topic, but focuses only on investment operations and foreign exchange reserves. The risk management solutions proposed therein seem to be largely applicable to any institutional investor. Bindseil, Gonzalez, and Tabakis (2009) covers both central bank investment and policy operations, and aims at elaborating on what makes the central bank special in terms of optimal management of its financial risks. Bindseil (2009b) notes first a number of specificities of any public investor, and then a number of specificities of central banks. Part II of the volume deals specifically with policy operations and risk measurement and management for collateralized credit operations of central banks undertaken in the context of monetary policy operations.

In this paper we analyze central bank lending policies and risk management in a financial crisis, with emphasis on risk-taking and economic efficiency. This topic has received increased public attention in recent years. In the U.S., criticism of the bailing out of wealthy Wall Street bankers with taxpayers' money has become a major issue (see e.g. Hollywood movies such as "Bailout", "Capitalism – a Love Story", and "Inside Job", or bestselling books such as Andrew Ross Sorkin (2009; "Too Big to Fail: The Inside Story of How Wall Street and Washington Fought to Save the Financial System–and Themselves"), being perceived as unfair, creating moral hazard, and supporting activities that are wasteful from a social perspective. The criticism is voiced independently of the fact that taxpayers have (in the meantime) recuperated their money from the special lending operations of the Fed. More recently, the critique has turned to large-scale asset purchase programs, which are thought to be helping leveraged institutions and over-leveraged governments survive the current economic difficulties at the eventual expense of taxpayers and/or savers, who get expropriated through inflation or at least negative real rates.

Risk-taking aspects of central bank lending have also become an issue of public debate in the context of the measures of the Eurosystem to address the euro area debt crisis. Some German critics of the ECB and of the euro have gone so far to conclude that the risk-taking has reached dimensions that make the European monetary union incompatible with the German constitution. For instance the German Bundestag member Peter Gauweiler argued in a constitutional complaint in June 2012 that "since the beginning of the financial crisis, the claims of the northern countries to the southern countries in the euro area [relating to central bank credit operations] are growing towards infinity.... A fundamental construction error of monetary union has been revealed. Therefore, monetary union has now become incompatible with the [German] constitution and with the democracy principle".[1]

The theoretical literature on central bank financial crisis management has so far focused mainly on providing rigorous rationale for the lender of last resort function of central banks as developed by Bagehot (1873)(see also Goodhart, 1999; Freixas, Giannini, Hoggarth, and Soussa, 2000; Bindseil, 2009a). For example, Diamond and Dybvig (1983), Diamond and Rajan (2001), as well as Rochet and Vives (2004) demonstrate the welfare enhancing effect of some form of liquidity insurance – as a backstop against potential coordination failures (bank runs) and contagion, following essentially from maturity and liquidity transformation inherent in banking operations.[2] Given that it is normally difficult to distinguish solvent from insolvent banks on a real-time basis (Goodhart, 1999), the question arises whether a lender of last resort is still efficient in such conditions, as it will probably lead to keeping some insolvent institutions afloat. Freixas, Rochet, and Parigi (2004) address this explicitly by introducing a model with both liquidity and solvency shocks that

[1] Dr. Peter Gauweiler, Verfassungsbeschwerde, 29 June 2012, http://www.peter-gauweiler.de/pdf/Verfassungsbeschwerde%20ESM.pdf

[2] More recently, Holmström and Tirole (2011) have introduced the concept of "inside" and "outside" liquidity to address more broadly the issue of how the economy at large can cope with liquidity shocks. They show that whenever liquidity cannot be endogenously generated within the corporate sector, outside liquidity – e.g. central bank money – needs to be provided.

are indistinguishable for the central bank which faces the problem that an insolvent bank may pose as an illiquid one and "gamble for resurrection", investing the loan in the continuation of a project with a negative expected net present value. It is shown that when it is costly to screen sound firms and solvent banks cannot be easily detected – as would be the case especially in a financial crisis – it is optimal for the central bank to offer emergency liquidity assistance to banks, however at a higher rate (lower than the market) and against collateral, which should serve to deter misuse of its facilities and protect against excessive risk-taking (see also Freixas and Parigi, 2008, for a review of results on lender of last resort and bank closure policy).

We extend the available literature by offering a stylized model capturing the effects of liberality in central bank liquidity provision (as specified through its collateral policy) on both central bank risk-taking and economic efficiency. The model provides what is to our knowledge the first formal backing of some of the key statements of Bagehot (1873).

The model is based on a comprehensive system of financial accounts, similar to the one in Bindseil and Winkler (2012), and follows Freixas, Rochet, and Parigi (2004) in allowing for solvency and liquidity shocks, which are correlated with each other and a priori indistinguishable from the central bank's perspective. It foresees two time periods and the possibility of bank and corporate default triggered by illiquidity and causing damage in the form of real asset value. Our analysis of how asset value shocks pass through the system of accounts is also inspired by that of Gray, Merton, and Bodie (2007), Gray and Malone (2008) and Castrén and Kavonius (2009), who study the interconnections and shock transmission channels between the risk-adjusted balance sheets of various sectors in the economy using Merton's (1974) structural credit risk model. Although we explicitly reflect the seniority of companies' liabilities structure, as in Gray, Merton, and Bodie (2007), we do not introduce the pricing of credit risk, as we assume the values of assets and liabilities are recorded at book values and fair values are established only at the end of period 2. Our approach allows us to explicitly address the central bank's problem of finding the right balance between the costs of default and the preservation of non-viable economic projects, and show that central bank and general economic efficiency considerations need not necessarily be aligned.

Our paper differs from Freixas, Rochet, and Parigi (2004) on a number of assumptions and results. First, we assume more realistically that liquidity and solvency shocks are correlated while in Freixas, Rochet, and Parigi (2004) a bank is either illiquid or insolvent, but not both. On a related note, we model liquidity shocks in a closed system of financial accounts to also capture such crucial concepts as the aggregate liquidity deficit of the banking system vis a vis the central bank, while Freixas et al include no such liquidity deficit in their analysis. Second, we assume for the sake of simplicity that there is a certain given ex ante distribution of the quality of economic projects while Freixas et al integrate investment decisions of banks into their model. Third, to reflect more closely the situation directly after the collapse of Lehman Brothers and experiences from the euro area debt crisis, we assume that both the secured and the unsecured money markets have broken down completely, while Freixas et al assume continued existence of such markets. Fourth, we explicitly model central bank risk-taking as a major central bank concern that may be relevant for the decisions taken by the central bank and for economic efficiency, while Freixas et al. do not consider this aspect. Finally, Freixas et al. focus on the pricing of emergency central bank credit as a means to discourage moral hazard, while in our view, in the case of a liquidity crisis, the availability of credit (not its price) is the overriding issue, and therefore constraining central bank lending to the right extent seems to be the more relevant parameter to address moral hazard.

The rest of this paper proceeds as follows. In Section 2, we develop the particularities of central bank lending and risk management policies in financial crises. We discuss the intuition that the central bank is particular on a number of dimensions, and that ignoring these particularities can lead to wrong policy conclusions, including potentially sub-optimal risk management. In Section 3 we develop a model of central bank liquidity provision, risk management and moral hazard that captures in a stylized way the recurrent themes in the debates surrounding central bank financial crisis management. It is shown how one crucial central bank risk control parameter, namely haircuts on central bank collateral, influences both central bank risk-taking and economic efficiency in a way that depends on economic circumstances. Section 4 draws conclusions.

2 Central bank lending and risk-taking in a financial crisis

In this section we show that central bank lending and risk management policies in a crisis are special for a number of reasons, and that ignoring these particularities may lead to sub-optimal central bank decisions both from the point of view of general economic efficiency and risk management.

2.1 Factors explaining higher central bank financial risk-taking during a financial crisis

Extended liquidity provision by central banks during a crisis comes at the cost of larger exposures compared with normal times. The increase in financial risk is driven by a number of factors, some of which can be illustrated in a simple system of financial accounts, similar in spirit to the one in Bindseil and Winkler (2012), where the approach is explained in detail (Table 1).[3] The economy is made up of four sectors – households, corporates, banks and the central bank. The household diversifies from its initially exclusive real asset holdings (E) into financial assets – banknotes (B) and deposits (D), divided equally among two *ex ante*

[3] Cf. also Bindseil and Jablecki (2011) who use the financial accounts setup to devlop a structural model of central bank intermediation.

identical banks. This diversification is also the source of real asset holdings of the corporate sector $(D + B)$, with the financial sector intermediating. Banknotes are issued by the central bank who provides them to banks through collateralized credit operations. The banks are, with regard to banknotes, intermediaries between the central bank and the households. The households' financial asset demand is however unstable, and in particular in a financial crisis households may want to substitute deposits with banknotes (shock d) or deposits in one bank with deposits in another bank (shock k). Consider now the following four reasons why central bank risk-taking may increase in financial crises, the last three of which have a direct representation in the system of financial accounts.

1. Probabilities of default of central bank counterparties and issuers of debt instruments used as collateral increase during a crisis. As illustrated e.g. by Standard&Poor's (2009), investment grade debtors (i. e. at least BBB-rated debtors) experience no default at all in good years (e.g. in 1992–1994, 1996, 2004, 2006, 2007 not even one single BBB rated debtor defaulted). In contrast, during bad years even higher rated companies default. For instance, in 2008 the default frequency for AA- and A-rated debtors was both 0.38%. Moreover, the correlation risks between central bank counterparties and collateral credit quality increase during a financial crisis. Generally, systemic crises create high correlation between debtors because common risk factors (instead of idiosyncratic risk factors) become predominant. Therefore, the likelihood of the worst case scenario for a repo operations, that of a simultaneous default of both the counterparty and the collateral issuer, increases significantly.

2. Central bank lending shifts towards stressed counterparties. During financial crises, stressed banks lose market access and experience funding gaps which are often addressed through increased recourse to central bank credit (this is the case of Bank 2 in the stylized system of financial accounts, which experiences liability outflows of k). This phenomenon may be called *relative central bank intermediation in the money market.* Hence, central bank lending becomes more concentrated on weaker counterparties (Bank 2) which implies that the asset side of its balance sheet becomes, on average, more risky and moreover less diversified. In the system of financial accounts, an increase of the average probability of default of counterparties (PD) arises if the deposit shifts by households k are correlated with potential solvency problems of banks. Then these shifts lead, on average, to a concentration of exposures of the central bank to weaker banks.

3. Central bank balance sheets may lengthen for two reasons. First, central banks start at some stage to take over the role of intermediary of the financial system in an *absolute* sense (*absolute central bank intermediation*). This occurs in the stylized model if the shock k reaches a certain level, namely if $k > \frac{1}{2}(B + d)$. Then Bank 1 is over-liquid and deposits its excess funds with the central bank.

Table 1: Financial accounts

Households/Investors			
Assets		Liabilities	
Real assets	$E-D-B$	Equity	E
Deposits Bank 1	$\frac{1}{2}D-\frac{1}{2}d+k$		
Deposits Bank 2	$\frac{1}{2}D-\frac{1}{2}d-k$		
Banknotes	$B+d$		

Corporates			
Assets		Liabilities	
Real assets	$D+B$	Bank loans	$D+B$

Bank 1			
Assets		Liabilities	
Loans to Corp. 1	$\frac{1}{2}(D+B)$	Households' deposits	$\frac{D}{2}-\frac{d}{2}+k$
CB deposit facility	$\max(0,-(\frac{B}{2}+\frac{d}{2}-k))$	CB borrowing	$\max(0,\frac{B}{2}+\frac{d}{2}-k)$

Bank 2			
Assets		Liabilities	
Loans to Corp. 2	$\frac{1}{2}(D+B)$	Household's deposits	$\frac{D}{2}-\frac{d}{2}-k$
		CB borrowing	$\frac{B}{2}+\frac{d}{2}+k$

Central bank			
Assets		Liabilities	
Credit to Bank 1	$\max(0,\frac{B}{2}+\frac{d}{2}-k)$	Banknotes	$B+d$
Credit to Bank 2	$\frac{B}{2}+\frac{d}{2}+k$	Deposit facility	$\max(0,-(\frac{B}{2}+\frac{d}{2}-k))$

4. The central bank balance sheet may also lengthen due to a flight of households out of bank deposits into banknotes (as it happened in Germany on 13 July 1931 when queues to withdraw banknotes emerged in front of all major banks at once, see Bindseil and Winkler, 2012). This would happen if households were generally worried about the solvency of the whole banking system. This is reflected as shock d in the system of financial accounts.

In Table 1, we assume for the sake of simplicity of the presentation that the liquidity shock $k > 0$, i.e. the Bank 1 is the "good" bank that experiences liquidity inflows, while Bank 2 is the "bad" bank that experiences liquidity outflows.

2.2 Why should central banks indeed be ready to accept higher risks?

We distinguish three main reasons for the central bank to act as the lender of last resort in a financial crisis and to provide elastic credit, even though this leads to higher and more concentrated exposures as argued above. Of course, this recognition does not imply that there are no draw-backs of a too supportive liquidity approach which may create moral hazard, support businesses that should be stopped as they generate social losses, or prevent the necessary price adjustments in markets for certain assets. In this sense, a too supportive central bank attitude can contribute to reduce the efficiency of the price system and the economy at large. We discuss both sides of the trade-off further in subsections 2.3-2.4 and offer a formal model in Section

3, replicating the result that a too elastic liquidity supply fails to stop loss-making businesses and hence undermines economic productivity over time.

1. Negative social externalities of funding liquidity stress and default due to illiquidity. Negative externalities potentially justify the intervention of a public authorities. As argued by Brunnermeier, Crocket, Goodhart, Persaud, and Shin (2009), the most important negative externality of bank default stems from the fire-sale spiral induced by liquidity problems of individual banks:

   > "In order to deal with such liquidity problems prior to failure, and in the course of liquidation after failure, the bank in difficulties will often be forced to sell assets (fire sales). But such sales will drive down the current market price of the same assets held on other banks' books, when these are valued on a mark-to-market basis. (...) In short, there is an internal amplifying process (liquidity spirals) whereby a falling asset market leads banks, investment houses, etc., to make more sales (deleveraging), which further drives down asset prices and financial intermediaries' assessed profit and loss and balance sheet net worth."

   By lending to banks against collateral and thereby eliminating the need for asset fire sales, the central bank can prevent the downward spiral and negative externalities of fire sales. This also implies that risk parameters such as counterparty default probabilities will not be exogenous to central bank measures as these measures will influence the stability of the system. Typically, central bank measures avoiding asset fire sales will help preserve solvency and reduce probabilities of default of counterparties and issuers, which also attenuates central bank risk-taking. Asset fire sales are not the only form of negative externalities of bank funding stress and illiquidity induced default that have been mentioned in the literature. Other forms of negative externalities are the spreading of depositors' panic in the form of a generalized bank run (such as observed in various countries in the early 1930s), and the generalized drying up of funding and market liquidity in the financial system as a consequence of hoarding driven by the experience that claims, including collateral, can get stuck in a default (relevant after the Lehman default). Generally, due to the systemic escalation inherent in most liquidity crises, it appears that many entities will find themselves to be (temporary) illiquid even though they would be in principle be solvent (i.e. if they survive the liquidity crisis without liquidity induced solvency damages). It is important to note in this context that it is lack of funding liquidity that typically triggers default, and not insolvency in the sense of negative capital: while solvency is an opaque concept and there is no objective way to be certain about any indebted economic agent's solvency, illiquidity is very concrete (the inability to meet a payment obligation). If capital and interbank markets are in a good state, then funding liquidity problems of a certain institution often reflect that investors and peers have information on actual solvency problems of that institution. In a general liquidity crisis, funding liquidity problems will be more widespread, and will correlate less with actual solvency problems of the concerned institutions. The model presented in Section 3 will allow to reflect the idea that the information content of funding liquidity problems with regard to underlying solvency problems will be higher in normal times than in general crisis times.

2. Unlike leveraged financial institutions, the central bank is not threatened by illiquidity in its own currency. Central banks have been endowed with the monopoly and freedom to issue the legal tender, central bank money. Therefore, they are never threatened by illiquidity in their own currency and it seems only natural that, in case of a liquidity crisis when all agents attach a high price to liquidity, the central bank remains more willing than others to hold (as collateral or outright) assets which are less liquid. This argument does not rely on the existence of negative externalities. Even if the central bank were a purely profit-oriented enterprise, its exemption from liquidity stress should make it ready to take over illiquid assets in a crisis (against a premium). After the crisis, liquidity operations can be wound down and balance sheet size of the central bank restored to normal levels, so as not to crowd out financial intermediation or fuel the build-up of inflationary pressure. The fact that bank and corporate defaults are costly in themselves even without externalities, as they destroy organizational capital and normally block the efficient use of the underlying resources at least for a while, should also be seen in the context. If a bank or a corporate are threatened by illiquidity (and associated default) in a financial crisis, and if in the case of default the (presumably positive) organizational capital would be destroyed, then saving this capital is part of the 'rent' that can be achieved through cooperation between the liquidity-stressed economic agent and the one that has unlimited liquidity. It is important to note that preventing costs of default in this sense through central bank liquidity does not invoke negative externalities, market failures and the public nature of the central bank. Empirical estimates of default costs in the corporate finance literature vary between 10% and 44% (see e.g. Glover, 2011, and Davydenko, Strebulaev, and Zhao, 2012). It should be noted that default costs in this sense are related, but not strictly identical to the concept of "Loss-given-default" as used by rating agencies (e.g. Standard&Poor's, 2009). Loss-given-default also reflects possible negative equity before default. Loss-given-default as reported by rating agencies typically ranges in the area of 40%-50%. In the model presented in Section 3, the cost of corporate default will be one crucial parameter for the optimal degree of elasticity of central bank credit provision. We will not model the negative externalities of default explicitly (although we could), but will simply assume that all costs of defaults (direct and externality-linked) can be captured in one parameter. The model will also allow for positive effects of default – namely to stop corporates/banks with low performance to continue operating in view of the

likely persistence in the future of their low performance (which may be viewed as a basic form of moral hazard).

3. Haircuts are a powerful risk mitigation tool if credit risk is asymmetric and the collateral provider (repo borrower) is more credit risky than the cash investor (repo lender). The power of haircuts is limited if cash taker and cash lender are equally credit risky since although haircuts protect the buyer, they expose the seller to unsecured credit risk which increases with the haircut level (Ewerhart and Tapking, 2008). Anecdotal evidence suggests that haircuts applied in repos between banks of similar credit quality tend to be rather low, while haircuts charged from other market participants, for example hedge funds, tend to be higher (see e.g. Fitch Ratings "Repo emerges from the shadow", 3 February, 2012, or ICMA, Haircuts and initial margins in the repo market, February 2012). Thus, banks would never question haircuts imposed by the central bank (repo lender), because the central bank cannot default. We will accordingly be able to assume in the model presented in Section 3 that banks will always be willing to pledge assets with the central bank if they are in need of funding.

2.3 Some historical illustrations

It is remarkable that the trade-off between central bank liquidity provision and risk-taking, and the related experience of central banks was already extensively discussed in the 19th century (e.g. Bagehot, 1873; King, 1936; Wirth, 1883). As the Bank of England's Jeremiah Harman explained in 1832 regarding the crisis of 1825: "We lent it (money) by every possible means and in modes we had never adopted before *consistent with the safety of the bank*. Seeing the dreadful state in which the public were, we rendered every assistance in our power" (quoted in Bagehot, op. cit., emphasis added). Bagehot also emphasized the importance of central bank liquidity provision, "(...) in time of panic it (the Bank of England) must advance freely and vigorously to the public". Hence, while Bagehot was well aware of the associated higher risk-taking of the central bank, he considered enhanced liquidity provision to be the only possibility to safeguard financial stability. Furthermore, he argued that such measures would be necessary *to minimize the central bank's eventual own financial risks*:

> "(M)aking no loans as we have seen will ruin it (Bank of England); making large loans and stopping, as we have also seen, will ruin it. The only safe plan for the Bank (of England) is the brave plan, to lend in a panic on every kind of current security, or every sort on which money is ordinarily and usually lent. This policy may not save the Bank; but if it do not, nothing will save it."

What Bagehot suggests would mean that in specific cases a tightening (loosening) of the collateral framework of the central bank could lead to an increase (decrease) of long-term expected central bank losses. Indeed, the aim of “loosening” measures should be to contribute to avoid worst-case scenarios by restoring confidence in a confidence crisis with negative feedback loops and multiple equilibria. If funding stress of banks, together with negative macroeconomic factors, lead to a continued credit crunch and economic downward spiral, solvency deteriorates over time and probabilities of default increase, such as to also increase expected losses of the central bank more and more. If restoring confidence through a more forthcoming collateral and risk control framework allows to prevent such a development from materializing, it could well be that it reduces long-term expected financial losses to the central bank (apart from the positive social welfare aspects of such measures).

It is interesting to note that indeed central banks often have not suffered large scale losses on their credit operations in financial. This could be explained first by the fact that central bank credit operations with banks are typically collateralized. The benefit of not being threatened by illiquidity, and hence having time for liquidation, allows the central bank to take its time with asset liquidation and to await an end of the crisis that triggered counterparty default, i.e. to await mean-reversion in collateral values.[4] As an illustration, neither the Federal Reserve nor the Bank of England, nor the Bank of Japan, although all having been involved heavily in non-standard forms of liquidity provision to stressed entities over the past few years, have so far faced any losses.

In the case of emergency liquidity measures offered by the Federal Reserve System via the Maiden Lane Facilities (the purpose of which was to facilitate the merger of JP Morgan with Bear Stearns and alleviate capital and liquidity pressures on American International Group), all credits have been repaid in full with a net gain for the US public. Also the RMBS and agency bond purchases of the Fed were profitable. The case of the AIG rescue is particularly instructive as it illustrates also the inherent endogeneity of risk with respect to central bank's emergency liquidity assistance. The profitable liquidation of the insurer's troubled assets (funded by the Fed and placed with special purpose vehicles called Maiden Lane Facilities) was possible largely due to the general recovery in asset prices stimulated by a combination of low interest rates, extensive liquidity provision and support for credit and mortgage markets. As explained by one Treasury official (quoted by the Financial Times): “We bought at the bottom of the market because we made it the bottom of the market... The bottom of the market would have been much deeper if there had been a fire sale of AIG's assets. We pulled back the markets from the brink and Maiden Lanes II and III were a big part of it.”[5]

In the case of the Eurosystem, following the default of Lehman Brothers, the Eurosystem was left with some 33 highly complex securities that the investment bank had pledged as collateral. A total of EUR 5.7 billion was provisioned to cover potential losses associated with the liquidation of these securities at subpar

[4] Mean reversion will obviously not materialize in case of the issuer's default or debt restructuring.

[5] Henry Sender, “AIG: An improbable profit”, The Financial Times, October 22, 2012. We are grateful to Witold Grostal for pointing out this news story to us.

prices. The Bundesbank has been in charge of liquidating the collateral and by end-2011, it has managed to dispose of 28 securities, bringing down the value of provisions gradually to EUR 2.2 billion by end-2010 and EUR 0.95 billion a year later. In April 2012 the Bundesbank managed to sell off the largest single remaining asset – a senior tranche in a securitization structure "Excalibur" with a notional value of EUR 2.16 billion – which puts it on track to sell off all the outstanding Lehman collateral by end-2012.[6]

A word of caution is needed, however. First, the fact that in some recent episodes central banks did not make losses does not imply that the opposite experience could not easily materialize. Moreover, some central banks seem to have solved the problem of large expected losses on their exposures through inflation, achieved by maintaining too low interest rates for a considerable period of time. The most famous example is the Reichsbank in the period 1914 to 1923. When after the loss of World War I large reparation payments were imposed on Germany, it was clear that the Reich was insolvent unless its domestic debt would continue to be inflated away, which did indeed happen. Remarkably, when in 1924 the mark was stabilized again, neither the Reich had defaulted, nor did the Reichsbank have to realize any losses on its claims on the Reich. However, costs to society were huge, as the society had eventually to carry both the costs of the war, and the damages inflation and hyperinflation inflicted on the efficiency of the economy (and on social cohesion). In the case of the euro area debt crisis, there is no reason to doubt the commitment of the (fully independent) ECB to maintain price stability and to take the necessary anti-inflationary measures (increases of central bank interest rates and absorption of liquidity), whenever inflationary pressures may build up. At the same time, the exposures of the Eurosystem towards weaker banks in weak economies can suffer losses in case of negative tail scenarios. For instance, the solvency of Greek banks had to be restored with official sector loans from other euro area countries, whereby these loans were dependent on program compliance by the Greek Government.

2.4 Two extreme approaches to the elasticity of central bank credit

To illustrate further the idea of endogeneity, consider for a moment two most extreme central bank lending frameworks that can be imagined:

1. **Loosest framework**: accept all balance sheet assets of banks as collateral at fair values (net book values for assets held to maturity), thus allowing banks to finance all their assets with the central bank. Such an approach would provide confidence to the markets in the sense that all market participants would know that no counterparty could ever default for liquidity reasons, as any liquidity outflow could be compensated by an increased recourse to central bank funding. It should be noted that what we call here the "loosest" framework would still rely on two fundamental risk management principles: (i) that

[6] Deutsche Bundesbank, Realisation of Lehman collateral: ABS "Excalibur" sold to Lone Star", Press Release, 2012-04-25.

the collateral should be accepted at adequate valuations, reflecting at least an unbiased medium- to long-term fair value; (ii) that counterparties should be solvent in the sense of having positive capital. In such a framework, pledging all assets (at fair values) with the central bank without haircuts is always sufficient to close any bank's funding gap resulting from loss of market access. Lending against over-valued collateral or towards insolvent counterparties should not be considered in any case. In this sense, one may want to call this framework the "loosest reasonable framework".

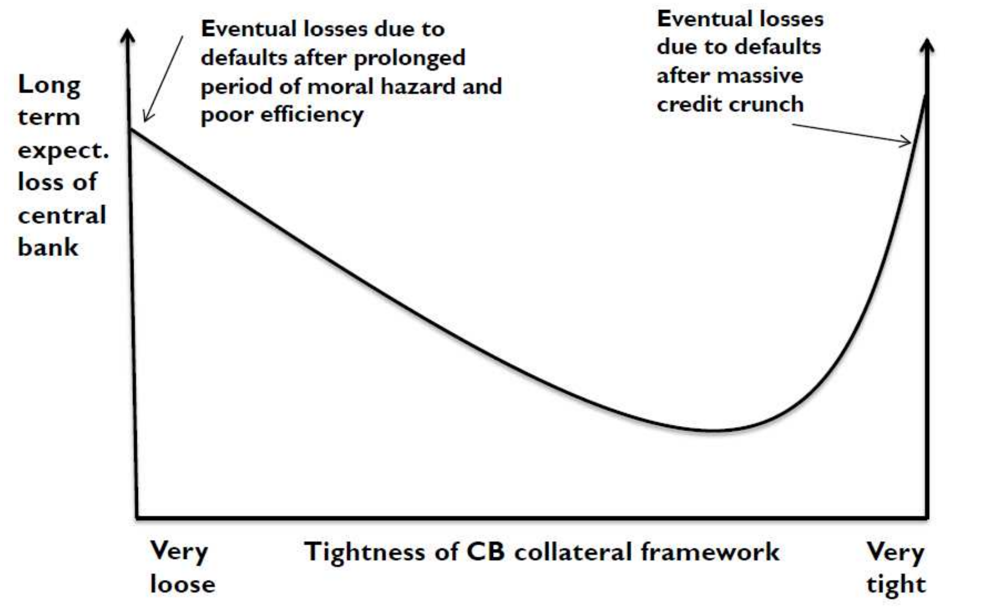
2. **Tightest framework**: the central bank implements monetary policy without any lending operations to banks. Instead, it holds exclusively credit risk-free assets, such as AAA-rated sovereign debt or gold, and merely adjusts the banking system's aggregate liquidity position and steers short-term interest rates through outright sales and purchases of the risk-free asset. The central bank may conduct at the margin some repos against risk-free assets, but in a bilateral way in which it chooses its counterparties and always goes for the most secure ones. In this framework, banks have no discretionary access to the central bank at all to close possible funding gaps.

One can conceptualize the move from one to the other extreme framework as a one-dimensional decision problem as follows. For instance, starting from the tightest framework, the central bank could first introduce a borrowing facility for banks against a very narrow set of eligible collateral (first, only the credit risk-free asset) and high haircuts. Then, the central bank could gradually widen the collateral set and eventually lower haircuts more and more, in a way to maintain risk equivalence of repo exposures across collateral types (risk equivalence would mean that after differentiated haircuts, residual risks are equal across collateral types). In the end, it would accept all bank assets with zero haircuts, which would be the loosest risk control framework.

For each of the intermediate risk control frameworks, one could calculate, at least theoretically, the long-term expected loss of the central bank (or any risk measure, such as the VaR of the central bank at various confidence levels). The curve obtained would certainly not be stable over time, i.e. its exact shape would depend in particular on the prevailing stability of the financial system. For instance, in a very resilient financial system, it could well be that the expected loss curve between the loosest and the tightest framework is monotonously downward sloping, as the system could cope with shocks thanks to highly elastic capital and interbank markets. In such a case, the expected loss function of the central bank would be similar to the one of any granular player in the market, i.e. one for which the own risk control framework does not influence the systemic properties of the financial system.

However, the normal case is that the tightest framework minimizes excessively the role of the central bank as the lender of last resort, which makes the system vulnerable to exogenous shocks and makes systemic financial crises and the associated economic contractions more likely. This at least seems to be the lesson from the history of central banking in financial crises, as summarized already by Bagehot. This would mean

Figure 1: Central bank expected losses under different collateral frameworks



that the long-term expected loss of the central bank, when moving from one extreme framework to the other, is first downward sloping, but also at some stage reaches an interior minimum and then is upward sloping (Figure 1).

While a move from the tightest to the loosest framework might plausibly have a positive impact on systemic stability in the short run, the long-run effects are less straightforward. For one thing, the loose framework is more supportive to moral hazard. By weakening the hard budget constraint (including the necessity of financial institutions and corporates to convince investors to lend to them), it may also undermine incentives for proper management and prolong projects that should end sooner rather than later, as they are loss-making. This also applies if the borrowing entity or the collateral issuer is a public entity. Allowing for moral hazard and an inefficient resource allocation may in the long run lead to higher eventual central bank losses, and again needs to be taken into account when identifying the E-loss curve between the loosest and the tightest framework.

Moving from these conceptual considerations to an application in practice would ideally require a precise identification of the expected loss curve. This is of course difficult as it depends on the exact knowledge of how the central bank's choices influence systemic properties and incentives over time. Moreover, the curve is unlikely to be stable over time as it depends on the inherent financial resilience of the system, and on the occurrence of exogenous shocks testing this resilience.

Moreover, the curve needs to be interpreted as a dynamic concept as it refers to some staring point in terms of collateral and risk control framework. Indeed, it could be argued that under the tightest framework, the central bank's expected losses must always be zero as it accepts only the risk-free asset as collateral. However, this no longer holds if the curve in Figure 1 is considered a response to the following question: "starting from a certain central bank collateral framework in a certain environment, what would be the expected central bank loss if the central bank changed the tightness of its framework moving to various points on the x-axis?". Thus, for instance, taking the Eurosystem collateral framework and euro area financial and economic conditions of the year, say, 2003, what would be the long-term (say over 20 years) expected central bank loss if: (i) the framework would stay unchanged; (ii) the framework would be moved within one year towards the loosest framework; (iii) the framework would be moved within one year to the tightest framework; etc.?" In this setting, moving to the tightest framework could lead to high losses, namely if the announcement leads to liquidity hoarding and forced deleveraging and the unfolding of a liquidity crisis in which eventually banks default vis-à-vis the central bank as they run out of funding and are also unable to substitute their previously used less liquid collateral with the risk free asset that the central bank now requires.

3 A simple model of central bank lending and risk management with real asset value shocks

In the present section we introduce a model that allows replicating the various effects discussed above.

3.1 The general setup

The model builds on the financial accounts representation introduced in Section 2. It innovates, also relative to previous papers using such models, by capturing asset value shocks, solvency and insolvency, default events and restructuring, and economic efficiency in a well-defined sense. The model contains both asset value (solvency) shocks which drive concerns regarding economic performance, and, like the model in Section 2, liquidity shocks that may lead to default. Default may occur despite an economic entity being solvent, and the insolvency of banks or corporates may remain unnoticed for an extensive period of time as they continue to receive funding of one or the other kind.

As noted by e.g. Bagehot (1873) or Kindleberger and Aliber (2005), a financial crisis almost always originates from an asset value shock, which in turn may be related either to systemic or idiosyncratic factors. Yet, solvency problems do not lead directly to default as it is assumed that they are discovered only with a significant time lag, reflecting the difficulties in valuing non-liquid assets and more generally the opaqueness of banks' balance-sheets as it could also relate to their significant off-balance-sheet activities or difficult-to-value derivatives transactions. However, liquidity problems are correlated with low quality of loan portfolios as investors receive noisy signals on asset values and tend to withdraw funding on the basis of these signals.

The model captures such features in the form of a closed system of financial accounts, similar to the one in Section 2. Needless to say, the exposition is highly stylized, but aims at capturing one key element of the central bank role in liquidity crises. The model assumes that: (i) the relevant interbank markets have broken down; and (ii) capital market access and deposit flows are uncertain and volatile. This assumption reflects recent experience in the post-Lehman period and the worst phases of the euro area sovereign debt crisis, as well as previous experience from the 1930s (see e.g. Bindseil and Winkler 2012 on Germany in 1931) or from the 19th century (King, 1936).

At the outset, households are endowed with real assets E (equity). They invest these real assets partially in corporate equity P and bank equity Q, and also exchange another part of their real assets into financial assets, namely banknotes B and bank deposits D (assumed to be divided equally between Bank 1 and Bank 2). Corporates finance their projects by bank loans (equal to $D + B + Q$) and the equity endowment from households (P). The financial sector, consisting of banks and the central bank, is the intermediary between households and corporates (apart from equity stakes in corporates). First, it offers deposits D to households and invests them in loans offered to corporates. Second, the banking sector is still an intermediary to the operation between the households and the central bank with respect to the issuance of banknotes B. Banks use banknotes to purchase real assets from households, which they sell on to corporates who finance them through a loan from the bank. Thus, total funding, and hence total assets held by banks amount to $B+D+Q$. The resulting financial structure of the economy is presented in Table 2.

When a bank defaults, this has some assumed direct costs. In the model, these costs materialize in the following concrete way: if the bank defaults, also the corporate that the bank was lending to defaults as the bank is no longer able to roll over its credit, and other banks cannot take over quickly enough because they cannot easily assess the quality and solvency of the enterprise.[7] When the corporate defaults, there is economic damage because its management is changed, assets have to be sold to new owners (possibly at distressed prices), changes to the assets have to be made to make them fit into new companies, there is a period of legal uncertainty and associated inertia, etc.[8]

If, thanks to the central bank's elastic liquidity provision, defaults of illiquid institutions are prevented, then this may be good since it ensures uninterrupted operation of business projects. However, it can also be bad since banks and corporates may default for good reasons – investors may have withdrawn funding as they

[7]This assumption is not supposed to reflect the empirically estimated default correlations which tend to be of the order of 1%-5% Moody's (2008a). Rather, it is meant to provide a clear way of including economic costs of default in the model and capturing that these costs ultimately materialize in the real sector by affecting the amount of real resources in the economy. Note that stochasticity could be introduced in a straightforward way by setting default correlation parameter between banks and corporates $\rho < 1$. Letting X and Y be default indicators of the bank and corporate respectively (that take the value 1 if a bank/corporate defaults and 0 otherwise), the conditional probability of a corporate defaulting following a bank default would be $\mathbb{P}(Y = 1|X = 1) = \mathbb{P}(X = 1, Y = 1)/\mathbb{P}(X = 1)$, with $\mathbb{P}(X = 1, Y = 1) = \rho\sqrt{\mathbb{P}(X = 1)(1 - \mathbb{P}(X = 1))\mathbb{P}(Y = 1)(1 - \mathbb{P}(Y = 1))} + \mathbb{P}(X = 1)\mathbb{P}(Y = 1)$. Such an assumption would however complicate the exposition without adding much explanatory value or altering the fundamental conclusions drawn below.

[8]See also Calvo (1998) pp. 41, 52 for a discussion of the costs of default.

Table 2: Financial accounts in the model

Households/Investors			
Assets		Liabilities	
Real assets	$E - D - B - Q - P$	Equity	E
Deposits Bank 1	$D/2$		
Deposits Bank 2	$D/2$		
Bank equity	Q		
Corporate equity	P		
Banknotes	B		

Corporate 1			
Assets		Liabilities	
Real assets	$(D + B + P + Q)/2$	Loans from Bank 1	$(D + B + Q)/2$
		Equity	$P/2$

Corporate 2			
Assets		Liabilities	
Real assets	$(D + B + P + Q)/2$	Loans from Bank 2	$(D + B + Q)/2$
		Equity	$P/2$

Bank 1			
Assets		Liabilities	
Loans to Corporate 1	$(D + B + Q)/2$	Households' deposits	$D/2$
		CB borrowing	$B/2$
		Equity	$Q/2$

Bank 2			
Assets		Liabilities	
Loans to Corporate 2	$(D + B + Q)/2$	Households' deposits	$D/2$
		CB borrowing	$B/2$
		Equity	$Q/2$

Central bank			
Assets		Liabilities	
Credit operations	B	Banknotes	B

receive (noisy) signals on solvency problems relating to bad management. In that case, preventing illiquidity through central bank credit may allow negative NPV projects to continue longer than necessary, and to continue wasting social wealth. It may sometimes be better for the society to discontinue a project through default and go through the one-off cost of reorganization, but then to allow again for a more productive use of the freed up resources.

The central bank in the model is assumed to have no particular information on solvency of banks and corporates, i.e. it does not even receive noisy signals, such as investors do. The central bank, however, can aim at providing liquidity to banks in a way that achieves the optimum with regard to minimizing the expected costs across two possible errors:

- Error 1: letting a bank default for liquidity reasons although it was viable in the sense that there was no reason to expect that it would produce sizable losses in the future;
- Error 2: preventing, through extensive liquidity provision, the default of a bank which is not sound and expected to generate future substantial losses if it is not wound down.

In the model, the parameter of the central bank to achieve the optimum is the haircut it imposes on collateral.[9] The optimum haircut will depend i.a. on the information content of liquidity shocks with regard to individual banks' solvency/efficiency problems. If this information content is high, then more conservative haircuts should be optimal, compared to the case of a low information content.

Concretely, we capture the issue of optimal central bank liquidity provision in a two period model with the following sequence of events.

Period 1:

1. Asset value ("solvency") shocks materialize, which are modeled as zero-mean random variables:

   μ – systemic asset value shock affecting assets held by all corporates;

   η_1 – idiosyncratic asset value shock affecting the assets held by Corporate 1;

   η_2 – idiosyncratic asset value shock affecting the assets held by Corporate 2;

2. Liquidity shocks materialize, which are correlated with asset value shocks, reflecting the intuition that liquidity shocks can have information content on debtors' economic performance and solvency. The correlation is controlled by the coefficients α, β:

   $d = \epsilon - \alpha\mu$ – outflow of deposits (across all banks) into banknotes;

[9] In practice, changes in the restrictiveness of the collateral framework can be brought about also by changes in eligibility. In the model, it has been assumed for the sake of simplicity that there is only one type of asset, and hence there is no scope to differentiate across different asset types in terms of eligibility. It may be an interesting model extension to differentiate across different asset types.

$k = \theta + \beta(\eta_1 - \eta_2)$ – deposit shift shock out of Bank 2 into Bank 1;

3. Funding liquidity shocks force banks to adjust (increase or decrease) their borrowing from the central bank. The banks pre-deposit all their assets with the central bank as collateral. Recourse to the CB cannot exceed available collateral after haircuts. The haircut level is h, so that the potential borrowing from the central bank is limited to $\frac{1}{2}(1-h)(B+D+Q)$.

4. If a bank hits its central bank collateral constraint, it defaults. This has two implications. First, the corporate defaults as it depended on the bank for financing (a credit crunch occurs). This is assumed to cause a damage to corporate asset value of x. On that basis, the values of the corporate liabilities can be established (assuming the juniority of equity relative to debt). Second, bankruptcy proceedings are initiated and banks' solvency is evaluated, whereby the value of remaining bank assets is divided between the creditors – the central bank and the households. First, the central bank will liquidate its collateral (in fact, by assumption, all assets of the bank), and the remaining asset value is then divided pari passu between the central bank (as far as it still has claims after the liquidation of collateral) and the household.

Period 2:

1. Banks and corporates that have not defaulted continue to exist, and it is assumed that the idiosyncratic real asset shock of period 1 repeats itself precisely. This reflects the assumed persistence of economic performance. Corporates that default are subject to a new draw of the idiosyncratic shock $\widetilde{\eta}_{1,2}$ which reflects the fact that they have received a new management and have been re-organized.

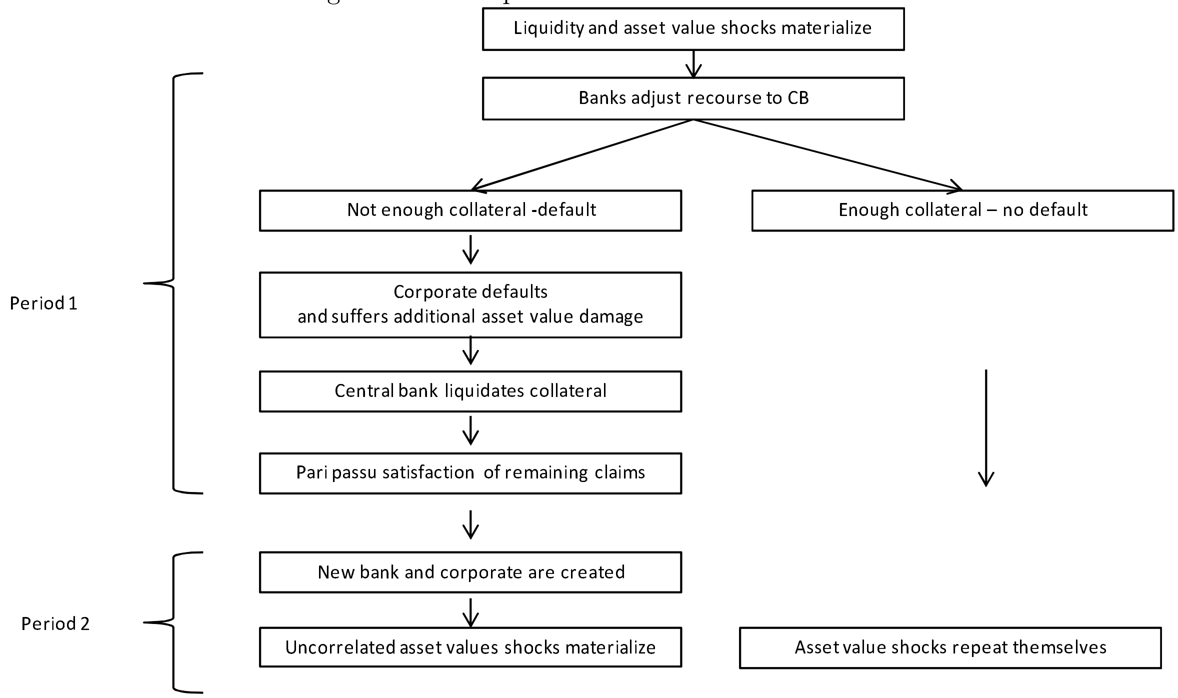
2. Economic efficiency and central bank losses are evaluated, as explained below.

This sequence of events is presented schematically in Figure 2.

3.2 Central bank risk-taking and economic efficiency – optimal haircut policy

The calculus of the cascading of asset value shocks and default events in the system of accounts is explained in detail in the Annex. In what follows, we employ the modeling framework developed above to illustrate the considerations in Section 2.4. Specifically, we analyze how (the expectation of) economic efficiency and central bank losses depend on the level of central bank haircuts and financial sector characteristics, such as the information content of liquidity shocks (originating from investors) with regard to the economic performance of issuers, the costs of default, volatility of symmetric and asymmetric liquidity and asset value shocks etc. Economic efficiency W is understood in the context of the performance of the corporate sector and expressed in terms of real assets in the economy. Formally, it is defined as the sum of asset value shocks

Figure 2: The sequence of events in the model



in period 1, costs of default (if any) and asset value shocks in period 2, equal to a new draw of asset value shocks (in case of default) or period 1 shocks (in case no default occurred):

$$W = \sum_{i=1,2} \left\{ \eta_i - \mathbb{1}_{\{fail,i\}} x + \mathbb{1}_{\{fail,i\}} \widetilde{\eta}_i + (1 - \mathbb{1}_{\{fail,i\}}) \eta_i \right\} \tag{1}$$

where $\mathbb{1}_{\{fail,i\}}$ is equal to 1 if default of Bank i occurs $(i = 1, 2)$ and 0 otherwise.

Central bank losses arise from the cascading of asset value shocks and defaults through the respective balance sheets as described in the Annex.[10] We will compare expected efficiency gains with the riskiness of the central bank balance sheet, expressed in terms of the expected losses on the collateral portfolio. Although strictly speaking expected loss is not a risk measure – since risk is by definition restricted to unexpected events – it has the most straightforward interpretation and exhibits greater stability than tail measures in the simulation exercise.[11] Moreover, since expected loss on an exposure is defined as the product of a counterparty's probability of default (PD) and the loss given default, changes in the central bank's expected losses will have a clear interpretation in terms of changes in counterparties' PD levels, thus reflecting also risk endogeneity. In this setup, the objective of the central bank is to find the optimum level of haircuts that

[10]Note that our definition of economic efficiency encompasses any potential losses borne by the central bank. This can be though to reflect the idea that the taxpayers are the ultimate stakeholders of the central bank and would have to cover the losses through taxes or by foregoing future seignorage income, which would be a sign of poor economic efficiency.

[11]A recent study of credit risk models applied by euro area central banks finds that expected loss is typically the starting point for assessing the riskiness of a portfolio (ECB, 2007). Other popular risk measures include the unexpected loss, i.e. the standard deviation of the loss distribution, and the VaR defined as a certain quantile of the loss distribution.

maximizes efficiency and minimizes central bank losses.

Expected efficiency is characterized analytically as follows:

Proposition 1. *Let efficiency W be defined as in (1), $N(\cdot)$ denote the cumulative standard normal distribution function and $A = \frac{1}{2}(D + B + Q)$. Furthermore, set $\sigma^2_{Y_1} = \frac{1}{\beta^2}(\sigma^2_\theta + \beta^2\sigma^2_{\eta_2} + \frac{1}{4}\sigma^2_\epsilon + \frac{\alpha^2}{4}\sigma^2_\mu)$, $\sigma^2_{Y_2} = \frac{1}{\beta^2}(\sigma^2_\theta + \beta^2\sigma^2_{\eta_1} + \frac{1}{4}\sigma^2_\epsilon + \frac{\alpha^2}{4}\sigma^2_\mu)$ and $\sigma^2_{d/2\pm k} = \sigma^2_\theta + \beta^2\sigma^2_{\eta_1} + \beta^2\sigma^2_{\eta_2} + \frac{1}{4}\sigma^2_\epsilon + \frac{\alpha^2}{4}\sigma^2_\mu$ (using the notation introduced in subsection 3.1). Then the expected value of W is given by:*

1. *if $\beta \neq 0$ and $\sigma_{\eta_i} \neq 0$ $(i = 1, 2)$*

$$\mathbb{E}(W) = \sum_{i=1,2} \frac{\sigma_{\eta_i}}{\sqrt{2\pi\left(\frac{\sigma^2_{Y_i}}{\sigma^2_{\eta_i}} + 1\right)}} \exp\left(\frac{-\left(-A(1-h) + \frac{1}{2}B\right)^2}{2\beta^2\sigma^2_{\eta_i}\left(\frac{\sigma^2_{Y_i}}{\sigma^2_{\eta_i}} + 1\right)}\right) - 2xN\left(\frac{-A(1-h) + \frac{1}{2}B}{\sigma_{d/2\pm k}}\right), \tag{2}$$

2. *if $\beta = 0$*

$$\mathbb{E}(W) = -2xN\left(\frac{-A(1-h) + \frac{1}{2}B}{\sqrt{\sigma^2_\theta + \frac{1}{4}\sigma^2_\epsilon + \frac{\alpha^2}{4}\sigma^2_\epsilon}}\right), \tag{3}$$

3. *if $\sigma_{\eta_i} = 0$, $\sigma_{\eta_j} \neq 0$ and $\beta \neq 0$*

$$\mathbb{E}(W) = \frac{\sigma_{\eta_j}}{\sqrt{2\pi\left(\frac{\sigma^2_{Y_j}}{\sigma^2_{\eta_j}} + 1\right)}} \exp\left(\frac{-\left(-A(1-h) + \frac{1}{2}B\right)^2}{2\beta^2\sigma^2_{\eta_j}\left(\frac{\sigma^2_{Y_j}}{\sigma^2_{\eta_j}} + 1\right)}\right) - 2xN\left(\frac{-A(1-h) + \frac{1}{2}B}{\sigma_{d/2\pm k}}\right). \tag{4}$$

Proof. See Annex. □

Expected efficiency $\mathbb{E}(W)$ will be driven by the relation between costs of default and the positive expected value of reoccurring asset value shocks. Intuitively, if a bank survives period 1 without being forced to default, it is more likely that it has funded sound projects, and the repetition of such business outcomes in period 2 is obviously associated with increased economic efficiency.

It follows from Proposition 1 that the first-order derivative of $\mathbb{E}(W)$ with respect to h is also given in closed form [12]:

[12] The formula is derived assuming $\beta \neq 0$. The calculations for the case when $\beta = 0$ are straightforward.

Table 3: Model specifications considered

	Specifications					
	I	II	III	IV	V	VI
$\sigma_{\eta_{1,2}}$	2	0/2/4	2	2	2	2
σ_{μ}	2	2	0/2/4	2	2	2
β	1	1	1	0.1/0.2/1	1	1
σ_{θ}	1	1	1	1	0/1/4	1
σ_{ϵ}	1	1	1	1	1	1
α	1	1	1	1	1	1
x	1	1	1	1	1	0/1/15/25

$$\frac{d\mathbb{E}(W)}{dh} = \sum_{i=1,2} \frac{\sigma_{\eta_i}}{\sqrt{2\pi\left(\frac{\sigma^2_{Y_i}}{\sigma^2_{\eta_i}}+1\right)}} \frac{2A\left(A(1-h)-\frac{1}{2}B\right)}{2\beta^2\sigma^2_{\eta_i}\left(\frac{\sigma^2_{Y_i}}{\sigma^2_{\eta_i}}+1\right)} \exp\left(\frac{-\left(A(1-h)-\frac{1}{2}B\right)^2}{2\beta^2\sigma^2_{\eta_i}\left(\frac{\sigma^2_{Y_i}}{\sigma^2_{\eta_i}}+1\right)}\right) - $$
$$- 2x\frac{A}{\sigma_{d/2\pm k}\sqrt{2\pi}} \exp\left(-\frac{(-A(1-h)+\frac{1}{2}B)^2}{2\sigma^2_{d/2\pm k}}\right). \tag{5}$$

Hence, $\mathbb{E}(W)$ can be both a monotonous and non-monotonous function of h, depending on the interplay of the various parameters describing the state of the financial system (e.g. costs of default, volatilities of idiosyncratic and systemic liquidity and asset value shocks etc.). For example, in a setting with no idiosyncratic solvency shocks ($\sigma_{\eta_1} = \sigma_{\eta_2} = 0$) and non-zero costs of default, $\mathbb{E}(W)$ will be decreasing, indicating a preference for a loose collateral framework. In what follows we consider a number of parameter sets that will illustrate the considerations in Section 2.4. For the sake of tractability, we introduce the following normalization of the structural variables. The initial endowment of real assets (household equity) is assumed to be fixed (at 100) and divided between real assets (50) and the following financial assets: banknotes (20), deposits (27), and equity stakes in corporates (2) and banks (1), whereby these different equity levels reflect the fact that banks are typically more leveraged than corporates. Solvency and liquidity shocks are assumed to be normally distributed, with zero means and volatilities varying in different environments.

3.2.1 Desirability to stop loss-making enterprises vs. cost of default – some examples

Table 3 shows the parameterization of the various cases considered.[13] In each of the six specifications (I – VI) we consider a number of sub-cases to see how robust is the functional relationship between haircuts and central bank risk-taking and economic efficiency in different environments. Since the central bank loss

[13]Simulation results including changes of var(ϵ) and the coefficient α are omitted for brevity as they are similar to those in Specifications V and IV, respectively. We also omit the specification with changing equity levels. It can be easily verified by considering the $\mathbb{E}(W)$ formula in Proposition 1 that increases in Q can result either in a parallel shift of the $\mathbb{E}(W)$ curve to the right (if capital is increased by diversifying away from banknotes) or no change at all (if capital increase is financed by a reduction of deposit levels – a more intuitive case).

function is not avaiable in closed form, we derive its distribution using Monte Carlo simulation. Each time the simulations entail 5,000 draws of a "state of the world", which gives basis for the calculation of the distribution of central bank losses and the calculation of the expected loss as a function of the haircut level, which increases in steps of 0.5% from 0% to 58%.[14]

We start with a baseline specification (I) featuring both key drivers of efficiency, namely firm-specific asset value shocks and costs of default (Figure 3). We also present three measures of central bank risk-taking: the expected loss, the unexpected loss (standard deviation of the loss distribution) and the 99% Value at Risk (i.e. the 99th percentile of the loss distribution). Since all risk curves have similar shapes, for ease of presentation, and in view of the considerations above, the remaining specifications will feature only the expected loss.

Consider first the shape of the expected efficiency function (left-hand panel). With the chosen parameter values:

$$\mathbb{E}(W) = \frac{4}{\sqrt{5.125\pi}} \exp\left(\frac{-1}{20.5}(14 - 24h)^2\right) - 2N\left(\frac{24h - 14}{\sqrt{10.25})}\right) \tag{6}$$

and

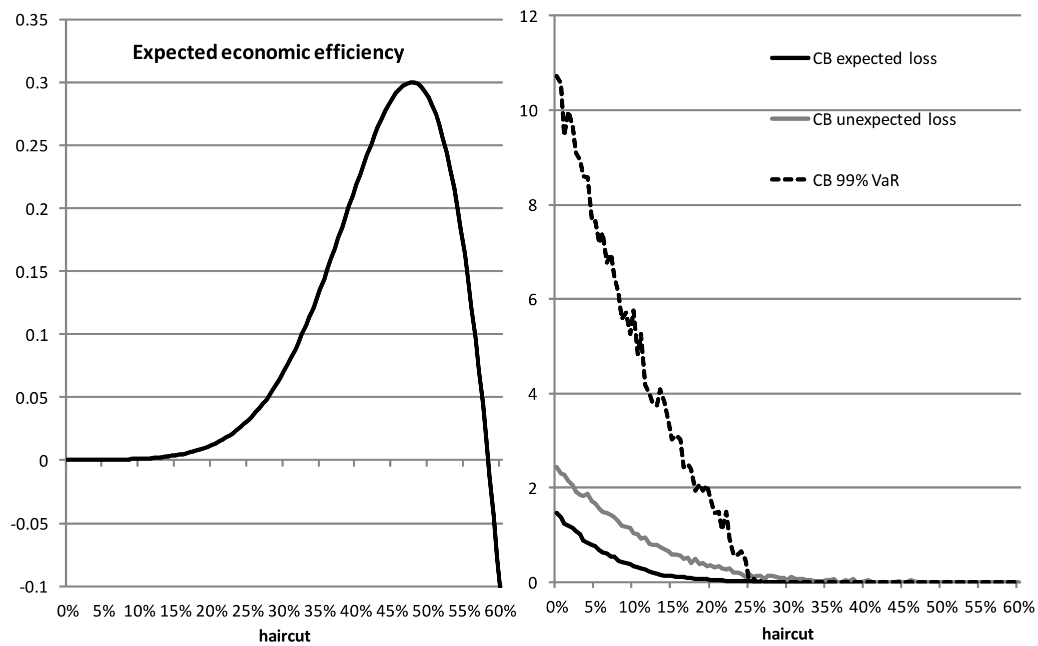
$$\frac{d\mathbb{E}(W)}{dh} = \exp\left(\frac{-1}{20.5}(14 - 24h)^2\right)\left(\frac{48(14 - 24h)}{20.5}\frac{4}{\sqrt{5.125\pi}} - \frac{48}{\sqrt{20.5\pi}}\right) \tag{7}$$

Thus, initially idiosyncratic asset value shocks produce an efficiency curve that increases with the haircut level. However, as default frequency increases, costs of default kick in and begin to weigh on economic efficiency. Ultimately, this generates a hump-shaped expected economic efficiency curve. Indeed, $\mathbb{E}(W)$ reaches a local maximum for $h = 0.48$. Turning to the central bank, Figure 3 (right-hand panel) shows that all risk measures decrease monotonously with the degree of restrictiveness of haircut policy, as could be expected in the case of any granular player whose risk management decisions have little systemic impact. Consequently, by $h = 0.48$ the balance sheet of the central bank is already fully protected, as all risk measures approach zero.

To verify the robustness of Specification I, we now investigate what happens to efficiency and central bank losses when the volatility of idiosyncratic asset value shocks first drops to zero and subsequently rises to four (Figure 4). When there are no firm-specific asset value shocks, $\mathbb{E}(W) = -2N((24h - 14)/\sqrt{2.25}))$ which is negative and falls further with the restrictiveness of the central bank collateral policy (Figure 4, left-hand panel). To see why this is so, observe that in such case economic efficiency is driven fully by the adverse effects of default and restructuring. The latter are initially very low as banks default very infrequently (e.g. for $h = 0.45$ bank's PD is still below 10% vs. almost 40% in the baseline specification). As haircuts increase, defaults become more prevalent, and the cost of going through reorganization weighs

[14] The upper bound for haircut levels has to be such that banks can at least finance their structural liquidity deficit, stemming from society's demand for banknotes, i.e. such that $B < (B + D + Q)(1 - h)$. Substituting for B, D and Q, we immediately get $h < 0.5833$.

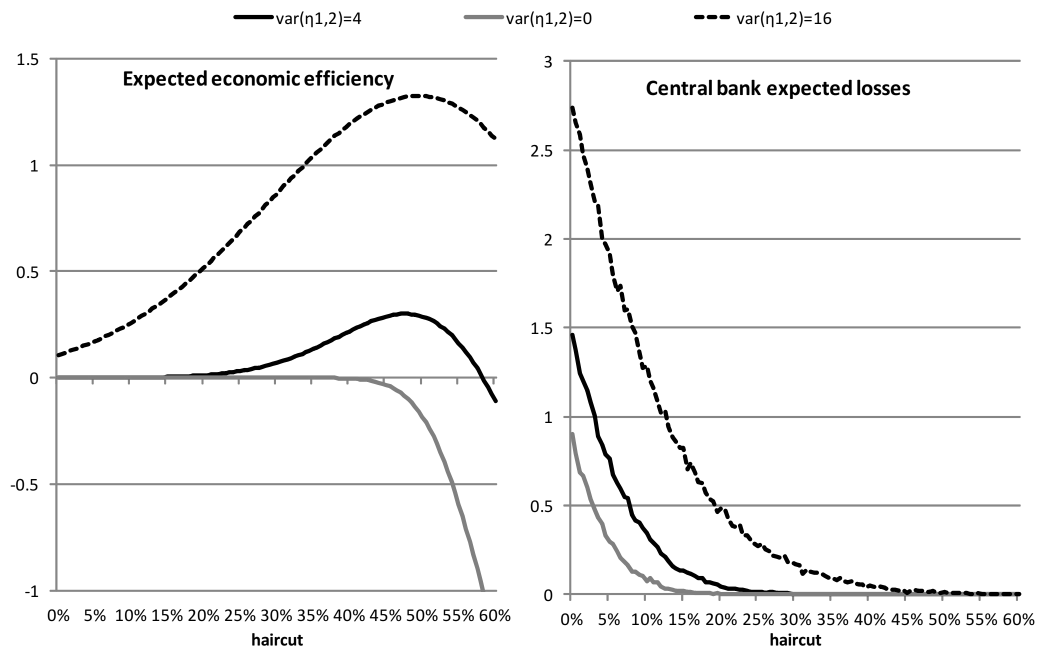
Figure 3: Expected economic efficiency and central bank risk-taking in Specification I (here and below see Table 3 for details of the specification)



on economic efficiency. When $\sigma_{\eta_i} = 4$, some 30% of solvency shocks will be such as to wipe out households' entire equity stake in corporates. On the other hand, with higher volatility, also sizeable positive shocks are increasingly likely. Thus, increasing the volatility of solvency shocks, while keeping the cost of default constant, produces greater economic efficiency and at the same time higher expected social losses, with the optimal haircut level virtually unchanged. Turning to the central bank (Figure 4, right-hand panel), higher volatility of idiosyncratic shocks clearly produces higher expected losses, both on account of higher PDs and more prevalent higher adverse shocks to corporate assets.

Specification III shows how systemic asset value shocks impact economic efficiency and central bank risk-taking. Recall from (1) and from Proposition 1 that systemic shocks do not impact efficiency directly, but instead affect the volatility of liquidity shocks and thus also banks' probabilities of default. When $\sigma_\mu = 0$, $\mathbb{E}(W) = \frac{4}{\sqrt{4.625\pi}} \exp\left(\frac{-1}{18.5}(14-24h)^2\right) - 2N((24h-14)/\sqrt{9.25}))$ and the efficiency-maximizing haircut increases to 54%. In contrast, with $\sigma_\mu = 4$, the optimal haircut drops to 44% (Figure 5, left-hand panel). This pattern is consistent with the observation that in a severe crisis, when systemic shocks dominate over solvency shocks, central banks tend to loosen the collateral framework. At the same time, higher systemic asset value shocks increase expected central bank losses for a given haircut level (right-hand panel), suggesting that the central bank should optimally tighten its collateral policy. That in this case the response of the central bank is not aligned with economic efficiency is due to the fact that with relatively low costs of default, the system can reorganize without disruptions and the risk management problem of the

Figure 4: Expected economic efficiency and central bank losses in Specification II



central bank can be approached in a similar way as that of a typical granular player. We will see below that this is not univerally true.

Specifications IV and V allow to analyze the impact of information content of liquidity shocks on economic efficiency. Note first that when $\sigma^2_{\eta_1} = \sigma^2_{\eta_2}$

$$\mathrm{corr}(\eta_i, k) = \frac{\mathrm{cov}(\eta_i, k)}{\sigma^2_{\eta_i}\sigma^2_k} = \frac{\mathrm{cov}(\eta_i, \beta(\eta_1 - \eta_2))}{\sigma^2_{\eta_i}(\sigma^2_\theta + \beta^2\sigma^2_{\eta_1} + \beta^2\sigma^2_{\eta_2})} = \frac{\pm\beta\sigma_{\eta_i}}{\sqrt{\sigma^2_\theta + 2\beta^2\sigma^2_{\eta_i}}}, \tag{8}$$

for $i = 1, 2$. Thus, as β decreases from 1 to 0.2 to 0.1 ($\sigma^2_\theta = 1$), the correlation between η_1 and the deposit shift shock k decreases from 0.66 to 0.34 and 0.19, respectively. At the same time, falling β depresses σ^2_k, which drops from 9 (with $\beta = 1$) to a mere 1.08 (with $\beta = 0.1$). Such a change also means that banks are initially less frequently forced to default and restructure, as the probability of default stays below 10% until the haircut level of about 45%. Beyond that point, defaults intensify, causing costly restructuring. Ultimately, the initially low PDs overshadow the decreased correlation of liquidity and asset value shocks, and as a result economic efficiency is markedly depressed, while the optimal haircut level is virtually unchanged (Figure 6, left-hand panel).

If the independent liquidity component θ of the deposit shift shock is eliminated, (8) shows that $\mathrm{corr}(\eta_i, k)$ increases to above 0.70. In contrast, increasing the variance of θ to 16, lowers $\mathrm{corr}(\eta_i, k)$ to 0.40 and at the same time markedly increases banks' PDs – a pattern consistent with the intensification of a financial crisis. In such a case, $\mathbb{E}(W) = \frac{4}{\sqrt{12.625\pi}} \exp\left(\frac{-1}{50.5}(14 - 24h)^2\right) - 2N((24h - 14)/\sqrt{25.25}))$ and the efficiency-maximizing

Figure 5: Expected economic efficiency and central bank losses in Specification III

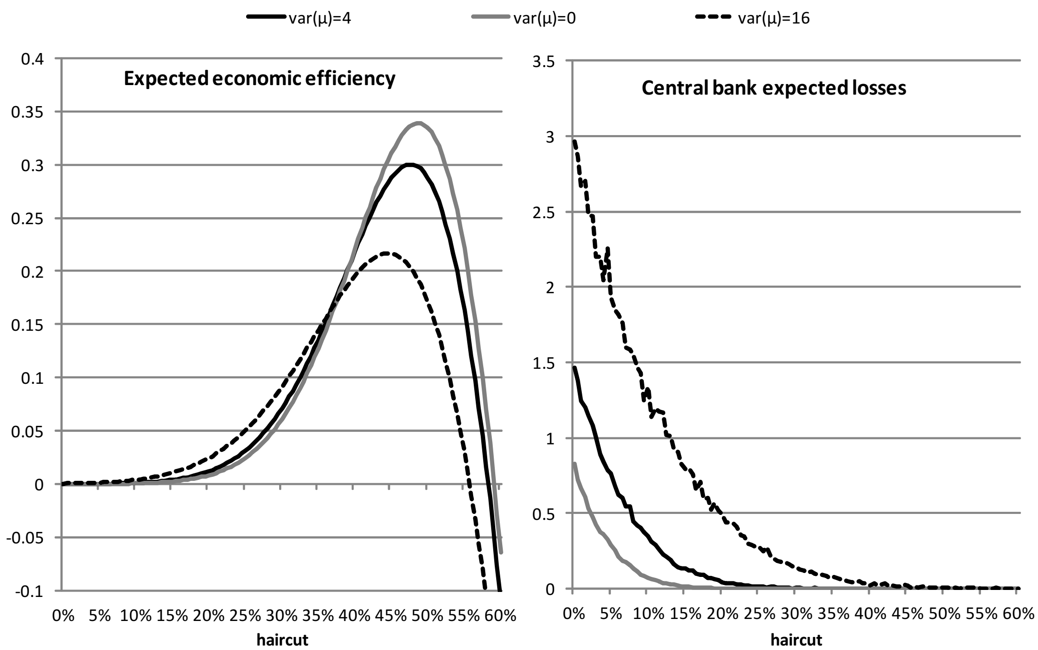


Figure 6: Expected economic efficiency and central bank losses in Specification IV

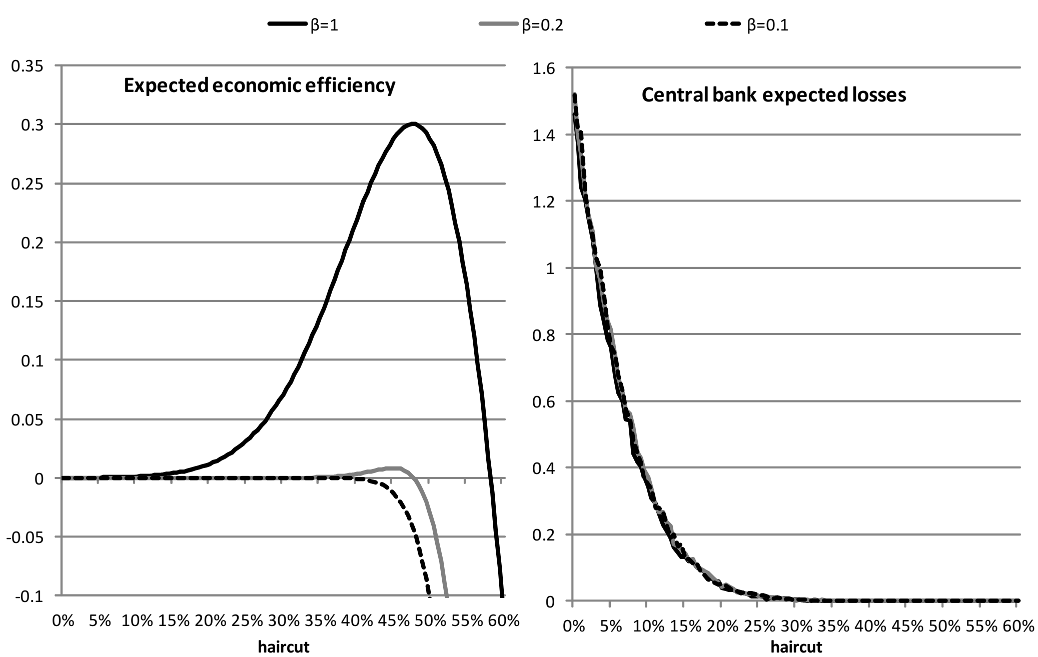
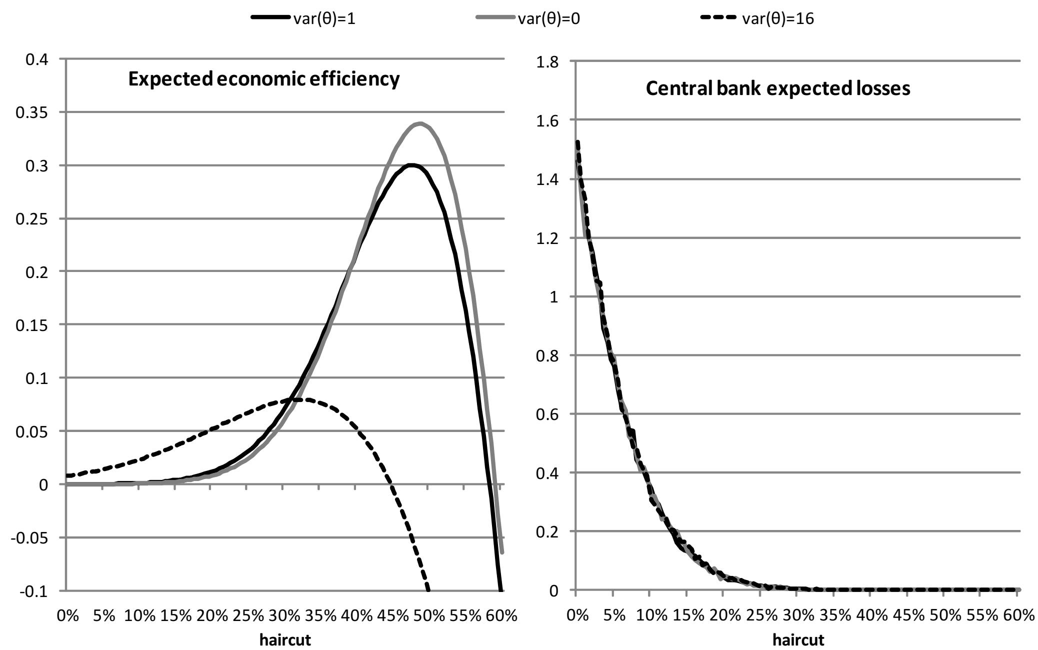


Figure 7: Expected economic efficiency and central bank losses in Specification V



haircut is just $h = 0.32$ (Figure 7).

Interestingly, in both specifications, the central bank's expected loss curves are largely unaffected, despite the changes in banks' PDs discussed above (Figures 6 and 7, right-hand panels). This is due to the fact that the key loss driver – i.e. volatility of asset value shocks – remains unchanged and the cost of default ($x = 1$) can still be absorbed by corporate and bank equity. Thus, in IV and V the risk management conclusion would be to keep the haircut unchanged, even though economic efficiency concerns (e.g. in the case when $\sigma_\theta^2 = 16$) would suggest loosening the collateral framework.

Finally, in Specificaion VI we investigate the impact of the cost of default on both economic efficiency and central bank's losses, whereby cost of default has the economic interpretation of loss given default (LGD)[15]. We run the simulations for four different levels of the cost of default which increases in steps from 0 to 1 (LGD=4%), to 15 (LGD=60%) and 25 (LGD=100%). Although these values are for illustrative purposes only, the latter two may indeed be higher than what one would expect of the pure cost of default. For example, in a recent study Moody's (2008b) reports that the average firm-wide LGD realized at the resolutions of defaults of 19 U.S. firms in 2007 was around 30%, slightly above 25.1% recorded in 2006 and well below the long-term average of 46% since 1987. Typically LGD ratios differ depending on the type of debt (loans vs. bonds), its seniority and collateralization. While the average LGD for junior subordinated debt in 1987-2007 was 84%, it was 34% for senior secured debt. The LGD for loans in 2006-2007 was merely

[15]Note, however, that this is the LGD on banks' loans and is in general not equivalent to the LGD on the central bank's exposure towards its counterparties.

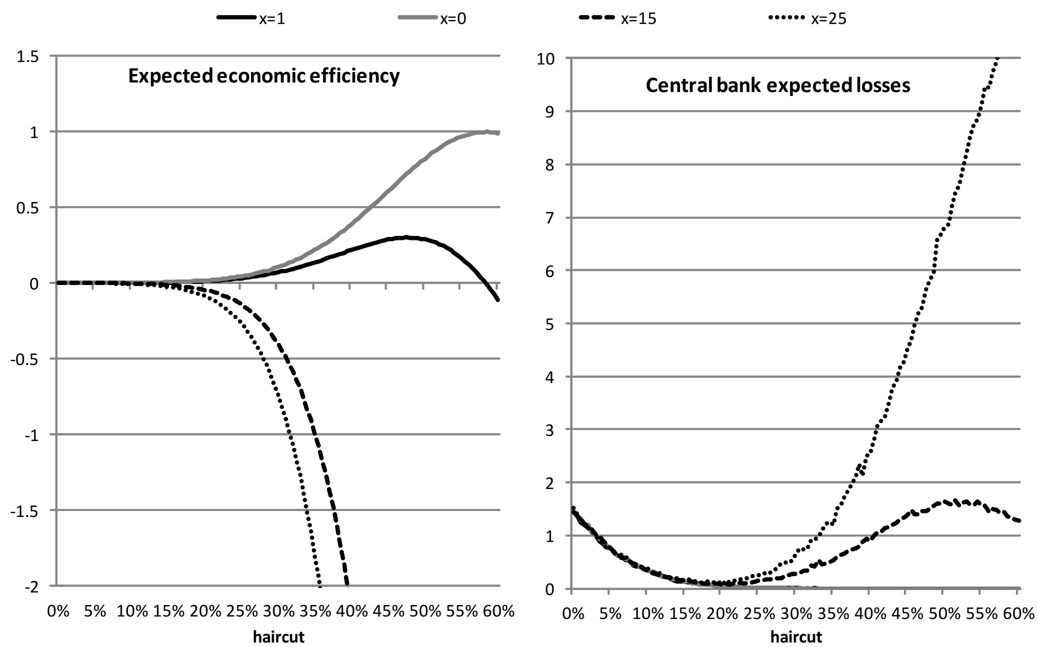
8%. Recall from subsection 2.2 however, that we assume parameter x to capture all costs of default, both direct and externality-related.

For $x = 0$, $\mathbb{E}(W) = \frac{4}{\sqrt{5.125\pi}} \exp\left(\frac{-1}{20.5}(14 - 24h)^2\right)$ and $\frac{d\mathbb{E}(W)}{dh} = \frac{48(14-24h)}{20.5} \frac{4}{\sqrt{5.125\pi}} \exp\left(\frac{-1}{20.5}(14 - 24h)^2\right)$. Thus, $\mathbb{E}(W)$ is monotonously increasing for $0 < h < 0.5833$, and reaches the maximum for the highest haircut level consistent with accommodating society's demand for banknotes (Figure 8). Intuitively, more restrictive haircuts ensure that more unsound business projects can be filtered out and costlessly wound down with a net gain for the society. As x increases, however, efficiency gains from killing loss-making businesses are offset by the cost of restructuring and the efficiency curve transforms from a hump-shaped one ($x = 1$) to monotonously decreasing one ($x = 15, 25$) as the negative externalities of default easily outweigh any positive effects of discontinuing malinvestments. Overall, the greater the cost of default, the lower is the optimal haircut from the point of view of expected economic efficiency.

Turning to the central bank, Figure 8 (right-hand panel) shows that when $x = 0$ expected losses decrease monotonously with the degree of restrictiveness of the haircut policy. Intuitively, the zero cost of default reflects a resilience of the financial system which can always re-organize without disruptions. Therefore, the system can cope even with a very conservative framework without being systemically destabilized. In such an environment, the central bank's risk management can in fact be approached similarly to the one of a typical granular player on the market, so that increasing haircuts allows it to effectively mitigate credit risk without influencing financial stability. However, when LGD is 60% or higher, the central bank expected loss curve changes from monotonously decreasing to a "U"-shaped one, whereby expected losses fall as the central bank moves from a very liberal framework ($h = 0\%$) to a moderate one ($h = 30\% - 40\%$), but pick up again once that point is breached. As the cost of default increases to 25, in which case reorganization entails almost total destruction of corporate assets (e.g. selling highly sophisticated machinery as metal junk), the losses expected in the most restrictive framework skyrocket. Interestingly however, even at maximum level, central bank losses are about twice lower than social losses.

Intuitively, these results reflect the fact that when the financial system is not perfectly resilient – as reflected e.g. by the high cost of default – the central bank can no longer be considered to have the same risk management problem as a granular player – the elasticity of its liquidity provision impacts the risk parameters of its counterparties with dramatic consequences for its balance sheet. For example, when $x = 25$ and $h = 15\%$ banks' PD is only about 15% and the associated LGD on the central bank's secured exposures is about 7%. In contrast, when $h = 40\%$, PDs rise to above 30% and the LGD increases to 20%, putting expected central bank losses back at the level they would have been in the loosest collateral framework characterized by PD at 8% and 100% LGD. Thus, excessive risk protection can be self-defeating as it increases default probabilities and expected default related losses for the central bank. The policy

Figure 8: Expected economic efficiency and central bank losses in Specification VI



conclusion is that, unlike in Specification I, when the cost of default needs to be factored into the analysis, a very restrictive haircut policy may neither be in the interest of society nor of the central bank. Instead, the optimal haircut level – i.e. one that allows striking the right balance between letting a viable bank default for liquidity reasons and preventing the default of a bank which is misallocating resources – is a moderate one.

Note that in the majority of specifications considered so far, reaching optimum efficiency and minimizing central bank losses required setting moderate, rather than very low haircuts. On the other hand, it should be taken into account that the model assumed that all bank assets were in principle eligible for central bank operations. In sum, the model is yet too stylized to derive conclusions on concrete optimal haircut levels that central banks should apply.

4 Conclusions

In a financial crisis, central banks play a crucial role as lenders of last resort. But to what extent should they extend credit to banks under funding stress, given that such elastic credit provision might increase their risk-taking and promote moral hazard? More specifically, what are the key trade-offs the central bank needs to consider in limiting the elasticity of its credit provision through collateral eligibility rules and haircuts?

In this paper, after reviewing a number of relevant considerations, we propose a simple model representing the key trade-offs and allowing to derive optimal central bank policies from a risk management and economic efficiency perspective. In particular, the model captures two possible central bank mistakes, namely: (i) letting productive, but temporarily illiquid business projects go bust; and (ii) preserving, through overly elastic liquidity provision, businesses that should default as they are loss-making. While the solvency of banks and corporates was assumed to be unobservable to the central bank prior to default, it is reflected in liquidity shocks, as investors (households) were assumed to receive noisy signals on solvency shocks and the quality of banks' loan portfolios.

The model provides a new formal backing for some of the key ideas of Bagehot (1873), grounded in a comprehensive system of financial accounts, capuring solvency, liquidity, and interaction between the two. The model shows that economic efficiency and central bank risk-taking are in many cases non-monotonous functions of haircuts, and even if the functions are monotonous, they can be either upward- or downward sloping. This means that depending on the haircut level and on economic circumstances, increasing haircuts can either increase or decrease central bank expected losses, and either increase or decrease economic efficiency, with the two not necessarily aligned and efficiency losses easily outweighing central bank losses. One key insight is that in stressed market conditions, characterized by high negative externalities of default, central bank losses can increase with the level of haircuts. Hence, paradoxically, loosening the collateral framework can be perfectly consistent with protecting the balance sheet of the central bank, as already implied by Bagehot's dictum that only the "brave" plan of the central bank is the "safe" plan. This is a specific consequence of a more general insight that financial sector risk tends to be endogenous with respect to central bank's emergency liquidity support. Going beyond model specification, this phenomenon can be illustrated by the following mechanism: if the funding stress of banks, together with other macroeconomic factors, lead to a continued credit crunch and economic downwards spiral affecting collateral values, counterparties' solvency will deteriorate over time and PDs will increase, eventually increasing also central bank's risk parameters. To the extent that the central bank's emergency liquidity operations manage to overcome the negative feedback loops characteristic of a systemic financial turmoil, these actions should then also reduce the central bank's long-term risk exposure. We believe this reasoning – illustrated formally by our model – goes a long way towards explaining why the major central banks have, over the course of the recent crisis, aimed at increasing the total post-haircut amount of collateral relative to the total balance sheet length of the banking system (Brunnermeier, Crocket, Goodhart, Persaud, and Shin, 2009). Indeed, this result is precisely replicated in the model, which shows that under parameter changes that are consistent with a financial crisis, i.e. when costs of default increase and liquidity shocks become more erratic and carry less information on solvency, the central bank should increase the total post-haircut amount of collateral.

We acknowledge that the functional forms of welfare and risk-taking derived within the model depend both on model assumptions and environmental parameters, which are not obvious to observe and difficult to calibrate using empirical data. Moreover, moving towards practical applications, the central bank expected loss curve would need to be interpreted as a dynamic concept, referring to some staring point in terms of collateral and risk control framework. However, difficulties with quantifying model results do not imply that the concept of risk endogeneity is too abstract to be used in practice and therefore can be ignored. To the contrary, the relevance of risk endogeneity inherent in the central bank's liquidity operations can be most readily appreciated by considering (in a dynamic sense) the two extreme collateral frameworks discussed in Section 2.4. Ignoring these effects may lead to sub-optimal policy decisions. Thus, risk endogeneity seems valid and relevant in central bank risk management, even if it makes it look more like "uncertainty management" in the Knightian sense.

Future research may develop the model along a number of dimensions. First, it would be useful – although challenging – to calibrate the model. Second, one may want to endogenize both the investment stage at the beginning of period 1 and the liquidity management decisions of the household, and how the two interact strategically. This would also allow to develop a more general concept of moral hazard in the model. An interesting extension would also involve moving to a multi-period setting in which the long-term costs of keeping insolvent banks afloat could be compared with the costs of default. On the other hand, the resulting complexity of the model may become very challenging with such extensions. Third, it could also be interesting to integrate the case of central bank outright asset purchases into the model, as such purchases have been of similar importance and had partially similar objectives as an elastic liquidity provision to banks against collateral in the recent crisis episode. Fourth, it could be interesting to consider heterogeneous bank assets and hence the possibility for the central bank to discriminate between bank assets in terms of collateral eligibility and haircuts. Fifth, the model may be varied to take into account the fact that often a lack of central bank collateral does not lead directly to bank default but to so-called "emergency liquidity assistance", which constitutes special central bank lending to banks that have run out of normal central bank eligible collateral.[16] ELA is typically associated with stigma in markets and pressure from the central bank on the bank to take specific efforts to reduce again central bank reliance. Both will also lead to extra social costs, although not in the form of default costs. Finally, it may be interesting to explore cases in which the banking system is heterogeneous ex ante, for instance in terms of what shares of the assets are funded by deposits and by central bank credit, before liquidity shocks arise.

[16] See e.g. the Review of the Bank of England's ELA by Ian Plenderleith available for download at: http://www.bankofengland.co.uk/publications/Documents/news/2012/cr1plenderleith.pdf.

Annex

Proof of Proposition 1

Assume first that $\beta \neq 0$ and $\sigma_{\eta_i} \neq 0$ $(i = 1, 2)$. Recall the form of the economic efficiency function:

$$W = \sum_{i=1,2} \left\{ \eta_i - \mathbb{1}_{\{fail,i\}} x + \mathbb{1}_{\{fail,i\}} \widetilde{\eta}_i + (1 - \mathbb{1}_{\{fail,i\}}) \eta_i \right\} \tag{9}$$

Since both η_i have zero expected value, the calculation of $\mathbb{E}(W)$ reduces to:

$$\mathbb{E}(W) = \mathbb{E}\left(\sum_{i=1,2} \left\{ -\mathbb{1}_{\{fail,i\}} x + \mathbb{1}_{\{fail,i\}} \widetilde{\eta}_i - \mathbb{1}_{\{fail,i\}} \eta_i \right\} \right) \tag{10}$$

Observe that

$$\mathbb{1}_{\{fail,i\}} = \begin{cases} 1 & \iff \frac{d}{2} \pm k > A(1-h) - \frac{B}{2} \\ 0 & \iff \frac{d}{2} \pm k \leq A(1-h) - \frac{B}{2} \end{cases}. \tag{11}$$

Since

$$\frac{d}{2} \pm k = \frac{\epsilon - \alpha\mu}{2} \pm \theta \pm \beta(\eta_1 - \eta_2) \sim N\left(0, \sqrt{\sigma_\theta^2 + \beta^2 \sigma_{\eta_1}^2 + \beta^2 \sigma_{\eta_2}^2 + \frac{1}{4}\sigma_\epsilon^2 + \frac{\alpha^2}{4}\sigma_\mu^2}\right), \tag{12}$$

thus, setting $\sigma^2_{d/2\pm k} = \sigma_\theta^2 + \beta^2 \sigma_{\eta_1}^2 + \beta^2 \sigma_{\eta_2}^2 + \frac{1}{4}\sigma_\epsilon^2 + \frac{\alpha^2}{4}\sigma_\mu^2$, we obtain:

$$\mathbb{E}\left(\sum_{i=1,2} \mathbb{1}_{\{fail,i\}} x \right) = 2xN\left(\frac{-A(1-h) + \frac{1}{2}B}{\sigma_{d/2\pm k}} \right). \tag{13}$$

Note that, by definition, $\widetilde{\eta}_i$ and η_i are independet (for $i = 1, 2$), and hence also $\mathbb{1}_{\{fail,i\}}$ and $\widetilde{\eta}_i$ must be independent. This implies that:

$$\mathbb{E}\left(\sum_{i=1,2} \mathbb{1}_{\{fail,i\}} \widetilde{\eta}_i \right) = 0. \tag{14}$$

To calculate $\sum_i \mathbb{E}(\mathbb{1}_{\{fail,i\}} \eta_i)$ let first $i = 1$. Then (11) can be restated as:

$$\mathbb{1}_{\{fail,1\}} = \begin{cases} 1 & \iff \eta_1 < \frac{1}{\beta}\left(-\theta + \beta\eta_2 + \frac{\epsilon - \alpha\mu}{2} - A(1-h) + \frac{B}{2} \right) \\ 0 & \iff \eta_1 \geq \frac{1}{\beta}\left(-\theta + \beta\eta_2 + \frac{\epsilon - \alpha\mu}{2} - A(1-h) + \frac{B}{2} \right) \end{cases} \tag{15}$$

Since $\mathbb{1}_{\{fail,1\}}$ and η_1 are not independent, the expectation of their product is a double integral and, by Fubini's theorem, can be calculated using iterated integrals. Thus, assume first that the right-hand-side expression is a constant and denote $Y_1 = \frac{1}{\beta}\left(-\theta + \beta\eta_2 + \frac{\epsilon - \alpha\mu}{2} - A(1-h) + \frac{B}{2}\right)$. Then $\mathbb{1}_{\{fail,1\}}\eta_1$ is a normal distribution function truncated to $(-\infty, Y_1)$. Thus,

$$\mathbb{E}(\mathbb{1}_{\{fail,1\}}\eta_1) = \int_{\infty}^{Y_1} \frac{z}{\sigma_{\eta_1}\sqrt{2\pi}} \exp\left(\frac{-z^2}{2\sigma_{\eta_1}^2}\right) dz. \tag{16}$$

Letting $w = z/\sigma_{\eta_1}$, we immediately obtain

$$\int_{-\infty}^{Y_1} \frac{z}{\sigma_{\eta_1}\sqrt{2\pi}} \exp\left(\frac{-z^2}{2\sigma_{\eta_1}^2}\right) dz = \sigma_{\eta_1} \int_{-\infty}^{\frac{Y_1}{\sigma_{\eta_1}}} \frac{w}{\sqrt{2\pi}} \exp\left(\frac{-w^2}{2}\right) dw = -\sigma_{\eta_1}\phi\left(\frac{Y_1}{\sigma_{\eta_1}}\right), \tag{17}$$

with $\phi(\cdot)$ being the normal PDF, which is the first of the iterated integrals.

Since $Y_1 \sim N((-A(1-h) + \frac{B}{2})/\beta, \sigma_{Y_1})$, it follows that

$$Y_1 = \sigma_{Y_1}U - \frac{-A(1-h) + \frac{B}{2}}{\beta} \tag{18}$$

for $U \sim N(0,1)$. Thus,

$$\phi\left(\frac{Y_1}{\sigma_{\eta_1}}\right) = \phi\left(\frac{\sigma_{Y_1}}{\sigma_{\eta_1}}U - \frac{-A(1-h) + \frac{B}{2}}{\beta\sigma_{\eta_1}}\right) = \phi(sU + t), \tag{19}$$

for $s = \sigma_{Y_1}/\sigma_{\eta_1}$ and $t = (-A(1-h) + \frac{B}{2})/(\beta\sigma_{\eta_1})$, and we are faced with the calculation of the following integral:

$$\int_{-\infty}^{\infty} \phi(su + t)\frac{1}{\sqrt{2\pi}} \exp\left(\frac{-u^2}{2}\right) du = \frac{1}{2\pi}\int_{-\infty}^{\infty} \exp\left(-\frac{s^2+1}{2}u^2 - stu - \frac{1}{2}t^2\right) du. \tag{20}$$

Since

$$\exp\left(-\frac{s^2+1}{2}u^2 - stu - \frac{1}{2}t^2\right) = \exp\left(-\frac{s^2+1}{2}\left(u + \frac{st}{s^2+1}\right)^2\right)\exp\left(\frac{s^2t^2}{2s^2+2} - \frac{1}{2}t^2\right) \tag{21}$$

we can complete the square and substitute $w = \sqrt{\frac{s^2+1}{2}}(u + \frac{st}{t^2+1})$, which leads to:

$$\int_{-\infty}^{\infty} \phi(su + t)\frac{1}{\sqrt{2\pi}} \exp\left(\frac{-u^2}{2}\right) du = \frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{s^2+1}} \exp\left(-\frac{t^2}{2(s^2+1)}\right). \tag{22}$$

Substituting for s and t, yields:

$$\mathbb{E}(\mathbb{1}_{\{fail,1\}}\eta_1) = \frac{\sigma_{\eta_i}}{\sqrt{2\pi\left(\frac{\sigma_{Y_i}^2}{\sigma_{\eta_i}^2} + 1\right)}} \exp\left(\frac{-\left(-A(1-h) + \frac{1}{2}B\right)^2}{2\beta^2\sigma_{\eta_i}^2\left(\frac{\sigma_{Y_i}^2}{\sigma_{\eta_i}^2} + 1\right)}\right). \tag{23}$$

The formula for $\sum_i \mathbb{E}(\mathbb{1}_{\{fail,2\}}\eta_2)$ is analogous, and combining (13) and (23) yields the desired formula.

When $\beta = 0$ and volatilities of idiosyncratic solvency shocks is non-zero, then obviously $\mathbb{1}_{\{fail,i\}}$ and η_i are independent, and $\mathbb{E}(W)$ reduces to:

$$\mathbb{E}(W) = -2xN\left(\frac{-A(1-h)+\frac{1}{2}B}{\sqrt{\sigma_\theta^2+\frac{1}{4}\sigma_\epsilon^2+\frac{\alpha^2}{4}\sigma_\mu^2}}\right). \tag{24}$$

Finally, if $\sigma_{\eta_i} = 0$, then $\mathbb{E}\left(\mathbb{1}_{\{fail,i\}}\eta_i\right) = 0$ and the $\mathbb{E}(W)$ will only be driven by $\mathbb{1}_{\{fail,j\}}\eta_j$, as in (23). This completes the proof. □

Cascading of asset value shocks

The annex shows how the sequence of events presented in Section 3 is reflected in the system of accounts.

Asset value shock

First, in period 1 asset value shocks materialize and affect corporate balance sheets. In case of default, these asset value shocks are revealed automatically as fair values are calculated for purposes of default proceedings. However, when no default occurs, fair values of assets are established only after period 2.

Liquidity shocks

Investors receive noisy signals on banks' and corporates' fundamentals and incorporate them to some extent in their demand for banknotes and deposits. Thus, asset value shocks combine with standard liquidity-demand shocks, forcing banks to adjust their borrowing from the central bank ($CBB_{1,2}$):

$$CBB_1 = \frac{1}{2}B – k + \frac{1}{2}d \tag{25}$$

$$CBB_2 = \frac{1}{2}B + k + \frac{1}{2}d \tag{26}$$

However, the central bank extends credit only to the extent the banks have sufficient liquidity buffers, i.e. only to the extent that:

$$\frac{1}{2}(D+B+Q)(1-h) \geq \left(\frac{1}{2}B – k + \frac{1}{2}d\right) \tag{27}$$

and

$$\frac{1}{2}(D+B+Q)(1-h) \geq \left(\frac{1}{2}B + k + \frac{1}{2}d\right) \tag{28}$$

If these conditions are not met, default and a bankruptcy procedure start, which also imply a corporate default. Assume first that one of the banks – without loss of generality, Bank 2 – had an insufficient liquidity buffer and was forced to default, causing also default of Corporate 2.

Corporate default

Table 4 presents the balance sheet Corporate 2, which due to default suffers an additional asset value damage of x. Table 4 shows also the balance sheet of Corporate 1, as it would look in period 1 if asset value shocks were recognized. However, as explained above, in case of no default, asset value shocks are revealed only some time later.

Bank 2 liquidation

Since the fair value of Bank 2's assets (FVA_2) is obviously equal to the liabilities of Corporate 2 to Bank 2 we must have:

$$FVA_2 = \frac{1}{2}(D+B+Q) - \max\left(0, -\left(\frac{P}{2} + \mu + \eta_2 - x\right)\right) \tag{29}$$

The first liability to be affected by possible losses up to depletion is the bank equity, which equals $\max\left(0, \frac{Q}{2} - \max\left(0, -\left(\frac{P}{2} + \mu + \eta_2 - x\right)\right)\right)$. Thus, the total loss to be suffered by the remaining liabilities is:

$$\max\left(0, -\frac{Q}{2} + \max\left(0, -\left(\frac{P}{2} + \mu + \eta_2 - x\right)\right)\right) \tag{30}$$

This loss now needs to be divided among the two creditors – the central bank and the households. This needs to be calculated in two steps. First, the central bank liquidates its collateral and, depending on the haircut, may achieve full recovery. In the second step, the remaining assets are used to satisfy the remaining unsecured claims (which may include claims of the central bank that could not be satisfied through the liquidation of collateral). The central bank has priority in so far as its claim against the bank is collateralized. At the moment of default, by definition, Bank 2 has a liability to the central bank equal to its total assets minus the haircut. That means it has pledged to the central bank all its assets, i.e. $1/2(D+B+Q)$. Taking into account the impact of solvency shocks, the collateral has fair value of:

$$FVC_2 = \frac{D+B+Q}{2}\left(\frac{\frac{1}{2}(D+B+Q) - \max\left(0, -\left(\frac{P}{2} + \mu + \eta_2 - x\right)\right)}{\frac{1}{2}(D+B+Q)}\right) \tag{31}$$

The central bank achieves full recovery if the fair value of collateral exceeds the value of borrowing, i.e.:

$$\frac{1}{2}(D+B+Q) - \max\left(0, -\left(\frac{P}{2} + \mu + \eta_2 - x\right)\right) \geq (1-h)\frac{D+B+Q}{2} \tag{32}$$

Table 4: Corporate balance sheets after period 1

Corporate 1			
Assets		Liabilities	
Real assets	$\frac{1}{2}(D+B+P+Q)+\mu+\eta_1$	Loan from Bank 1	$\frac{1}{2}(D+B+Q)-\max\left(0,-\left(\frac{P}{2}+\mu+\eta_1\right)\right)$
		Equity	$\max\left(0,-\left(\frac{P}{2}+\mu+\eta_2\right)\right)$

Corporate 2			
Assets		Liabilities	
Real assets	$\frac{1}{2}(D+B+P+Q)+\mu+\eta_2-x$	Loan from Bank 2	$\frac{1}{2}(D+B+Q)-\max\left(0,-\left(\frac{P}{2}+\mu+\eta_2-x\right)\right)$
		Equity	$\max\left(0,-\left(\frac{P}{2}+\mu+\eta_2-x\right)\right)$

Or equivalently, if the haircut is greater than the percentage loss on the loan portfolio:

$$h \geq \frac{\max\left(0, -\left(\frac{P}{2} + \mu + \eta_2 - x\right)\right)}{\frac{1}{2}(D + B + Q)} \tag{33}$$

Call Δ the difference between the collateral value and the central bank's claim:

$$\begin{aligned} \Delta &= \frac{D + B + Q}{2} - \max\left(0, -\left(\frac{P}{2} + \mu + \eta_2 - x\right)\right) - (1 - h)\frac{D + B + Q}{2} = \\ &= \frac{D + B + Q}{2}\left(h - \frac{\max\left(0, -\left(\frac{P}{2} + \mu + \eta_2 - x\right)\right)}{\frac{1}{2}(D + B + Q)}\right) \end{aligned} \tag{34}$$

If $\Delta > 0$, then the central bank has full recovery and the depositors suffer a loss of $\frac{D}{2} - k - \frac{d}{2} - \Delta$. If $\Delta < 0$, then the central bank suffers a loss of $-\Delta$ and households lose all of their deposits, since the entire pool of assets was utilized to satisfy (incompletely) the central bank's claim.

The case of no default

Consider now what happens if the liquidity shock does not exhaust the borrowing potential of Bank 2 and as a result no defaults occur. If the liquidity shock is lower than the bank's borrowing potential, book value of Bank 2 exposure (BVE_2) towards the central bank is: $\frac{B}{2} + k + \frac{d}{2}$. The fair value of collateral is:

$$FVC_2 = \left(\frac{B}{2} + k + \frac{d}{2}\right)\left(\frac{\frac{1}{2}(D + B + Q) - \max\left(0, -\left(\frac{P}{2} + \mu + \eta_2 - x\right)\right)}{\frac{1}{2}(D + B + Q)}\right)\frac{1}{1 - h} \tag{35}$$

If $FVC_2 > BVE_2$, then the central bank recovers BVE_2 and households recover $\min(\frac{D}{2} - k - \frac{d}{2}; FVC_2 - BVE_2)$. If $FVC_2 < BVE_2$, then the central bank first recovers FVC_2, and the remaining assets are divided pari passu (i.e. in proportion to book value of remaining exposures). Since the value of remaining assets is:

$$\begin{aligned} FVA_2 &= \frac{D + B + Q}{2}\left(1 - \frac{\max\left(0, -\left(\frac{P}{2} + \mu + \eta_2 - x\right)\right)}{\frac{1}{2}(D + B + Q)}\right) - \left(\frac{B}{2} + k + \frac{d}{2}\right)\left(1 - \frac{\max\left(0, -\left(\frac{P}{2} + \mu + \eta_2 - x\right)\right)}{\frac{1}{2}(D + B + Q)}\right) = \\ &= \left(1 - \frac{\max\left(0, -\left(\frac{P}{2} + \mu + \eta_2 - x\right)\right)}{\frac{1}{2}(D + B + Q)}\right)\left(\frac{D + Q}{2} - k - \frac{d}{2}\right) \end{aligned} \tag{36}$$

The central bank recovers:

$$CB_{RR} = FVC_2 + \frac{(BVE_2 - FVC_2)}{(\frac{D}{2} - k - \frac{d}{2} + BVE_2 - FVC_2)}FVA_2 \tag{37}$$

while the household recovers:

$$HH_{RR} = \frac{\frac{D}{2} - k - \frac{d}{2}}{(\frac{D}{2} - k - \frac{d}{2} + BVE_2 - FVC_2)} FVA_2. \tag{38}$$

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